Oral Presentation

The 7th TSME International Conference on Mechanical Engineering
13-16 December 2016

Mechanical Engineering Innovation for Green Society

AEC0022

Optimum Design of Scroll Expander Wrap for Organic Rankine Cycle (ORC) Power Generation

Kritsada Lowrungsi[1,*] and Monsak Pimsarn[2]

[1,2] Department of Mechanical Engineering, Faculty of Engineering,
King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand
[*] Corresponding Author: E-mail: krits.low@gmail.com, Telephone Number 0894414169

Abstract

The Organic Rankine Cycle (ORC) is an emerging technology for power generation through heat recovering from different thermal sources. In the 1-10 kW ORC power generation, it is difficult to find the suitable equipment to expand the gas, reduce pressure and converse heat to be mechanical energy. Considering from thermodynamics and mechanics principle, it is obviously found that the scroll mechanism is the most suitable for kW scale power generation. The reason for this is that it can operate without the high mass flow rate condition. This paper presents an optimization design of scroll expander wrap to assess the maximum isentropic power generation (brake horse power) from the expansion ratio (discharge volume / suction volume). Using fixed thickness scroll formulas from the previous work, in this design study, the base radius and scroll height were chosen to be design variables and the gradient-based methods and the direct search methods were utilized to search for the maximum expansion ratio. Additionally, some of constraints such as scroll wrap radius within 90 mm to be set in Copeland's frame radius 120 mm, within slenderness ratio of scroll height and over the minimum scroll height / base radius ratio were also included in the design. The design result exhibits that expansion ratio is 5.7155 which is better than the standard scroll expander, 3.9061. Therefore, this leads to 44.3% improvement of expansion ratio. The results of this work can be considered as the first process of ORC system design to maximize performance of expander machine corresponding to the built in volumetric ratio within the desired dimension.
Keywords: Scroll, Expander, Organic Rankine Cycle, ORC, Power

1. Introduction

The low grade waste heat recovery (Temperature lower than 200 °C) from cold and hot cycle for power generation is another one of system efficiency improving. Various technologies had been introduced with this waste heat but the result is not satisfied. Even though the new emerging technology for this waste heat level have been invented. The Organic Rankine Cycle (ORC) is the Rankine cycle which use organic fluid instead of ordinary water to be convective heat transfer media in the system. Thus, this is the most popular alternative at this time. The organic substance which has been chosen is our well known fluid, refrigerant, in regard to required temperature and phase. R245fa is the most preferable because the narrow phase transition energy and suitable temperature level.

In this cycle, electric generators will be driven by prime mover. Expansion machines have been used in this function which can be classified in 2 types, turbo machines and positive displacement expanders for the high and low gas flow rate accordingly.

In this paper, the author will focus to the positive displacement mechanism, scroll expander which is suitable for the low mass flow rate (lower than 1 kg/s) for low grade waste heat recovery. These mass flow rate and temperature can be applied with the waste heat of factory and building by using the concept of optimum design to choose the suitable dimensions of scroll expander for the maximum power generation from expansion ratio (volume ratio). By set up the assumption of expansion ratio varied to the number of involute cycle, leads to establish the key variable to be inner ending angle, φ_{ie} and base circle radius, r_b.

2. Development of Scroll Wrap

Scroll theory has been invented by Leon Creux on 1905 [1]. But it was impossible to produce a working pair of scrolls until the middle 1970's due to a very small tolerance required [2]. After that, the scroll compressor become more and more popular because of its unique advantages, low level of noise, high efficiency and high reliability.

The geometry of scroll compressor is one of the main factors affecting the efficiency of the compressor. The geometry of the scroll has to be completely understood in order to establish a thermodynamic model.

The wraps of scrolls directly affect the built-in compression ratio and the performance of the compressor. Many researchers have done a lot of work in this field which emphasis on the inner portion because the outer portion of the scroll wraps is the involute and simple. Furthermore, many following papers described the development of geometrical scroll model from this basis until Baolong Wang [2] was develop the scroll wrap model which have thickness variable inside.

