The IMC Interaction Measure for a Large System with Block Decomposition

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Abstract

An interaction measure is introduced for a large linear multivariable system with block decomposition. Its derivation is based an the recently developed Internal Model Control (IMC) design procedure and the IMC interaction measure. The interaction measure is a function of frequency and bears a rigorous relation to closed loop stability and performance. Two examples from the literature are used to illustrate its merits.

Introduction

The problem of interactions arose naturally when control engineers started concerning themselves with multivariable systems. The first approach to the interaction measure was proposed by Bristol [1] whose Relative Gain Array (RGA) had considerable very wide acceptance. According to Bristol, interactions are characterized by a constant matrix. It measures steady state interactions only and leads to erroneous conclusion about desirable pairings. In 1974, Rosenbrock [2] proposed the approach to measure system interaction through the Direct and Inverse Nyquist Arrays (DNA and INA respectively). From 1982 to 1983, Garcia and Morari [3],[4],[5] developed a new design concept of feedback control, Internal Model Control (IMC) as shown in Fig. 1 which is theoretically a good method for control system design. In 1983, Economou and Morari 6 developed a new interaction measure for linear stationary multivariable systems. Based on the recently developed Internal Model Control design procedure, the interaction measure cansists of two frequency dependent curves for each selected input/output pair. The curves which can vary between 0 and 1. The interaction measure is not very convenient for "large" systems. Therefore, the interaction measure for a large system with block decomposition is proposed.

The IMC Interaction Measure for a large system with Block Decomposition

The IMC design procedure has been theoretically applied to many control problems. However as the system dimension increases, the design procedure depends directly on the system model. Considering the IMC structure as shown in Fig. 1, the plant is described by the following large system transfer matrix G :

$$G = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & & \vdots \\ G_{N1} & G_{N2} & & G_{NN} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $G_{11}, G_{12}, \ldots, G_{1N}, G_{21}, G_{22}, \ldots, G_{2N}, G_{N1}, \ldots,$ G_{NN} are n X n matrices.

The diagonal block matrices model is :

$$\tilde{G} = \begin{bmatrix} \tilde{G}_{11} & & O \\ & \ddots & \\ O & & \tilde{G}_{NN} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The diagonal model transfer matrix can be factorized as :

$$\tilde{G} = \tilde{G}_{+} \cdot \tilde{G}_{-} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where $\tilde{G}_+$ contains time delay and/or RHP Zeros

$\tilde{G}_-$ is a stable and realizable model transfer matrix

For a diagonal block matrices model, the corresponding IMC controller, G_c and robustness filter, F will also be diagonal block matrices :

$$G_c = \tilde{G}_-^{-1} = \begin{bmatrix} \tilde{G}_{11-}^{-1} & & 0 \\ & \ddots & \\ 0 & & \tilde{G}_{NN-}^{-1} \end{bmatrix} \qquad \text{..........(4)}$$

and the robustness filter

$$F = \begin{bmatrix} F_1 & & 0 \\ & \ddots & \\ 0 & & F_N \end{bmatrix} \qquad \text{..........(5)}$$

where $F_1, F_2, \ldots, F_N$ are also n X n matrices

The filter is Simplified as :

$F_1 = f_1 I_{nn}$, $F_2 = f_2 I_{nn}$, ... , $F_i = f_i I_{nn}$, ... , $F_N = f_N I_{nn}$

where the f_i's are such that the roots of the characteristic equation lie in the Left half plane and I_{nn} is a n X n identity matrix.

