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Study of Aircraft Design Problem Using Multivariate Optimization

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Abstract

This paper presents more insight into the aircraft design problem through the use of the multivariate optimization method (MVO). A simple aircraft sizing and optimization code for civil transport aircraft, an MVO program, has been developed and employed for this study. The design of regional transport aircraft is utilized as the case study to investigate the interaction among the design disciplines, requirements, and constraints. Three wing mass estimation methods are also evaluated to study their influences on the aircraft configuration. The results show consistency of the wing methods on the aircraft configuration although there is disagreement on mass prediction.

Keywords: Aircraft Design, Multivariate Optimization

1. Introduction

Aircraft design is a multidisciplinary problem involving complex interaction of a large number of variables, constraints and analyses. Therefore, searching for the optimum design has beset all aircraft designers since, for a given set of requirements, there can be many feasible solutions. The multivariate optimization method (MVO) –the integration of a numerical optimization (optimizer) into the aircraft design synthesis– has been widely used to tackle the problem. The basic algorithm of the MVO is shown in Figure 1. Formulation of MVO problem begins with identifying: a set of design variables to be altered by the optimizer, the objective function to be minimized, and constraint functions to be satisfied. As MVO is capable of handling a large number of design variables and constraints, more details of the problem can be setup and details analyses can then be performed. The use of the numerical optimization provides automated, logical and non-bias design, and it is more efficient to locate the optimum design. Several sophisticated aircraft sizing and optimization codes have been developed and widely used in both industry and education [1-4]. Although extremely valuable, these codes are mostly complicated and difficult to comprehend owning to the involvement of considerable numbers of variables, constraints and sophisticated analyses. Therefore, to gain insight into the design problem, a simpler code which can provides a rapid means of understanding the multidisciplinary interaction is required.

The purpose of this study is to provide insight into the interaction among the design disciplines, requirements and constraints during the early design phase of the configuration feasibility study, with minimum effort. A simple MVO code for the design of civil transport aircraft has been developed and employed for the study. Within the synthesis part, three wing weight estimation methods are investigated to demonstrate their influences on the aircraft configuration.

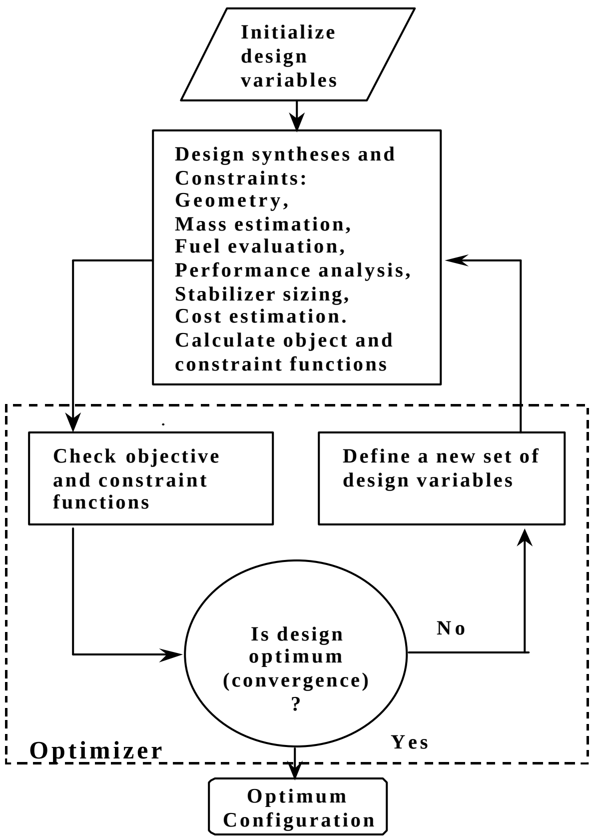


Figure 1 Basic algorithm of the MVO

2. Design Problem

A simple aircraft sizing and optimization program (MVO) for civil transport aircraft has been developed and employed for this study. The sizing part (aircraft synthesis) is mainly based on the empirical formula given in Howe [5], and LSGRG2C optimization program [6] is integrated as the optimizer. Three wing mass methods for transport aircraft were evaluated to study their influences on the design as follows:

1. Howe's formula [5]:

$$M_{wing} = \frac{C_1}{C_5}\left[A^{0.5} S^{1.5} \sec\Lambda_{1/4}\left(\frac{1+2l}{3+3l}\right) \frac{M_{to}}{S}\bar{N}^{0.3}\left(\frac{V_D}{t/c}\right)^{0.5}\right]^{0.9} \quad \text{(kg)} \qquad (1)$$

where C_1 is a coefficient depending on the type of aircraft, C_5 is the secondary lifting surface factor accounting for the tailplane and fin, A is the wing aspect ratio, S is the gross wing planform area (m^2), $L_{1/4}$ is 0.25 chord sweep (deg), λ is the taper ratio, M_{to} is the aircraft takeoff mass (kg), $\bar{N}$ is 1.65 times the limit load factor, V_D is the design maximum diving speed (m/s EAS), t/c is the thickness to chord ratio at the wing centerline, and M_{wing} is the wing mass (kg).