At this stage, the new formula of working chamber volume has been developed based on discretional initial angles of involute. And the expression of the volume during all the suction, compression and discharge processes in a general subsection function are disclosure too.

After that, the paper "Derivation of Optimal Scroll Compressor Wrap for Minimization of Leakage Losses" by Ian H. Bell [3] uses this concept to develop volume ratio by substitute depth and inner ending angle in this ratio. By using fixed displacement volume (suction volume of scroll compressor or exhaust volume of scroll expander), set up radial loss and leakage loss have the formula belonged to inner ending angle then calculate the minimum total loss by Phyton software.

3. Geometric Parameters

3.1 Scroll Wrap Definition

The scroll expander geometry is formed of involutes of a circle. Details of these geometric parameters are beyond the scope of this paper, nevertheless the full derivations are presented by Baolong Wang [2]. The involute of a circle is the line that would be formed if unwrap a circle with radius r_b. The coordinates of a point on the involute are presented in Ian H. Bell [3] as the follow:

$$x = r_b(\cos\varphi + (\varphi - \varphi_0)\sin\varphi)$$
$$y = r_b(\sin\varphi - (\varphi - \varphi_0)\cos\varphi) \quad (1)$$

The thickness of the scroll wrap can be written by:

$$t_s = r_b(\varphi_{i0} - \varphi_{o0}) \quad (2)$$

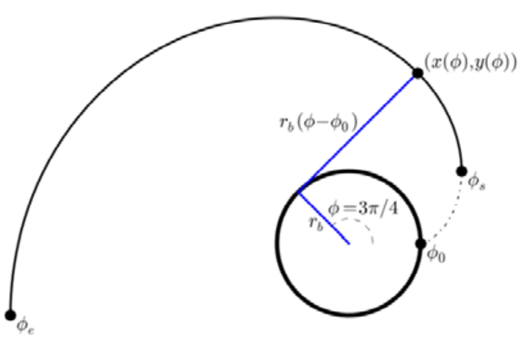

Fig. 1 Basic terminology of scroll involute, reprinted from [3]

The displacement volume, inner chamber volume and volume ratio can be given as the follow:

$$V_{disp} = -2\pi h_s r_b r_o(3\pi - 2\varphi_{ie} + \varphi_{i0} + \varphi_{o0}) \quad (3)$$

$$2V_{c,d} = 2\pi h_s r_b r_o(2\varphi_{os} + 3\pi - \varphi_{i0} - \varphi_{o0}) \quad (4)$$

$$V_{ratio} = \frac{V_{disp}}{2V_{c,d}} = \frac{3\pi - 2\varphi_{ie} + \varphi_{i0} + \varphi_{o0}}{-2\varphi_{os} - 3\pi + \varphi_{i0} + \varphi_{o0}} \quad (5)$$

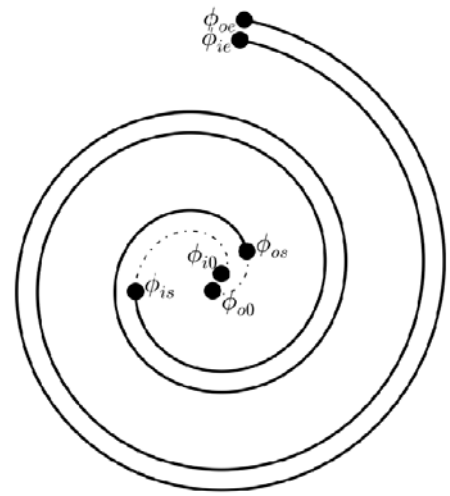

Fig. 2 Derivation of scroll expander wrap, reprinted from [3]

