The 33rd Conference of The Mechanical Engineering Network of Thailand
2 - 5July 2019,Udon Thani

TSF - 017

Determination of the Critical Mass Flow Rate Through Supersonic Steam Nozzles Using a Two-phase Fluid Model

Chatchai Chiraporncha[1], Nithipun Prajong[1], Pasin Sangsukiam[1], and Satha Aphornratana[1]

[1]*School of Manufacturing Systems and Mechanical Engineering, Sirindhorn International Institute of Technology, Thammasat University, Pathumthani, 12120, Thailand*

*Corresponding Author: E-mail: satha@siit.tu.ac.th, Tel: 02-986-9009

Abstract

Gaining knowledge on properties of critical flow through a supersonic nozzle is essential to optimize performance of ejector refrigeration systems. Generally, critical mass flow rate through nozzles is calculated using the ideal gas assumption. However, this assumption is not applicable in the cases concerning wet fluids, so computations of critical mass flow rate have to be based on the two-phase mixture theory. Multiple previous studies have proposed models for two-phase mixture flow. However, these models are extremely complex and some of the parameters, such as slip ratio and void fraction of the two-phase mixture, are difficult to obtain. Therefore, the objective of this study is to propose a simplified, alternative approach to estimating the critical mass flow rate of a two-phase mixture through small supersonic nozzles. Comparisons are then made between the calculated critical mass flow rate from the mathematical model presented and the experimental values acquired from an ejector refrigeration system, using water as a working fluid. Nozzle with throat sizes of 1.7, 2.0, 2.4 and 2.8 mm are employed in this study, with the nozzle inlet temperature ranging from 110-155°C. For every case, a small difference of less than 10% between the calculated and experimental mass flow rate is obtained, verifying the validity of the simplified model introduced in this study. This study will not only be useful in designing and modeling systems composed of supersonic nozzles, but will also contribute to further understanding of related analyses on two-phase and critical flow behaviors.

***Keywords**:* Supersonic Nozzle; Critical Mass Flow Rate; Two-phase Mixture; Ejector Refrigeration System

1. Introduction

Supersonic nozzle, or De Laval nozzle, was first invented by Swedish inventor Gustaf de Laval in 1888. It was first used to produce supersonic jet streams driving a steam turbine. Today, supersonic nozzles are utilized in sprinkles, small-scale wind tunnels, the chemical industry, the materials coating industry, and the production of vacuum environments equipped with steam ejectors. Figure 1 displays a diagram of a supersonic nozzle designed for steam. High pressure, high temperature, and dry steam from a boiler enters the nozzle on the left-hand side. It is accelerated and expanded in the elliptical profile duct until the steam's velocity reaches sonic value and choke occurred at the throat of the nozzle. Then, the steam is further expanded and accelerated in the conical profile duct and discharges out as a supersonic jet stream on the right-hand side.

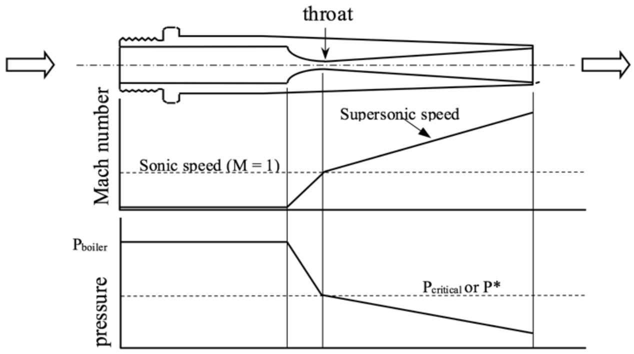


Fig 1 : De Laval nozzle and the variation of Mach number and pressure along the nozzle.

For a specified pressure and temperature of the inlet steam, when the nozzle is choked, the mass flow rate of the steam reaches its maximum value and cannot be increased even if the nozzle's downstream pressure isreduced. This maximum mass flow rate is usually called the critical mass flow rate ($\dot{m}_{cri}$). The pressure and the temperature at the throat, where the steam is choked, are called the critical pressure (P_{cri}or P*) and critical temperature (T_{cri}or T*). Normally, when flow meters are not available at hand to directly measure the mass flow rate or when it is difficult to do so, critical mass flow rate is computed by assuming that steam is an ideal gas, :

$$\dot{m}_{cri} = \rho^* \cdot C^* \cdot A_{throat} \qquad (1)$$

where ρ^* is the static density of steam at the throat where the steam's velocity is equal to sonic value (C^*), where:

$$\rho^* = \frac{P^*}{R \cdot T^*} \qquad (2)$$

and

$$C^* = \sqrt{k \cdot R \cdot T} \qquad (3)$$

where T^* and P^* are the static temperature and static pressure of the steam:

$$T^* = T_{boiler} - \frac{(C^*)^2}{2 \cdot C_p} \qquad (4)$$

$$P^* = P_{boiler} \cdot \left[\frac{T^*}{T_{boiler}}\right]^{\frac{k-1}{k}} \qquad (5)$$