2. Nicolai's formula [7]:

$$W_{wing} = 0.00428 S^{0.48} \frac{A M_{sl}^{0.43}}{[100(t/c)_{max}]^{0.76}} \frac{(N_{ult} W_{to})^{0.84} l^{0.14}}{(\cos\Lambda_{1/4})^{1.54}} \quad \text{(lbs)} \qquad (2)$$

where W_{to} is the aircraft takeoff weight in lbs, S is the wing area in ft^2, N_{ult} is the design ultimate load factor or 1.5 times the limit load factor and M_{sl} is the maximum Mach number at sea level.

3. Raymer's formula [8]:

$$W_{wing} = 0.0051(W_{to}N_{uit})^{0.557} S^{0.649} A^{0.5} (t/c)_{max}^{-0.4} (1+l)^{0.1} (\cos\Lambda_{1/4})^{-0.1} S_{csw}^{-0.1} \quad \text{(lbs)} \qquad (3)$$

where S_{scw} is the control surface area in ft^2.

The regional transport aircraft having the specified mission as shown in Table 1 was used as the baseline aircraft for comparison and further tradeoff study.

Table 1 Mission requirements of the baseline aircraft

Items	Requirements
No. of passengers	80 (5 seat abreast)
Range (km)	2500
Takeoff field length (m)	≤ 1800, ISA sea level
Landing field length (m)	≤ 1800, ISA sea level
Approach speed	≤ 70 m/s
Cruise Mach number	0.8
Cruise Altitude (km)	11
Propulsion	2 engines, T/W_{eng} = 6 Bypass ratio = 5.5

As the fuselage dimension is mainly determined by the volume required to accommodate passengers, the main design process, therefore, is to size and optimize the wing and engines under constraints to meet the specified requirements. The design variables are selected as shown in Table 2, and the design constraints are listed in Table 3. The takeoff mass is chosen as the objective function to be minimized.

Table 2 Design variables

	Design variables
1.	Aspect ratio
2.	Quarter-chord sweep, $\Lambda_{1/4}$ (deg)
3.	Thickness-chord ratio, t/c
4.	Taper ratio
5.	Fuel mass fraction
6.	Takeoff thrust-weight ratio, T/W
7.	Takeoff wing loading, W/S (N/m^2)
8.	Thrust at sea level per engine (N)

Table 3 Design constraints

	Design constraints	Value
1.	Fuel volume remaining (m^3)	= 0
2.	Takeoff field length (m)	≤ 1800
3.	2nd segment climb gradient (%)	≥ 2.4
4.	Initial cruise rate of climb (m/s)	≥ 1.5
5.	Cruise CL buffet	≤ 0.65 $\cos\Lambda_{1/4}$
6.	Approach speed (m/s)	≤ 70
7.	Landing field length (m)	≤ 1800
8.	Sensitivity to turbulence	≥ 0
9.	Maximum wing span (m)	≤ 40
10.	SEP at diving speed (m/s)	≥ 0

3. Results and Discussions

The optimized values of the design variables and constraints of the baseline aircraft are shown in Table 4, and the aircraft characteristics according to the three wing methods are compared in Table 5. It can be seen that all wing methods provided almost the same aircraft dimension which also agrees well with the same class of existing regional aircraft. However, the Howe's method produced the largest wing mass and, hence, takeoff mass at all flight conditions as shown in Figure 2. As can be seen from Figure 2, all three wing methods give similar trend of takeoff mass variation with cruise conditions.