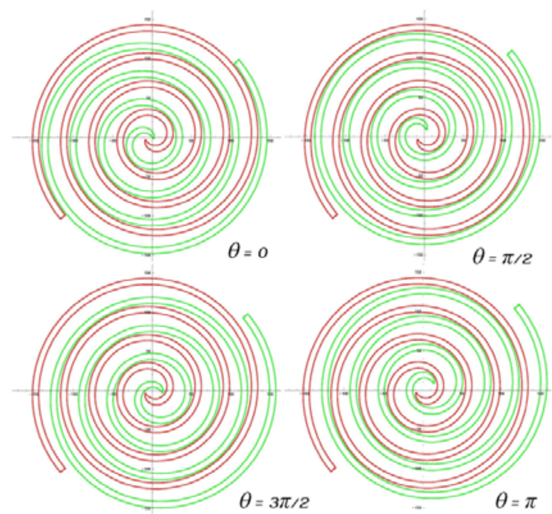

Fig. 3 Cycle of scroll expander wrap

3.2 Derived Terms

In order to calculate displacement volume and volume ratio, some constraints need to be clarified. [3]

The orbiting radius of the orbiting scroll can be given by:

$$r_o = r_b\pi - t_s \quad (6)$$

The outer involute initial angle is given by:

$$\varphi_{o0} = -t_s / r_b \quad (7)$$

The inner involute initial angle, φ_{i0} is set to 0.

The outer starting angle, φ_{os} is equal to $1.57 + \varphi_{o0}$ radians in Vincent Lemort [4] as the follow:

$$\varphi_{os} = 1.57 - t_s / r_b \quad (8)$$

After passed some simplification from (4) - (7), the height of the scroll wrap can be given by:

$$h_s = \frac{Vdisp}{2\pi rb^2 Vratio(\pi + \varphi o0)(2\varphi os + 3\pi - \varphi i0 - \varphi o0)} \quad (9)$$

4. Optimization Methodology

4.1 Mathematical Model

In this point of view, the nonlinear, constrained optimum design problem can be expressed mathematically as follows [5], [6]:

Find	$X = [x_1, x_2, \ldots x_n]^T$	(10)
to maximize	$f(X)$	(11)
subject to	$h_i(X) = 0, \quad i = 1,\ldots,m = 1$	(12a)
	$g_j(X) \le 0, \quad j = 1,\ldots,n = 3$	(12b)
	$x_{k,min} \le x_k \le x_{k,max}$	(12c)

Based on the above discussion, the design variables are defined as the following: base circle radius r_b, inner ending involute angle φ_{ie}, scroll thickness t_s, scroll height h_s.

Fig. 4 shows the implementation process. The scroll thickness and scroll height have been set up to 4.56 mm and 28.65 mm accordingly same as the standard scroll expander of Vincent Lemort [4].

Thus, the two design variables are shown in (13), [7]

$$X = [r_b, \varphi_{ie}]^T \quad (13)$$

4.2 Objective function

The objective function can be expressed as shown in (14) using the design variables from (13).

$$V_{ratio} = f(r_b, \varphi_{ie}) \quad (14)$$

In computation process, (5) - (8) will be used for evaluating to be the objective function.

4.3 Constraint conditions

In this optimal design of scroll expander wrap for organic Rankine cycle, the constraint conditions mainly consist of four aspects,

(h_1): Geometric relationship between h_s & V_{disp}
In computation process, (9) will be used for this constraint.

(g_1): Within allowable scroll radius, $R_{oe} \le 90$
Scroll Radius, R_{oe} is the maximum radius of scroll orbit. This parameter is can be calculated as the follow:

$$R_{oe} = r_b + r_b\varphi_{ie} + t_s / 2 \quad (15)$$

(g_2): Under limited $h_s \le 12t_s$
To control the slenderness ratio of scroll height versus scroll thickness for the smooth operation of gas flow and prevent effect of transverse vibration.

(g_3): Acceptable $h_s \ge 2r_b$
To control the ratio of scroll height versus base circle radius for the smooth operation of gas flow.