Considering the small gain theorem for the sufficient stability condition :

$$\left\| F\, G_c(G - \tilde{G}) \right\|_\infty < 1 \qquad \text{..........(6)}$$

where $\| \cdot \|$ stands for any appropriately ∞ - Norm Substituting equations (1), (2), (4) and (5) into equation (6), we get :

$$\left\| \begin{bmatrix} 0 & F_1 G_{11}^{-1} G_{12} & \cdots & F_1 G_{11}^{-1} G_{1N} \\ F_2 G_{22}^{-1} G_{21} & 0 & \cdots & F_2 G_{22}^{-1} G_{2N} \\ \vdots & & & \vdots \\ F_i G_{ii}^{-1} G_{i1} & \cdots & \cdots & F_i G_{ii}^{-1} G_{iN} \\ \vdots & & & \vdots \\ F_N G_{NN}^{-1} G_{N1} & \cdots & \cdots & 0 \end{bmatrix} \right\|_\infty < 1 \qquad \text{..........(7)}$$

$$\max_i |f_i| \; \| G_{ii}^{-1} (G_{i1} \cdots G_{i,i-1} \; G_{i,i+1} \cdots G_{iN}) \|_\infty < 1 \; ;$$

$$i = 1,2, \ldots , N \qquad \text{..........(8)}$$

Equation (8) can be rewritten as a set of N simultaneous inequalities:

$$|f_i| < f_{Ri} = \frac{\| G_{ii} \|_\infty}{\sum_{j, j \neq i}^{N} \| G_{ij} \|_\infty} \; ; \; i = 1,2, \ldots , N \qquad \text{..........(9)}$$

Starting with a relation equivalent to equation (6)

$$\| (G - \tilde{G}) G_c \cdot F \|_\infty < 1 \qquad \text{..........(10)}$$

We can obtain the dual set of N simultaneous in equalities ;

$$|f_i| < f_{Ci} = \frac{\| G_{ii} \|_\infty}{\sum_{j, j \neq i}^{N} \| G_{ji} \|_\infty} \; ; \; i = 1,2,3, \ldots , N \qquad \text{.......(11)}$$

This two sets of inequalities in equations (9) and (11) give the upper bounds on the filtering action that can be taken so that loop stability is preserved. The following quantities (R_i, C_i) are defined to constitute the new interaction measure ;

The i th block (row) IMC interaction measure (R_i) is the quantity :

$$R_i = 1 - \frac{|f_{Ri}|}{1 + |f_{Ri}|} \qquad \text{..........(12)}$$

and the i th block (column) IMC interaction measure (C_i) is the quantity :

$$C_i = 1 - \frac{|f_{ci}|}{1 + |f_{ci}|} \qquad \text{..........(13)}$$

By definition R_i and C_i vary from 0 to 1 , low values corresponding to low interactions (high gain filter allowed). **The row and** column of the large system (block) interaction measures combined give the information about system interactions.

Low R_i and Low C_i : very good input/output pairing.

Low R_i and Hight C_i : Input block i is dominant to the system. Loop i affects all others and is affected by none

High R_i and Low C_i : Input block i is insignificant to the system. Loop i affects no other loops.

High R_i and High C_i : Low quality input/output pairing.

Examples :

Case 1: The plant transfer function matrix is

$$G_I = \begin{bmatrix} \frac{10}{(s+1)} & \frac{1}{(3s+1)} & 0 & 0 \\ \frac{0.01}{(4s+1)} & \frac{0.1}{(2S+1)} & \frac{0.05}{(3s+1)} & 0 \\ 0 & \frac{0.05}{(5s+1)} & \frac{1}{(3s+1)} & \frac{0.01}{(4s+1)} \\ 0 & 0 & \frac{0.1}{(2s+1)} & \frac{5}{(s+1)} \end{bmatrix} \qquad \text{......(14)}$$

In this case

$$G_I = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \qquad \text{.............(15)}$$

where

$$G_{11} = \begin{bmatrix} \frac{10}{(s+1)} & \frac{1}{(3s+1)} \\ \frac{0.01}{(4s+1)} & \frac{0.1}{(2s+1)} \end{bmatrix} \qquad \text{.............(16)}$$

$$G_{12} = \begin{bmatrix} 0 & 0 \\ \frac{0.05}{(3s+1)} & 0 \end{bmatrix} \qquad \text{.............(17)}$$

$$G_{21} = \begin{bmatrix} 0 & \frac{0.05}{(5s+1)} \\ 0 & 0 \end{bmatrix} \qquad \text{.............(18)}$$

$$G_{22} = \begin{bmatrix} \frac{1}{(3s+1)} & \frac{0.01}{(4s+1)} \\ \frac{0.1}{(2s+1)} & \frac{5}{(s+1)} \end{bmatrix} \qquad \text{.............(19)}$$

