Passive vibration control of an automotive component using evolutionary optimisation

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Abstract

In this paper, the use of multiobjective evolutionary optimisers for passive vibration suppression of an automotive component is demonstrated. The component is used to connect a car engine to some point of a car body between the front seats. Under such a circumstance, the structure is subject to several mechanical phenomena e.g. stress failure, fatigue, vibration resonance, and vibration transmissibility. The optimisation problem is posed to find structural shape and size such that maximising structural natural frequency and simultaneously minimising structural mass while constraints include stress failure and displacement. The multiobjective optimiser employed is the multiobjective version of Population-Based Incremental Learning (PBIL) with and without using a surrogate model**.** The optimum results obtained are illustrated and discussed. It is found that the proposed design scheme is effective and efficient for an automotive component design.

Keywords: multiobjective evolutionary algorithm; shape optimisation; Pareto optimal front; automotive component; Vibration suppression

1. Introduction

Due to highly increasing competitiveness in automotive industry, many car manufacturers require to develop new products to offer to customers. Therefore, automotive components are always improved by means of design optimisation [1-2].

Practical engineering design problems are usually assigned to find the best solutions of design variables that lead to optimised design objectives whilst fulfilling all the predefined constraints. Often, the design problem has more than one objective which is called multiobjective optimisation. The most popular method used for the multiobjective optimisation is Evolutionary Algorithms (EAs) [3-6]. The method can explore a Pareto optimum front within a single run and without requiring function derivatives. However, a lack of search consistency and low convergence rate are the inevitable drawbacks of the multiobjective evolutionary algorithms (MOEAs) [5]. For this reason, the hybridisation of a surrogate model method and multiobjective optimisers has been invented and this approach is found to be very powerful and effective [6].

This paper presents the multiobjective evolutionary optimisation of an automotive component. The component is used to connect a car engine to some point of the body between the front seats. The structure is subject to several mechanical phenomena such as stress failure, fatigue, vibration resonance, and vibration transmissibility. The design problem is posed to find structural shape and size such that maximising structural dynamic stiffness while, at the same run, minimising structural mass. Design constraints include stress and displacement. Three dimensional finite element analysis (FEA) is employed to evaluate the objective and constrain function values. The optimum solutions called Pareto solutions are explored by using PBIL incorporating with a Gaussian process surrogate model and a Latin Hypercube Sampling technique. The proposed design approach is found to be numerically powerful and effective.

2. Surrogate model method

The term' surrogate model' used in an optimisation process is an approximate model which is used to approximate the objective and constrain functions in optimisation problems [7]. Such a design strategy is useful when dealing with optimisation problems with expensive function evaluation, limited function values available, and problems that need to perform an experiment to evaluate their function values. The hybrid of the surrogate model with an optimiser can be achieved in several ways. One of the commonly used strategies is that, during the main optimisation process, some design solutions have been evaluated. Those solutions and their corresponding objective and constraint values are used to build a surrogate model. This model is then used as an approximate function evaluation. The optimisation with the surrogate model is performed with significantly less running time when compared with using the actual function evaluation. The obtained optimum solution of this design phase is brought to the main optimisation process where its actual function value is determined. With a highly accurate surrogate model, this design strategy is far superior to purely using an evolutionary algorithm. The computational steps are repeated until the termination conditions are fulfilled. The commonly used surrogate models for optimisation are Kriging model [8], radial basis interpolation [6], polynomial interpolation [9] and neural network [10]. In this paper, only the Kriging model is employed.

2.1. Kriging Model

A Kriging model (also known as a Gaussian process model) used herein is the famous MATLAB toolbox named Design and Analysis of Computer Experiments (DACE) [8]. The estimation of function can be thought of as the combination of global and local approximation models i.e.