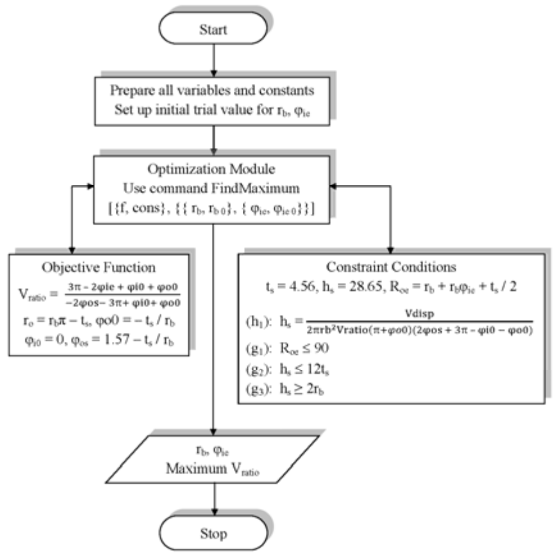

Fig. 4 Work flow of the calculation processes

5. Software Implementation Process

One equality and 3 non-equality constraints are involved in the optimal design of scroll expander wrap for organic Rankine cycle, and this problem is a typical constraint nonlinear programming problem. To solve this type of problems, can be broadly categorized into gradient-based methods and direct search methods. Gradient search methods use first derivatives (gradients) or second derivatives (Hessians) information. Examples are the sequential quadratic programming (SQP) method, the augmented Lagrangian method, and the (nonlinear) interior point method.

Commercial numerical software such as Wolfram Mathematica can support existing optimization algorithms. When the objective function, constraint conditions and the initial values are defined, the optimum design can be preceded by optimization function, FindMaximum (Wolfram command).

FindMaximum used to solve numeric local optimization problem by linear programming methods, nonlinear interior point algorithms and utilize second derivatives. This command continues until either of the goals specified by AccuracyGoal or PrecisionGoal (Wolfram command) is achieved [8].

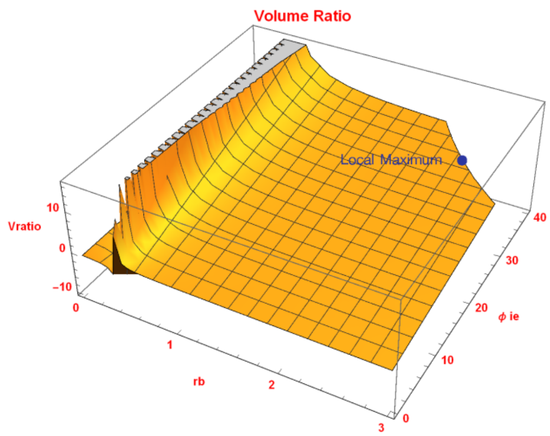

Fig. 5 Isometric perspective of volume ratio based on base circle radius and inner ending angle

6. Result and discussion

The optimization problem under consideration in this study is to find the optimal values of the design variable, x_i that maximize the volume ratio, V_{ratio}. From the computation process, the result of optimum design compared with standard scroll expander has been shown as in Table. 1 - 3. By fixing scroll thickness and scroll height 4.56 mm and 28.65 mm accordingly, in order to compare with standard scroll expander of Vincent Lemort [4]. That scroll wrap has volume ratio 3.9061 at base circle radius r_b 3.2640 mm, inner ending involute angle φ_{ie} 25.8250 radian (4.1102 involute cycle). The maximum volume ratio is 5.7155 (efficiency increase 44.3%) at base circle radius r_b 2.4756 mm, inner ending involute angle φ_{ie} 34.4345 radian (5.4804 involute cycle). This optimal point has shown in Fig. 5 - 6 for isometric perspective and contour of volume ratio based on base circle radius and inner ending angle accordingly. The significant constraints are (h_1): Geometric relationship between h_s & V_{disp}, (9) and (g_1): Within allowable scroll radius, $R_{oe} \leq 90$. Both of these criteria make the local maximum of volume ratio.