Fig. 2 is the plot of the robustness filter as a function of frequency, when block (1,1) and block (2,2) pairing is used.

Fig. 3 is the plot of the R_i and C_i as a function of frequency, when block (1,1) and block (2,2) pairing is used. It shows that, R_i and C_i are very low and the magnetude of the filter is very high. Therefore, the system is a very good input/output pairing.

<u>Case 2</u> : The plant transfer function

$$G_{II} = \begin{bmatrix} \frac{0.05}{(s+1)} & \frac{1}{(s+1)(2s+1)} & \frac{1}{(2s+1)} & \frac{1}{(3s+1)(2s+1)} \\ \frac{1}{(s+1)(2s+1)} & \frac{1}{(2s+1)} & \frac{1}{(3s+1)(s+1)} & \frac{1}{(s+1)} \\ \frac{0.05}{(3s+1)} & \frac{1}{(s+1)} & \frac{1}{(3s+1)} & \frac{1}{(2s+1)(s+1)} \\ \frac{1}{(3s+1)(s+1)} & \frac{1}{(3s+1)} & \frac{0.05}{(s+1)} & \frac{1}{(3s+1)(s+1)} \end{bmatrix} \quad ..(20)$$

In this case

$$G_{II} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \quad ....(21)$$

$$G_{11} = \begin{bmatrix} \frac{0.05}{(s+1)} & \frac{1}{(s+1)(2s+1)} \\ \frac{1}{(s+1)(2s+1)} & \frac{1}{(2s+1)} \end{bmatrix} \quad ....(22)$$

$$G_{12} = \begin{bmatrix} \frac{1}{(2s+1)} & \frac{1}{(3s+1)(2s+1)} \\ \frac{1}{(3s+1)(s+1)} & \frac{1}{(s+1)} \end{bmatrix} \quad ....(23)$$

$$G_{21} = \begin{bmatrix} \frac{0.05}{(3s+1)} & \frac{1}{(s+1)} \\ \frac{1}{(3s+1)(s+1)} & \frac{1}{(3s+1)} \end{bmatrix} \quad ....(24)$$

$$G_{22} = \begin{bmatrix} \frac{1}{(3s+1)} & \frac{1}{(2s+1)(s+1)} \\ \frac{0.05}{(s+1)} & \frac{1}{(3s+1)(s+1)} \end{bmatrix} \quad ....(25)$$

Fig. 4 is the plot of the robustness filter as a function of frequency, when block (1,1) and block (2,2) pairing is used.

Fig. 5 is the plot of the R_i and C_i as a function of frequency, when block (1,1) and block (2,2) pairing is used. It shows that, the system is a good input/output pairing at low frequency. At high frequency, the system is low quality input/output pairing.

Conclusions

The introduced IMC interaction measure is applied very well to large systems. The use of the ∞-norm in conjunction with the small gain theorem as a sufficient condition for closed loop stability is very conservative.

References

[1] Bristol, E.H., "On a New Measure of Interaction of Multivariable Process Control", IEEE Trans. Autom. Contr. AC-11 ; No.7 , PP. 133-134 ; 1966.

[2] Rosenbrock , H.H., "Computer Aided Control System Design" Academic Press , 1974.

[3] Garcia, G.E. , Morari, M., "Internal Model Control 1. A unifying review and some New Results", Ind. Eng. Chem. Process. Des., 21, PP. 308-323 , 1982.