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$$y(\mathbf{x}) = \bar{f}(\mathbf{x}) + Z(\mathbf{x}) \quad (1)$$

where $\bar{f}(\mathbf{x})$ is a global regression model, $Z(\mathbf{x})$ is a stochastic Gaussian process with zero mean and non-zero covariance representing a localised deviation, and **x** is a design variables vector. In this work, a linear function is use for a global model, which can be expressed as:

$$\bar{f} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i = \boldsymbol{\beta}^{\mathrm{T}}\mathbf{f} \quad (2)$$

where $\boldsymbol{\beta} = [\beta_0, \ldots, \beta_n]^T$, $\mathbf{f} = \mathbf{f}(\mathbf{x}) = [1, x_1, x_2, \ldots, x_n]^T$. The covariance of $Z(\mathbf{x})$ is expressed as:

$$Cov(Z(\mathbf{x}^p), Z(\mathbf{x}^q)) = \sigma^2 \mathbf{R}[R(\mathbf{x}^p, \mathbf{x}^q)] \quad (3)$$

for p, q = 1, …, N where R is the correlation function between any two of the N design points, and **R** is the symmetric correlation matrix size $N\times N$ with the unity diagonal [8]. The correlation function used in this paper is

$$R(\mathbf{x}^p, \mathbf{x}^q) = \exp(-(\mathbf{x}^p - \mathbf{x}^q)^T \boldsymbol{\theta}(\mathbf{x}^p - \mathbf{x}^q)) \quad (4)$$

where θ_i are the unknown correlation parameters to be determined by means of the maximum likelihood method. Having found $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$, the Kriging predictor can be achieved as

$$\bar{y} = \mathbf{f}(\mathbf{x})^T \boldsymbol{\beta} + \mathbf{r}^T(\mathbf{x}) \mathbf{R}^{-1}(\mathbf{y} - \mathbf{F}\boldsymbol{\beta}) \quad (5)$$

Where $F = [\mathbf{f}(\mathbf{x}^1), \mathbf{f}(\mathbf{x}^2), \ldots, \mathbf{f}(\mathbf{x}^n)]^T$ and $\mathbf{r}^T(\mathbf{x}) = [R(\mathbf{x}, \mathbf{x}^1), R(\mathbf{x}, \mathbf{x}^2), \ldots, R(\mathbf{x}, \mathbf{x}^N)]$. For more details, see [8].

3. Multiobjective Population-Based Incremental Learning (MOPBIL)

PBIL algorithm is an evolutionary optimiser based upon binary searching space. The PBIL approach evolves its population based upon the so-called probability vector, the probability of having '1' elements on each column of a binary population. The example of how the probability vector works is shown in Fig.1 which implies that one probability vector can produce a variety of binary populations.

In the multiobjective optimisation, more probability vectors should be used in order to obtain a more diverse population; therefore, it is called a probability matrix. Starting with an initial probability matrix that have all elements as "0.5", and an initial Pareto archive, the binary population according to the initial probability matrix is then created. The binary population is decoded and objective values are evaluated. The best binary solutions, whether it is based on minimisation or maximisation, is chosen to update the probability vector $P_{i,j}^{new}$ for the next iteration using the relation

$$P_{i,j}^{new} = P_{i,j}^{old}(1 - L_R) + \mathrm{b}_j L_R \quad (6)$$

where L_R is called the learning rate, a value between 0 and 1, to be defined and b_i is the mean value of the j^{th} column of the randomly selected non-dominated binary solutions. For this study, L_R is set as:

$$L_R = 0.5 + rand(+0.1 \,\mathrm{or}\, -0.1) \quad (7)$$

where $rand \in [0,1]$ is a uniform random number. Mutation on the i^{th} row of the probability matrix is allowed to take place by a predefined probability and it can be expressed as:

$$P_{i,j}^{new} = P_{i,j}^{old}(1 - m_s) + rand(0 \,\mathrm{or}\, 1) m_s \quad (8)$$

where m_s is the amount of shift used in the mutation.

population 1	population 2	population 3
0 0 1 1	0 1 1 0	0 1 0 1
1 1 0 0	1 1 0 0	1 0 0 1
0 0 1 1	1 0 1 0	0 0 0 1
1 1 0 0	0 0 0 1	0 1 0 0
Probability Vectors		
[0.5,0.5,0.5,0.5]	[0.5,0.5,0.5]	[0.25,0.5,0,0.75]

Fig.1 Probability vector and their corresponding populations

The updating process is completed when all rows of the probability matrix are changed. The probability matrix is updated and the external Pareto archive is improved iteratively until convergence is achieved.