The result proof the assumption of expansion ratio varied to the number of involute cycle. But if no geometric relationship between h_s & V_{disp}, the scroll wrap will be impractical design from the incomplete geometrical characteristic.

Table. 1 Size dimension of simplified standard scroll expander of Vincent Lemort [4] and optimum design model

Size Dimension	t_s [mm]	h_s [mm]	r_o [mm]	r_b [mm]	R_{oe} [mm]
Standard	4.56	28.65	5.6942	3.2640	89.8368
Optimum	4.56	28.65	3.2172	2.4756	89.9999

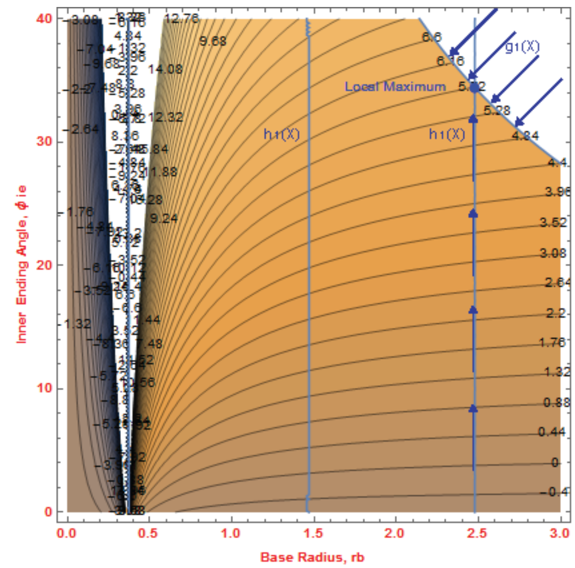

Fig. 6 Contour of volume ratio based on base circle radius and inner ending angle

Table. 2 Angle dimension of simplified standard scroll expander of Vincent Lemort [4] and optimum design model

Angle Dimension	φ_{o0} [rad]	φ_{i0} [rad]	φ_{os} [rad]	φ_{is} [rad]	φ_{ie} [rad]
Standard	-1.3971	0	0.1729	2.1029	25.8250
Optimum	- 1.8420	0	- 0.2720	1.6580	34.4345

Table. 3 Performance of simplified standard scroll expander of Vincent Lemort [4] and optimum design model

Performance	V_{disp} [m^3]	$2V_{c,d}$ [m^3]	V_{ratio}	Eff. [%]	Involute Cycle
Standard	145,946.0	37,363.6	3.9061	100.0	4.1102
Optimum	87,864.9	15,372.9	5.7155	144.3	5.4804

Table. 4 Comparing expansion ratios in the same shape (t_s, h_s and R_{oe}) between current ORC scroll expander research with optimum design model

Size Dimension	t_s [mm]	h_s [mm]	R_{oe} [mm]	r_b [mm]	V_{ratio}
Zhu Wu [9]	5.08	51.55	64.3776	4.0860	1.774
Optimum	5.08	51.55	64.3776	2.1152	4.865
Jen-Chieh Chang [10]	4.00	41.00	56.8956	2.5900	2.950
Optimum	4.00	41.00	56.8956	2.1568	3.859

The current ORC scroll expander research compared with optimum design model has been shown as in Table. 4. The new one has higher expansion ratio (volume ratio) for both papers. Zhu Wu's [9] scroll wrap has volume ratio 1.774, Later, it can be increased to 4.865 and Jen-Chieh Chang's [10] scroll wrap can be changed from 2.950 to be 3.859 as well.