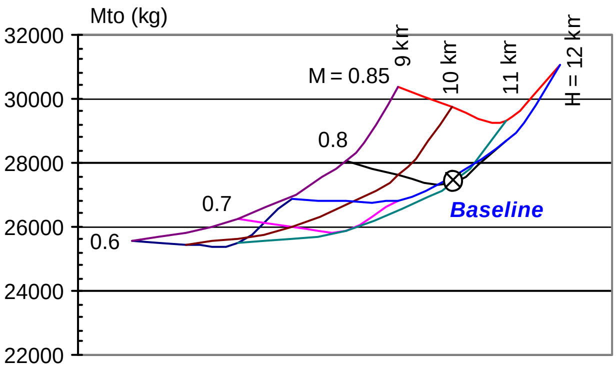
It was shown that the Nicolai's formula provided accurate wing weight prediction for large (long haul) transport aircraft [9]. However, it was suggested in Howe [5] that the wing formula for large aircraft would under predict the wing weight of medium and small aircraft; therefore, a correction factor for aircraft type was introduced. As can be seen from the above formulas, the Howe's method provides a correction factor C_1 for aircraft type (i.e. short/medium/long haul), whereas the others are applied universally for all transport aircraft type. Note that the Raymer's formula does not take account of the design diving speed (or Mach number) explicitly as the others. The design diving speed for a typical transport aircraft could vary from 180 to 210 m/s EAS for a typical cruise Mach number range of 0.7 to 0.85 [10]. This could contribute some error to the method.

Table 4 Optimized design variables of the baseline aircraft

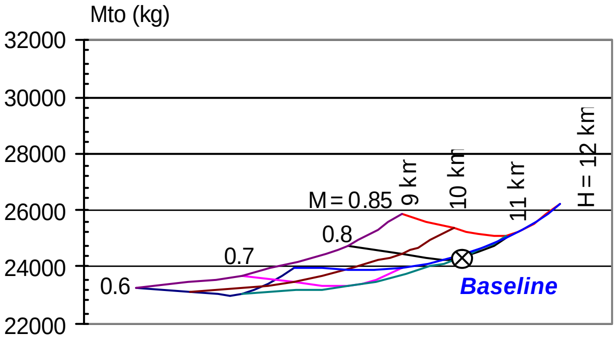
Design Variables	Value	Lower bound	Upper bound	*Howe*	*Nicolai*	*Raymer*
1. Aspect ratio	8	5	40	8.07	8.34	9.77
2. 1/4c sweep (deg)	30	0	50	12.55	17.15	16.07
3. t/c ratio	0.2	0.05	0.25	0.12	0.12	0.12
4. Taper ratio	0.4	0.25	1	0.25	0.25	0.25
5. Fuel mass fraction	0.3	0.1	0.5	0.16	0.16	0.15
6. T/W ratio	0.3	0.1	1	0.34	0.32	0.32
7. W/S (N/m^2)	5000	1000	10000	5930.31	5558.50	5448.27
8. Thrust per engine (N)	50000	10000	200000	45084.83	38572.09	38124.29
9. Cruise Mach number				0.8	0.8	0.8
10. Cruise altitude (m)				11000	11000	11000
Design Constraints [>= 0]						
1. Objective function Mto (kg)				***27398.05***	***24279.12***	***24499.05***
2. Mto matching [= 0]				0	0	0
3. Fuel matching [= 0]				0	0	0
4. Fuel volume available				0	0.02	0
5. Takeoff field length (m)				0	0	0
6. 2nd segment climb gradient (%)				0	0	0.01
7. Initial cruise rate of climb				2.30	1.89	2.26
8. Cruise CL buffet				0.07	0.09	0.10
9. Approach speed (m/s)				0.00	1.51	2.38
10. Landing field length (m)				0.14	0.17	0.19
11. Sensitivity to turbulence				0.36	0.33	0.25
12. Max. wing span (m)				20.88	21.10	19.25
13. SEP at cruise diving speed				0	0	0