In cases where the total number of the non-dominated solutions is greater than the archive size, the archiving operator called the normal line method [4] is activated to remove some solutions from the archive. The archiving technique is used to prevent excessive use of computer memory during an optimisation process. The basic idea of the normal line technique is used to remove some non-dominated design solutions while maintaining population diversity in the archive. For more details of multiobjective PBIL, see [11].

4. Design Problem

This paper presents a multiobjective optimisation design problem for an automotive part as shown in Fig. 2. The component is used to connect the car engine with the car body. Under the working conditions, this structure is subject to several mechanical phenomena e.g. stress, fatigues, vibration resonance, and dynamic force transmissibility. Also, the structural displacement due to a number of loading conditions should not exceed the predefined limit.

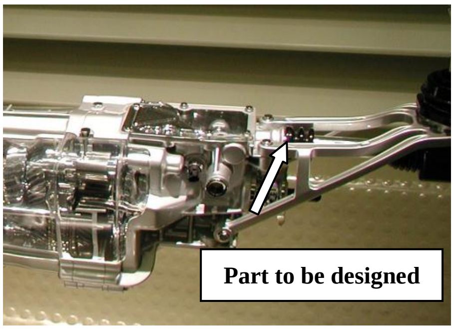

Fig. 2 Automotive part

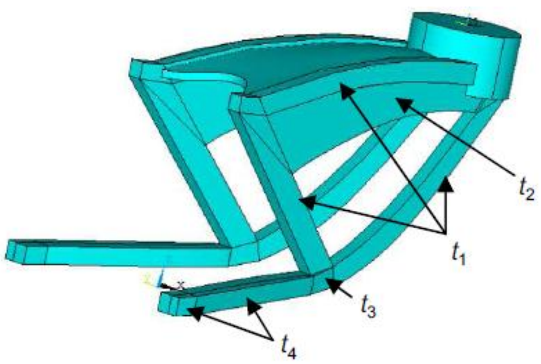

Fig. 3 a. Sizing variables

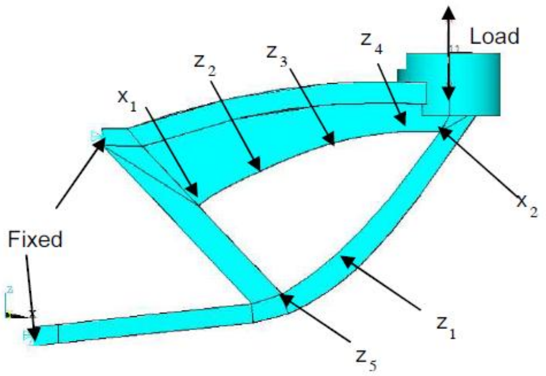

Fig. 3 b. Shape variables

The multiobjective optimisation problem is posed to find structural shape and size such that maximising structural natural frequencies and minimising mass whereas constraint include stress failure and displacement, which can be expressed as

Min: $\mathbf{f} = [f_1(\mathbf{x}), f_2(\mathbf{x})]$ (9)

Subject to

$\sigma_{max} \le \sigma_{allowable}$

$u_{max} \le 0.005$

$0.0015 \le t_1 \le 0.0115$

$0.0015 \le t_2 \le 0.0115$

$0.0015 \le t_3 \le 0.01$

$0.0025 \le t_4 \le 0.015$

$-0.003 \le z_1 \le 0.01$

$-0.0025 \le z_2 \le 0.0028$

$-0.01 \le z_3 \le 0.005$

$-0.01 \le z_4 \le 0.005$

$0 \le z_5 \le 0.03$

$-0.01 \le x_1 \le 0.01$

$-0.005 \le x_2 \le 0.01$

where $\mathbf{x}$ is a design variable vector (all variables are displayed in Fig. 3). f_1 is a function of mass. f_2 is a function of dynamic stiffness (or natural frequencies). x , f_1 and f_2 can be express as :

$$\mathbf{x} = \{t_1, t_2, t_3, t_4, z_1, z_2, z_3, z_4, z_5, x_1, x_2\}^T$$

and

$$f_1 = mass \quad (10)$$

and

$$f_2 = \frac{1}{\omega_1 + \omega_2 + \omega_3 + \omega_4 + \omega_5}. \quad (11)$$