[4] Garcia , C.E. Morari , M., "Internal Model Control 2", Ind. Eng. Chem. Process. Des., 1983.

[5] Garcia , C.E., Morari , M., "Internal Model Control 3" , Ind. Eng. Chem. Process Des., 1983.

[6] Morari , M., Economou , C.G., "The IMC Interaction Measure" AIChE Journal, 1983.

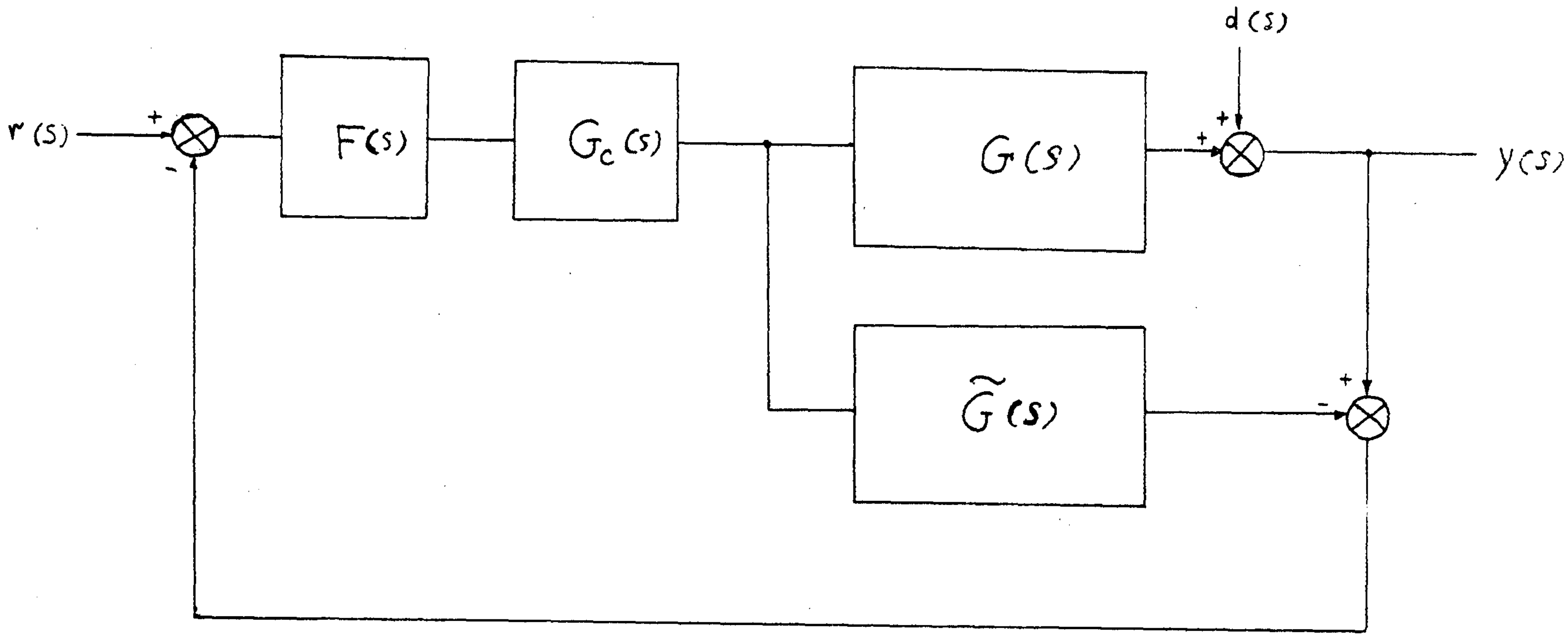

Fig. 1 The complete IMC structure

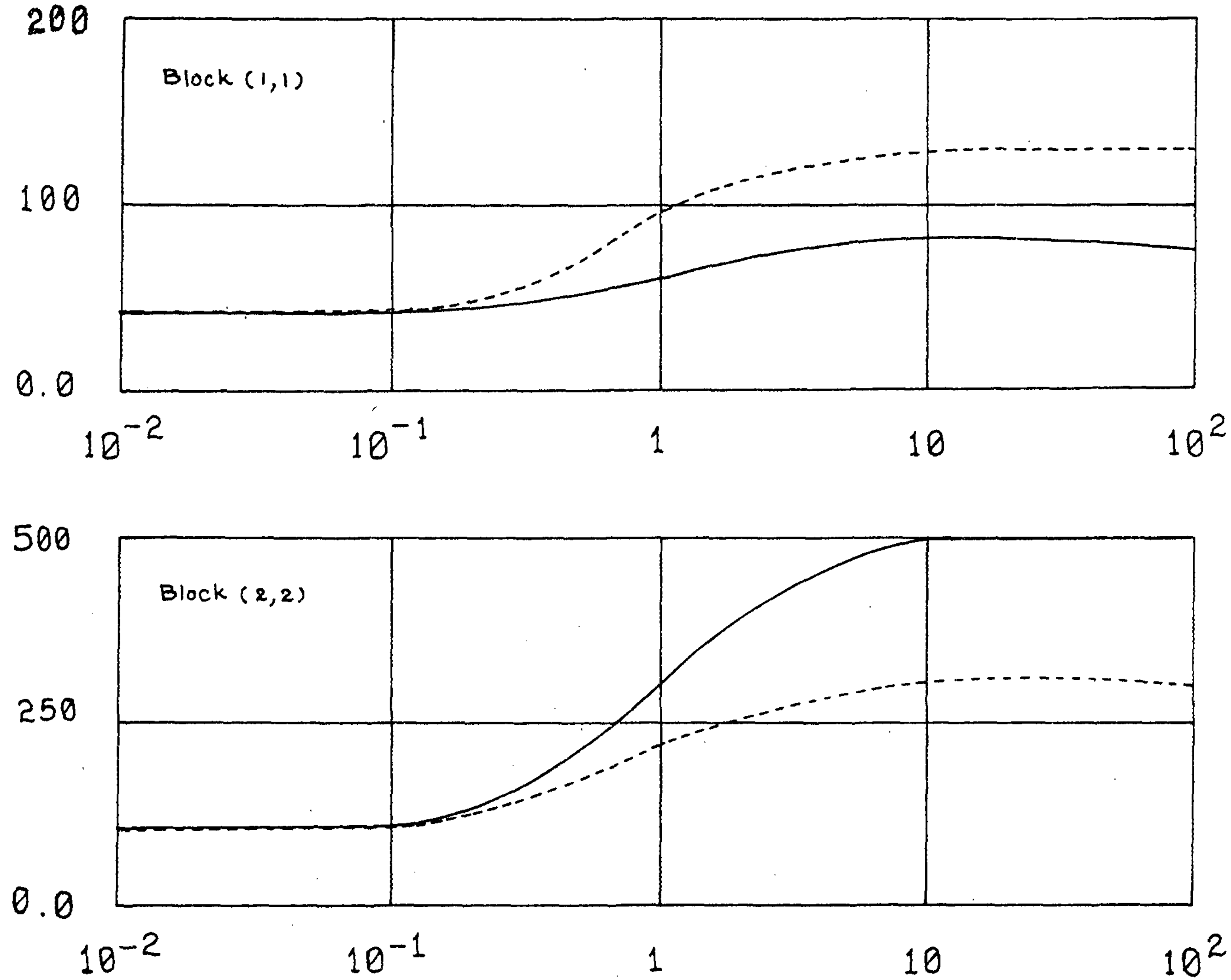

Fig. 2 The robustness filter for system G_I

(———— f_{Ri} , - - - - - - f_{Ci})