Determination of critical mass flow rate is necessary when designing and modeling systems involving supersonic nozzle to achieve optimum performance and enhance the safety aspect of the system. For instance, one wrong decision in choosing a pressure relief valve can lead to dangerous overpressure circumstances.

However, the assumption of ideal gas may not be perfectly correct. When saturated steam is expanded isentropically in the nozzle, some vapour will condense. Therefore, the expanded steam becomes a two-phase fluid whose characteristics are far from that of an ideal gas. In this paper, steam is treated as a two-phase fluid. Previously published studies concerning two-phase critical flow, namely Levy [1], Milan & Vladimir [2], Fauske [3], Henry [4] and Lemonnier&Sel-mer-Olsen [5], all took complex parameters such as slip ratio and void fraction into consideration in order to calculate the two-phase critical mass flow rate. However, in most cases, these parameters are difficult to derive and obtain. Thus, a simplified mathematical model has been developed in this study. The critical mass flow rate is calculated by using the developed model and then compared with the experimental critical mass flow rate acquired by using water as the working fluid flowing in an ejector refrigeration system.

This study will not only be useful in designing and modeling systems composed of supersonic nozzles, but will also contribute to further understanding of related analyses on two-phase and critical flow behaviors.

2. Mathematical model

High temperature and high pressure saturated steam (may be a few degree Celsius superheated to ensure that the steam is dry) enters the nozzle with negligible speed (1). The steam is expanded (isentropic expansion 1 − 2′ or irreversible expansion 1 – 2) in a semielliptical profile converging duct. The steam is accelerated to sonic speed and choked at the throat of the nozzle (2). It is then further expanded and accelerated in the conical profile diverging duct to supersonic speed (not of concern in this analysis).

State properties of the inlet steam (1), nozzle's isentropic efficiency, nozzle's throat area and the steam's critical mass flow rate ($\dot{m}_{exp}$: obtained experimentally) are all required. Since supersonic nozzles always operate with choking condition, the critical mass flow rate and the state of steam at the throat are both independent of the condition downstream of the throat. The critical pressure (P_2) is first assumed and the properties of saturated vapour and saturated liquid are then defined. If the expansion process is isentropic, state of steam at the nozzle throat is:

$$s_{2'} = s_1 \qquad (6)$$

Since the steam is wet vapour, the dryness quality is calculated from:

$$x_{2'} = \frac{s_{2'} - s_{f@P_2}}{s_{fg@P_2}} \qquad (7)$$

The enthalpy is then obtained from:

$$h_{2'} = h_{f@P_2} + x_{2'} \cdot h_{fg@P_2} \qquad (8)$$

By assuming the nozzle's isentropic efficiency to be 95% as recommended by Huang et al. [6], the enthalpy (h_2) is:

$$\eta_{nozzle} = \frac{h_1 - h_2}{h_1 - h_{2'}} \qquad (9)$$

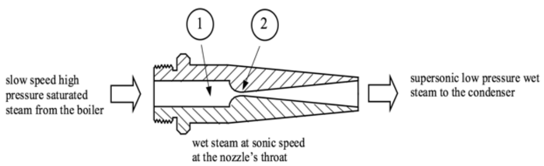
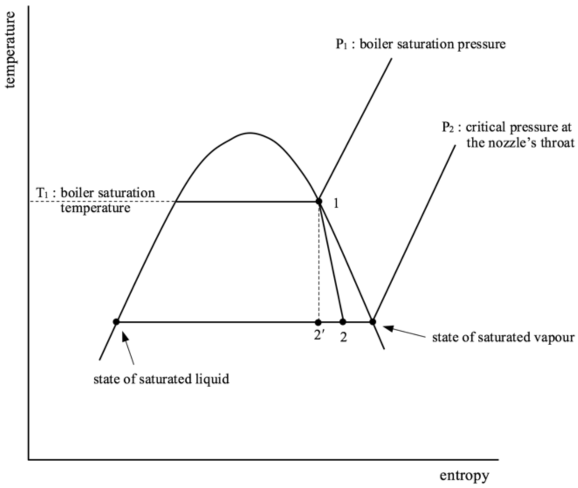


Fig 2: Nozzle's control volume and Mollier chart (Temperature – Entropy chart).