The meaning of higher expansion ratio is that expander can reduce higher pressure. Xinjing Zhang [11] and Sebastien Declaye [12] described pressure ratio analysis in their research. The summary for ORC system is (i) normal expansion, of which the outlet pressure is almost close to the system required pressure; (ii) under expansion, of which the discharging pressure is higher than the system required pressure, thus the flow of refrigerant is not smooth from the over pressure; (iii) over expansion, of which the discharging pressure is lower than the system required pressure thus it would cause reverse flow during discharging process. It can be called the pressure ratio corresponding to the built in volumetric ratio of the machine. The under/over expansion losses then occur as illustrated on the P-V diagram presented in Fig. 7.

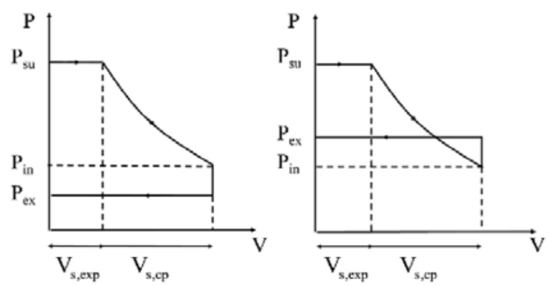

Fig. 7 Under/over expansion losses, reprinted from [12]

Thus, this can be the first process of ORC system design to maximize performance of expander machine within the desired dimension. Then set up the system suction/exhaust pressure of expander from this build in volumetric ratio to be closed to isentropic expansion.

Then use some equations from Zhu Wu [9] and Jen-Chieh Chang [10] to evaluate expander as the following.

The power output of the scroll expander is determined by the conservation equation of energy:

$$\dot{W}_{exp} = \eta_{mec}\, \dot{m}\, (h_{exp,in} - h_{exp,out}) \quad (16)$$

In which, $h_{exp,in}$ and $h_{exp,out}$ represent the enthalpy at the inlet and exit of the expander, respectively. The overall mechanical efficiency, η_{mec} is represents the affect from the transmission loss of the pulley and belt, and the mechanical loss of the power generator. Expander efficiency per unit flow rate is defined as:

$$\eta_{exp} = \frac{h_{exp,in} - h_{exp,out}}{h_{exp,in} - h_{exp,out,s}} \quad (17)$$

where, $h_{exp,out,s}$ means the state of fluid, calculated by isentropic expansion process. The volumetric efficiency could be determined the leakage level of a scroll expander. This is defined as the ratio of theoretical mass flow rate and the actual one:

$$\eta_V = \frac{\dot{m}_{th}}{\dot{m}_{actual}} \quad (18)$$

The higher expansion ratio results in the higher difference between enthalpy at the inlet and exit of the expander for the higher power output.

7. Conclusion

The optimal values of the design variables of the scroll expander wrap for organic Rankine cycle (ORC) power generation were acquired numerically under the required thickness and height of scroll wrap. Expansion ratio in term of volume ratio and its constraints were analyzed using linear programming methods, nonlinear interior point algorithms and utilize second derivatives by FindMaximum (Wolfram command). The maximum volume ratio is 5.7155 (efficiency increase 44.3%) at base circle radius r_b 2.4756 mm, inner ending involute angle φ_{ie} 34.4345 radian (5.4804 involute cycle), while thickness and height were set up to 4.56 mm and 28.65 mm accordingly. Because these 2 variables were relatively similar weight with the chosen 2 variables, to control their dimensions to be matched in Copeland's frame for the next prototype plan.

The comparison with another ORC scroll expander to validate the procedure with the different dimension still shows the higher expansion ratio. This higher expansion ratio leads to the higher difference between enthalpy at the inlet and exit of the expander for the higher power output.

The results of this work can be considered as the first process of ORC system design to maximize performance of expander machine corresponding to the built in volumetric ratio within the desired dimension. It is helpful for the design of scroll expanders wrap and prediction of the whole ORC system performance.