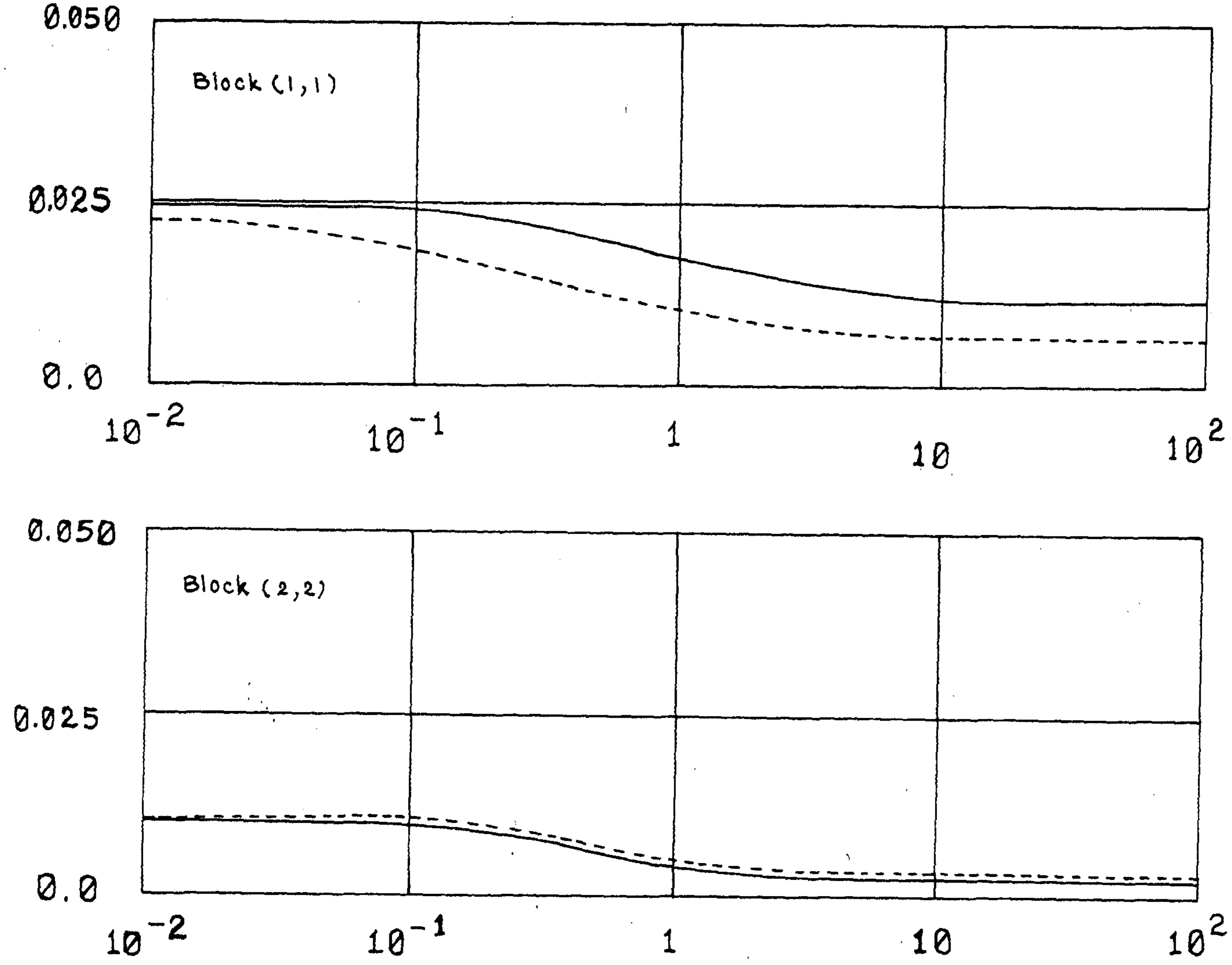

Fig. 3 The IMC interaction measure for system G_I

(———— R_1 , - - - - - - C_1)