The velocity of steam at the throat is obtained from:

$$h_1+\frac{V_1^2}{2}=h_2+\frac{V_2^2}{2} \tag{10}$$

Since flow speed of the inlet steam is negligible, then:

$$\frac{V_2^2}{2}=h_1-h_2 \tag{11}$$

The dryness quality is:

$$x_2=\frac{h_2-h_{f@P_2}}{h_{fg@P_2}} \tag{12}$$

The specific volume of the steam is:

$$v_2=v_{f@P_2}+x_2\cdot v_{fg@P_2} \tag{13}$$

The nozzle cross-sectional area is:

$$A_2=\frac{\dot{m}_{cri}\cdot v_2}{V_2} \tag{14}$$

To compute for the critical mass flow rate ata fixed nozzle throat size, Eqs. (6) to (13) can be utilized and Eq. (14) is modified into:

$$\dot{m}_{cri}=\frac{A\cdot V_2}{v_2} \tag{15}$$

This calculated mass flow rate will then be compared to the experimental mass flow rate calculated from:

$$\dot{m}_{exp}=\frac{A_{boiler}\cdot z}{v_{f@P_1}\cdot t} \tag{16}$$

3. Experimental apparatus

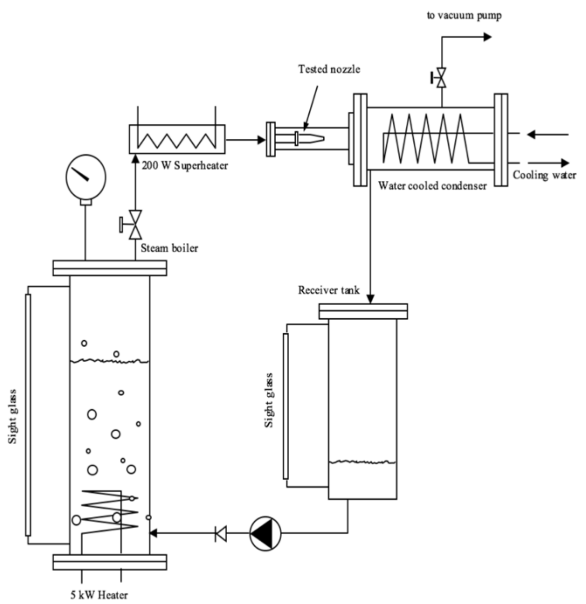


Fig 3: The experimental apparatus.

An experimental apparatus designed and constructed for testing the critical mass flow rate through various supersonic nozzles is shown in Figure 3. The nozzles are tested with boiler saturation temperature ranging from 110 to 155°C. Four nozzles with throat diameter of 1.7, 2.0, 2.4, and 2.8 mm are tested. During the test, the steam critical mass flow rate is obtained by switching off the feed pump and measuring the change of liquid level within the boiler vessel at its sight glass indicator after its valve is open. The valve is fully opened and pressure loss due to friction is assumed to be zero. The superheater is also used to superheat the entering steam by 5°C to ensure that the steam is perfectly dry.

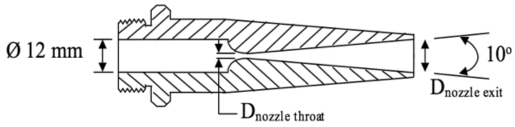


Fig4 : The tested nozzle.

The tested nozzles (Fig. 4), designed to produce the exit Mach number of 3.0, were originally used with a steam ejector. However in this study, the main focus is the critical mass flow rate, which is only dependent on throat size and boiler saturation temperature. Therefore, the conical converging duct has no effect on the test results.