The other parameters are defined as follows:

σ_{max} = Maximum von Misses stress

$\sigma_{allowable}$ = Allowable stress

t_i =Shape thickness

z_i=Position of the key points in *z*-axis direction

x_i= Position of key point in *x*-axis direction

ω_i = mode i^{th} natural frequency of a structure

Figs. 3a. & 3b. display all of the sizing and shape design variables. The thicknesses (t_i in Fig. 3 a.) are the thickness of the sub-regions of the automotive component as shown. The z_i parameters determine the key points in vertical direction as located in Fig. 3 b. These key points are used to generated a spline curve so as to define the shape of the part. The x_i parameters define the horizontal position of the key points on the component.

The structure is acted upon by three load cases (bending, twisting and swaying loads) at the right-hand cylinder part. The objective and constraint function values are evaluated by using FEA. The evaluation process is carried out in such a way that, with the given input design variables as defined, the shape and dimensions of the structure are created. The finite element analysis is then performed. Finally the computational results can be obtained. Function evaluation is somewhat time-consuming, which means it is difficult to apply a common evolutionary algorithm to solve the optimisation problem (9). As a result, the surrogate-assisted evolutionary algorithm is developed to deal with such a difficulty.

To tackle multiobjective optimisation as defined in (9), the MOPBIL algorithm and the surrogate-assist MOPBIL (MOPBIL-SM) are used to find Pareto optimal solutions. MOPBIL-SM is a design strategy that exploits the surrogate model to create an initial Pareto archive rather than starting with a randomly generated population as with the traditional multiobjective PBIL.

The computational steps for generating an initial Pareto archive by using the surrogate model are as follows:

I. Sample a set of design variable vectors from design experiment by using the LHS technique.
II. Evaluate design functions by FEA.
III. Constructing a surrogate model by using the Kriging technique.
IV. Use MOPBIL find Pareto optimal set based on the surrogate model.
V. Find the real function values of the Pareto optimal front obtained from optimising the approximate Kriging model (step IV).
VI. Use a non-dominated sorting technique to find the initial Pareto archive

The LHS is used to sample 100 design solutions for constructing a surrogate optimisation model. Subsequently, with this initial Pareto archive, the common MOPBIL is operated where the population size is 30, the number of iterations is 10, and archive size is set as 30.

5. Results and Discussion

The progress of Pareto optimal solutions of the optimisation design problem by using the hybridisation of a surrogate model method and the MOPBIL is displayed in Fig. 4. It can be seen that the Pareto front from iteration 1 to iteration 10 has slight improvement. This means that the initial front generated by means of a surrogate-assisted approach is very powerful.

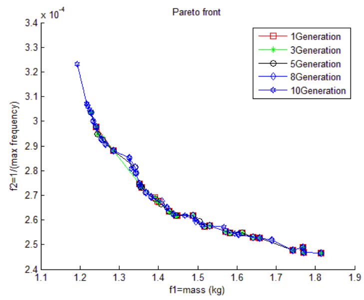

Fig.4 Pareto front of the MOPBIL-SM

In order to verify the effectiveness of the hybrid approach, the original MOPBIL without the use of a surrogate approach is performed with the same population and archive sizes while the total generation number is set to be 30. This implies that the original MOPBIL uses 30×30 actual function evaluations which is approximately twice the number of evaluation used by MOPBIL-SM (100 + 10×30 evaluations). The results from the former are termed as MOPBIL whereas the results obtained from the later are named MOPBIL-SM. Figs.5-7 compare the Pareto fronts obtained from using MOPBIL-SM at the generations of 1, 3 and 5, and using MOPBIL at the generations of 10, 20 and 30 respectively. It can be found that the results from using MOPBIL-SM are better than those obtained from using the original MOPBIL even with a far smaller number of finite element analyses. That means the hybrid approach is far superior to the original optimiser.