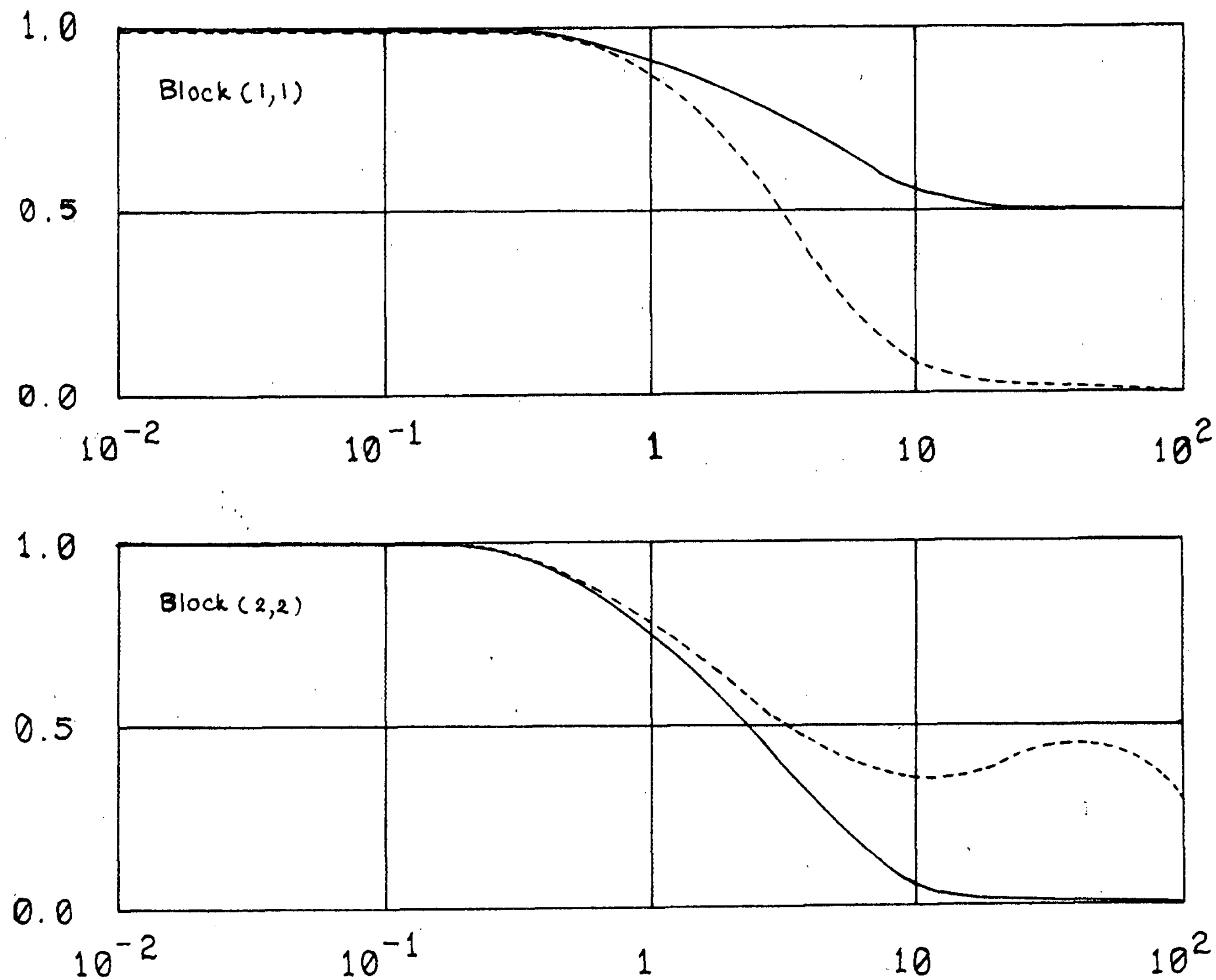

Fig. 4 The robustness filter for system G_{II}

(———— f_{Ri} , - - - - - - f_{Ci})