4. Discussion of the results

Table 1 : Calculation results

Boiler's saturation temperature	130°C
Boiler's saturation pressure	270.28kPa
Nozzle throat diameter	2.8 mm
Nozzle throat area	6.16 mm^2
Critical mass flow rate ($\dot{m}_{exp}$)	8.748 kg/hr
Isentropic efficiency	95%

P_2 (kPa)	x_2	υ_2 (m^3/kg)	V_2 (m/sec)	M	A (mm^2)
270.28	1.000	0.668	0	0	∞
250	0.996	0.716	158.70	0.373	10.95956
200	0.984	0.871	316.00	0.743	6.70072
157	0.971	1.079	422.64	0.994	6.20321
156	***0.971***	***1.085***	***425.06***	***1.000***	***6.20286***
155	0.970	1.091	427.48	1.006	6.20293
150	0.969	1.123	439.57	1.034	6.20966
100	0.949	1.608	565.33	1.330	6.91282
50	0.919	2.978	722.93	1.701	10.01100

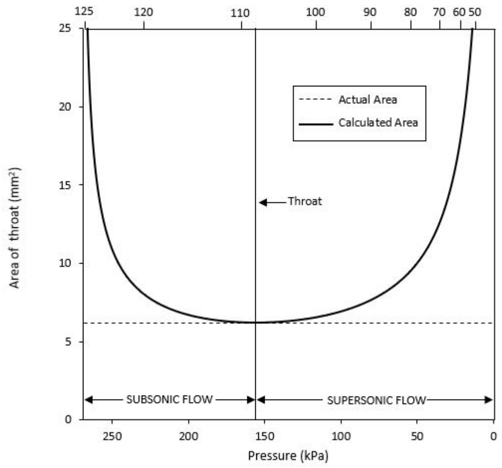


Fig 5: Plots of calculated and actual flow area against pressure and temperature

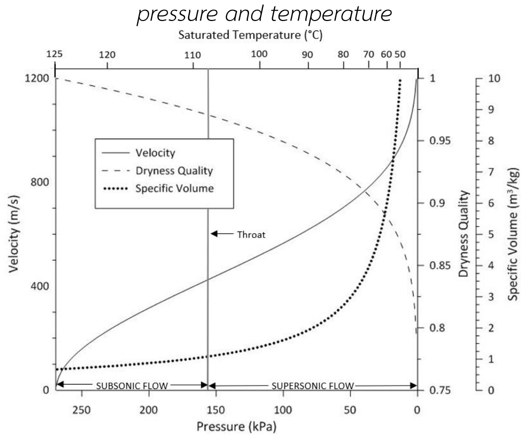


Fig 6: Plots of flow velocity, dryness quality, and specific volume against pressure.

When the steam is expanded through the supersonic nozzle, the static pressure of the steam decreases from the stagnation value (at the boiler or nozzle inlet) to the lowest value at the nozzle exit and the valve is fully opened and pressure loss due to friction is assumed to be zero..Table 1 shows the calculation results of the flow dryness quality, specific volume, velocity, and cross-sectional area of nozzle using Eqs. (6) to (14) by fixing the mass flow rate to be equivalent to the experimental value (8.748 kg/hr) and assuming that liquid droplets and vapours travel at the same velocity. The boiler saturation temperature of 130°C and saturation pressure of 270.28 kPa are used in these calculations as an example. As the pressure decreases along the nozzle, the calculated cross-sectional area of the nozzle also decreases until it reaches the minimum value, representing the throat section of the nozzle,at pressure of 156 kPa (cross-sectional area of 6.2 mm^2). The Mach number or the ratio of local velocity to the throat sonic velocity increases from 0 to 1 at the throat of the nozzle (at pressure equals to 156 kPa) and continues to increase downstream of the nozzle's throat, resulting in supersonic velocity of flow in the diverging section of the nozzle.

A graph plot of the calculated nozzle's cross-sectional area along the nozzle (from inlet to exit) is shown in Figure 5. Upstream of the nozzle's throat, the flow is in subsonic region and the nozzle profile demonstrates a converging duct. Downstream of the nozzle's throat, the flow is in supersonic region and the nozzle profile represents a diverging duct. At the nozzle's throat, the flow is choked and the flow speed is equal to local sonic value.

The velocity, dryness quality, and specific volume along the nozzle are depicted in Figure 6. The velocity at the inlet is very low and doesn't affect the calculation, so it can be negligible. The flow is accelerated from negligible velocity at the nozzle's inlet to sonic value at the nozzle's throat section, which corresponds to the point where the cross-sectional area is the lowest (pressure 156 kPa). Steam is then further accelerated along the nozzle to supersonic velocity in the diverging section of the nozzle. The specific volume displays a gradually increasing trend as steam further expands along the nozzle. The dryness quality of steam decreases along the nozzle to values lower than one, emphasizing the two-phase characteristic of the flow. This shows theinvalidity of employing ideal gas assumptions incritical mass flow rate calculations since the dryness quality is less than one.

For the experimental mass flow rate of 8.748 kg/hr, the nozzle's throat area, calculated using the developed mathematical model, is 6.20 mm^2.Upon comparison with the actual value of 6.16 mm^2,a difference of only 0.65 % is found.