8. Nomenclature

Variables			Subscripts	
A	mm^2	Area	0	Initial
F	-	Flow correction parameter	s	Starting
h_s	mm	Scroll height	e	Ending
N	-	Number of	i0	Inner initial
r_b	mm	Base circle radius	is	Inner starting
r_o	mm	Orbiting Radius	ie	Inner ending
t_s	mm	Scroll thickness	o0	Outer initial
x	mm	x-Coordinate	os	Outer starting
y	mm	y-Coordinate	oe	Outer ending
φ*	rad	Effective involute angle	exp	Expander
φ	rad	Involute angle	mec	Mechanical
δ	mm	Gap width	in	Inlet
V_{disp}	mm^3	Displacement Volume	out	Outlet
$2V_{c,d}$	mm^3	Inner Chamber Volume	s	Isentropic
V_{ratio}	-	Volume ratio	th	Theoretical
R_{oe}	mm	Scroll Radius	V	Volumetric
$\dot{W}$	kW	Power		
$\dot{m}$	kg/s	Mass flow rate		
h	kJ/kg	Enthalpy		
η	-	Efficiency		

ICOME2016-0042

9. References

9.1 Article in Journals

[2] Baolong Wang, Xianting Li, Wenxing Shi (2005). A general geometrical model of scroll compressors based on discretional initial angles of involute, *International Journal of Refrigeration*, vol.28 (2005), pp. 958–966.

[5] Kyoungwoo Park, Seungjae Moon (2005). Optimal design of heat exchangers using the progressive quadratic response surface model, *International Journal of Heat and Mass Transfer,* vol.48 (2005), pp. 2126–2139.

[6] Jiangfeng Wang, Zhequan Yan, Man Wang, Maoqing Li, Yiping Dai (2013). Multi-objective optimization of an organic Rankine cycle (ORC) for low grade waste heat recovery using evolutionary algorithm, *Energy Conversion and Management,* vol.71 (2013), pp. 146–158.

[7] Xueyi Li, Shoubo Jiang and Qingliang Zeng (2013). Optimization of Two-Stage Cylindrical Gear Reducer with Adaptive Boundary Constraints, *Journal of Software,* vol.8, no. 8, August 2013, pp. 2052-2057.

[9] Zhu Wu, Deng Pan, Naiping Gao, Tong Zhu, Feibo Xie (2015). Experimental testing and numerical simulation of scroll expander in a small scale organic Rankine cycle system, *Applied Thermal Engineering*, vol.87 (2015), pp. 529-537.

[10] Jen-Chieh Chang, Chao-Wei Chang, Tzu-Chen Hung, Jaw-Ren Lin, Kuo-Chen Huang (2014). Experimental study and CFD approach for scroll type expander used in low-temperature organic Rankine cycle, *Applied Thermal Engineering,* vol.73 (2014), pp. 1444-1452.

[11] Xinjing Zhang, Yujie Xu, Jian Xu, Yong Sheng, Zhitao Zuo, Jimin Liu, Haisheng Chen, Yaodong Wang, Ye Huang (2016). Study on the performance and optimization of a scroll expander driven by compressed air, *Applied Energy,* xxx (2016) xxx–xxx.

[12] Sébastien Declaye, Sylvain Quoilin, Ludovic Guillaume, Vincent Lemort (2013). Experimental study on an open-drive scroll expander integrated into an ORC (Organic Rankine Cycle) system with R245fa as working fluid, *Energy*, vol.55 (2013), pp. 173-183.

9.2 Proceedings

[3] Bell, Ian H., Groll, Eckhard A., Braun, James E., and Horton, W. Travis (2012). Derivation of Optimal Scroll Compressor Wrap for Minimization of Leakage Losses, *International Compressor Engineering Conference (2012),* Paper 2111.

[4] Vincent Lemort, Sylvain Quoilin, Jean Lebrun (2008). Numerical Simulation of a scroll expander for use in a rankine cycle, *International Compressor Engineering Conference at Purdue,* July 14-17, 2008.

9.3 Patent

[1] L. Creux (1905). Rotary engine. *US Patent No. 801182;* 1905.

9.4 Software

[8] *Wolfram Mathematica 10.0* Software (2014).