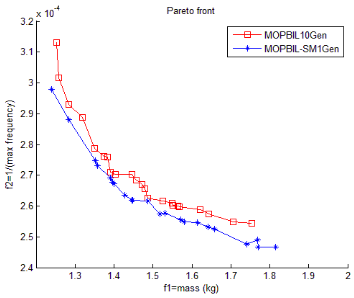

Fig.5 Comparative Pareto fronts: MOPBIL 10 Generations versus MOPBIL-SM 1 Generation

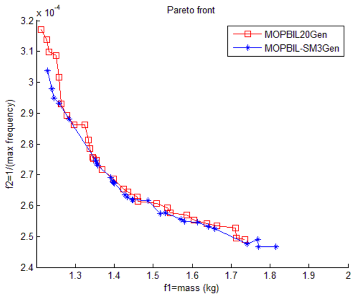

Fig.6 Comparative Pareto fronts: MOPBIL 20 Generations versus MOPBIL-SM 3 Generations

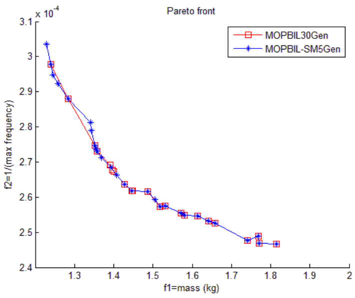

Fig.7 Comparative Pareto fronts: MOPBIL 30 Generations versus MOPBIL-SM 5 Generations

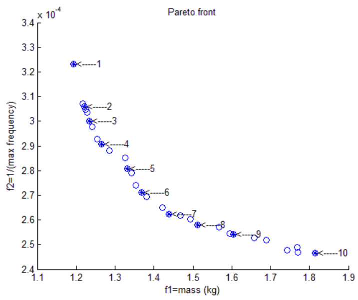

Fig.8 Pareto front from MOPBIL-SM

The Pareto optimal solutions of the MOPBIL-SM shown in Fig. 8 have the corresponding design solutions as shown in Fig. 9. The optimum components have an obvious variation for the design variables t_2 , z_3 and z_5, while the other variables have a slight variation. It can be seen that, with one optimisation run, we can have a number of optimum components for decision making.

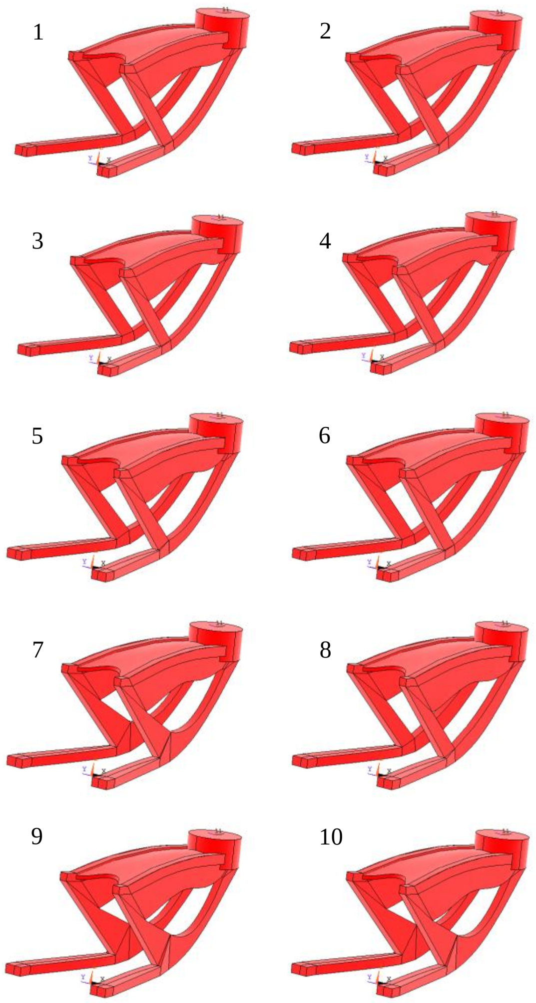

Fig.9 3D automotive parts corresponding to selected solutions in Fig. 8

6. Conclusions

The multiobjective 3D shape and sizing optimisation problem of an automotive component using the hybridisation of a surrogate Kriging model and MOPBIL is demonstrated. The results show that the proposed approach is efficient and effective for solving the design problem. The new design strategy outperforms the original PBIL optimiser based upon the total number of function evaluations. An improved design strategy employing much less function evaluations is the target for future work.

7. Acknowledgments

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