This method is also used to calculate the critical mass flow rate through various nozzle sizes (throat diameter of 1.7, 2.0, 2.4, and 2.8 mm). The boiler saturation temperature ranges from 110 to 155°C. The calculation results are compared with the experimental values obtained from the testing apparatus developed.

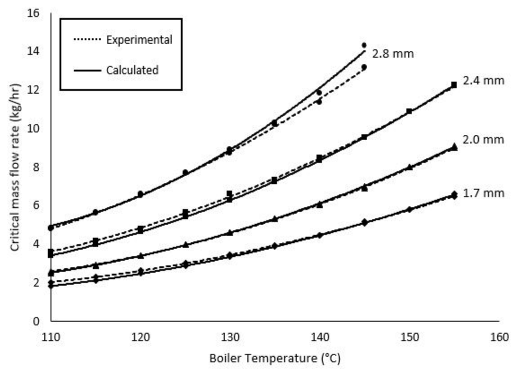


Fig 7: Comparison between calculated and experimental mass flow rate.

Figure 7 shows the comparison between the critical mass flow rate obtained experimentally and mathematically, using the proposed model in this study. The proposed model is validated through observing a good agreement between the calculated and experimental values, and all errors are found to be less than 10%.

5. Conclusion

Since the ideal gas assumption is not applicable for cases where the working fluid is a wet fluid such as steam, and where the existing two-phase model may prove to be too complex for simplified two-phase flow modeling, this paper's proposed model has been established to calculate the critical mass flow rate of two-phase flow through supersonic steam nozzles. Nozzles sizes of 1.7, 2.0, 2.4, and 2.8 mm and nozzle inlet temperatures ranging from 110-155℃ are employed in this model.

The effectiveness of the model was verified by comparing the calculated values with the experimental results of critical mass flow rate obtained from ejector refrigeration system. With the assumption of 95% nozzle isentropic efficiency, the results showed slight differences of less than 10% in every case, verifying the validity of this model.

The proposed model not only allows the determination of critical mass flow rate, but also velocity, dryness quality, specific volume, and cross-sectional area profile along the nozzles. When cross-sectional area of the nozzle is at its minimum, referred to as the nozzle throat, the sonic velocity is obtained. This model is suitable for designing and modeling system comprised of supersonic nozzles. However, in cases where more accurate results are required, the more-complex two-phase theories should be employed to compute for critical mass flow rate values.

6. References

[1] Levy S. (1964). Prediction of two-phase critical flow rate,Journal of Heat Transfer, vol.87(1), February 1965, pp. 53-57.
[2] Milan M.P. and Vladimir D.S. (2016). Two-component two-phase critical flow, FME Transaction, vol.44(2), pp. 109-114.
[3] Fauske H.K. (1962). Contribution to the theory of two-phase, one-component critical flow, ANL-6633, Argonne National Laboratory, 1962.
[4] Henry RE, Fauske H.K. (1971). The two-phase critical flow of one-component mixtures in nozzle, orifices, and short tubes, Journal of Heat Transfer, vol.93(2), May 1971, pp. 179-187
[5] Lemonnier, H. and Selmer-Olsen, S. (1992). Experimental investigation and physical modelling of two-phase two-component flow in a converging-diverging nozzle, International Journalof Multiphase Flow, vol.18(1), January 1992,pp. 1-20.
[6] Huang B.J., Chang J.M., Wang C.P., andPetrenko V.A. (1999). A 1-D analysis of ejector performance. International Journal of Refrigeration, vol.22(5), August 1999, pp. 354-364.

7. Nomenclature

Nomenclature

A	cross sectional area (m^2)
C*	speed of sound (m/s)
C_p	specific heat at constant pressure (J/kg K)
h	specific enthalpy (J/kg)
k	specific heat ratio
$\dot{m}$	mass flow rate (kg/hr)
M	Mach number
P	pressure (kPa)
R	gas constant (J/kg K)
s	specific entropy (J/kg K)
T	temperature (K)
t	time (s)
V	speed of steam (m/s)
x	dryness quality
z	height of water loss in boiler (m)

Greek symbols

η	isentropic efficiency
ρ	density (kg/m^3)
υ	specific volume (m^3/kg)

Subscripts

cri	critical
exp	experimental
f	saturated liquid
g	saturated vapor
1	condition at inlet
2	condition at throat in actual expansion
2'	condition at throat in isentropicexpansion