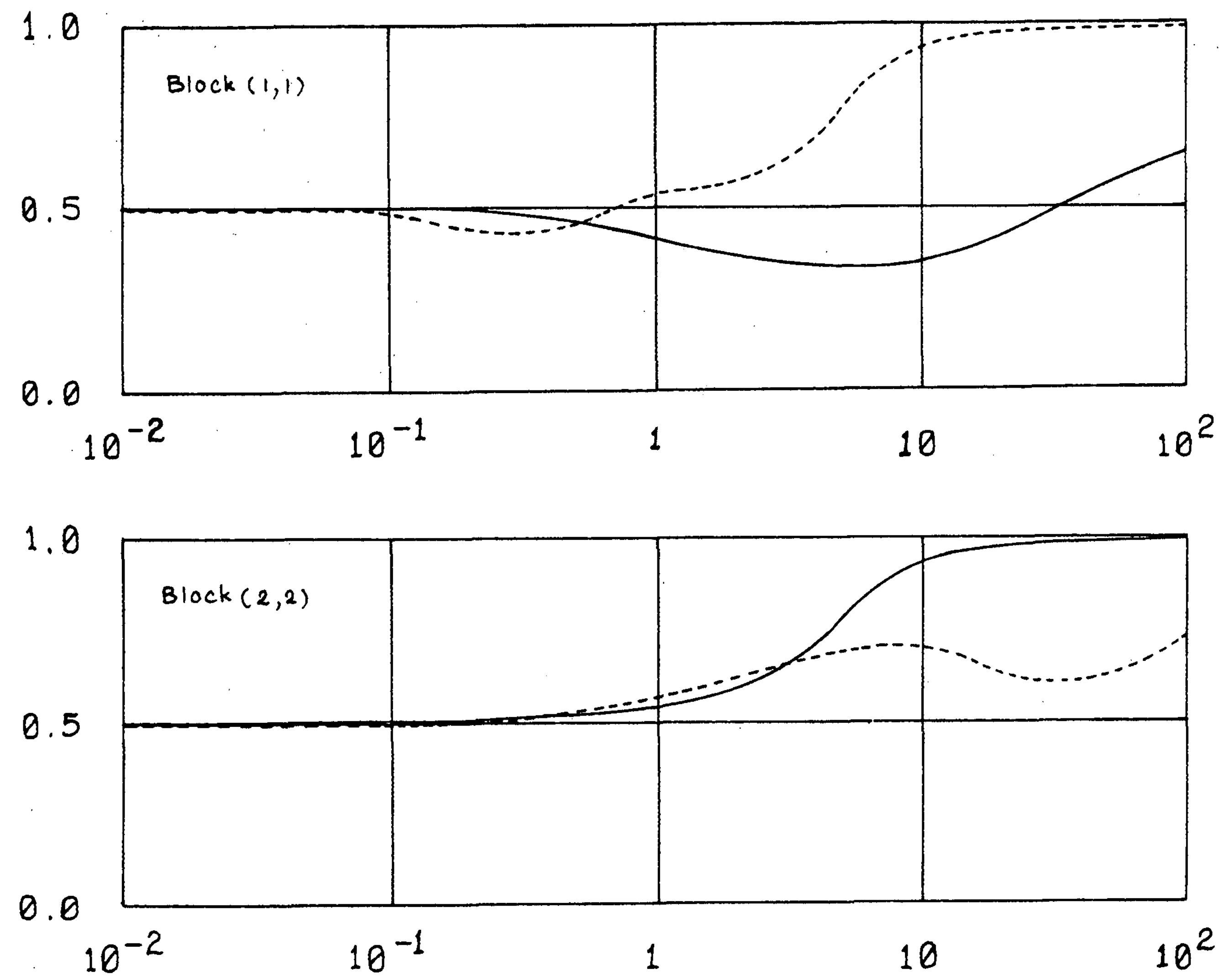

Fig. 5 The IMC interaction measure for system G_{II}

(———— R_i , - - - - - - C_i)