Table 5 Comparison of the baseline aircraft characteristics

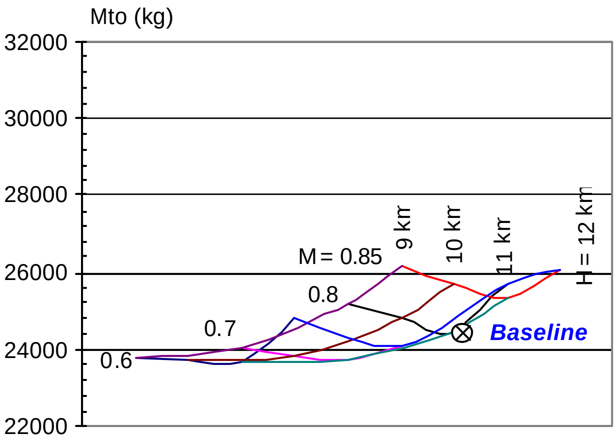
	Howe	*Nicolai*	*Raymer*		*Howe*	*Nicolai*	*Raymer*
Aircraft Dimension :				Landing gear	1219.21	1080.42	1090.21
Wing				Fuel	4284.12	3761.26	3677.79
Aspect ratio	8.07	8.34	9.77	Operating empty mass	15353.94	12757.86	13061.26
1/4c sweep (deg)	12.55	17.15	16.07	Zero fuel mass	23113.94	20517.86	20821.26
Wing area (m^2)	45.31	42.84	44.10	*Takeoff mass*	*27398.05*	*24279.12*	*24499.05*
Wing span (m)	19.12	18.90	20.76	OEM fraction	0.56	0.53	0.53
t/c ratio	0.12	0.12	0.12	Fuel mass fraction	0.16	0.16	0.15
Taper ratio	0.25	0.25	0.25				
Flap type	F/Slat	F/Slat	F/Slat	**Performance:**			
Fuselage				Takeoff field length (m)	1800	1800	1800
No. of seat abreast	5	5	5	Average cruise CL/CD	15.131	15.343	16.084
Fuselage width (m)	3.55	3.55	3.55	Approach speed (m/s)	70	68.495	67.624
height (m)	3.7	3.7	3.7	Landing field length (m)	1579.277	1539.49	1516.77
length (m)	28.58	28.58	28.58	Cruise speed (m/s)	236.056	236.056	236.056
Mass Breakdown (kg):				Vd max (m/s EAS)	205.778	205.778	–
Fuselage mass	4687.48	4687.48	4687.48	Cruise Mach number	0.8	0.8	0.8
Operational mass	1320	1320	1320	Cruise altitude (m)	11000	11000	11000
Payload mass	7760	7760	7760	CLmax clean wing	1.5	1.5	1.5
Wing mass	*2435.04*	*937.01*	*1179.66*	CLmax takeoff	2.538	2.484	2.498
Lifting surface	2922.05	1124.41	1415.60	CLmax landing	3.172	3.106	3.123
Propulsion	2191.40	1874.84	1853.08	T/W (takeoff)	0.336	0.324	0.317
System & Equip.	3013.79	2670.70	2694.90	W/S (N/m^2)	5930.306	5558.497	5448.268



a) Howe's wing mass



b) Nicolai's wing mass



c) Raymer's wing mass

Figure 2 Variation of takeoff mass with cruise conditions

The wing methods were then been evaluated with various aircraft types by the author. It was found that for large transport aircraft type, all three methods gave similar wing weight, within ± 10% difference. However, for medium haul aircraft, the Howe's method gave the result closer to existing aircraft than the others did. This was mainly due to the correction factor for aircraft type introduced in the method. Accordingly, only the Howe's formula had been further employed in this study.

The results in Figure 2 clearly show the impact of the cruise speed and altitude on the takeoff mass and, in fact, on the overall design. For a given cruise Mach number, the optimum cruise altitude is at 11 km where the engine and aerodynamic efficiencies are best compromised. For a given cruise altitude, reduction of takeoff mass can be gained by decreasing cruise speed. This, however, will increase the journey time, essentially for a long haul aircraft.

The variations of quarter-chord sweep and wing aspect ratio with cruise conditions are illustrated in Figures 3 and 4, respectively. It can be seen from Figure 3 that the wing sweep is employed at Mach number greater than 0.7 to avoid the drag rise at such high speed. The swept wing design can always be seen from all high speed aircraft. However, increasing sweep will increase the wing weight, as can be seen from the wing mass formulas. As shown in Figure 4, the aspect ratio reduces progressively as the cruise speed increases. At low speed associated with the unswept wing design, the aspect ratio can be set high to reduce the induced drag without significant increase in wing weight. However, at higher speed, the design diving speed also increases resulting in greater wing weight; and with combined effect of sweep angle the wing mass can increase substantially. Therefore, the aspect ratio was kept smaller to reduce the weight penalty.

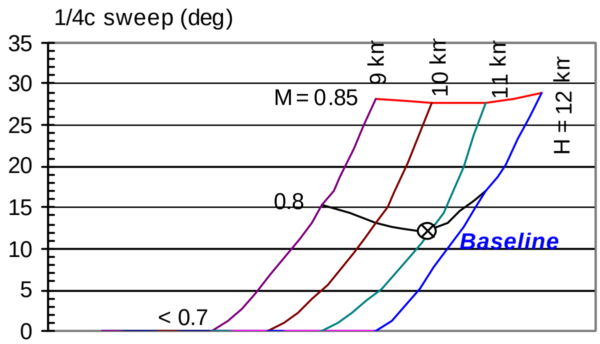


Figure 3 Variation of wing sweep with cruise conditions

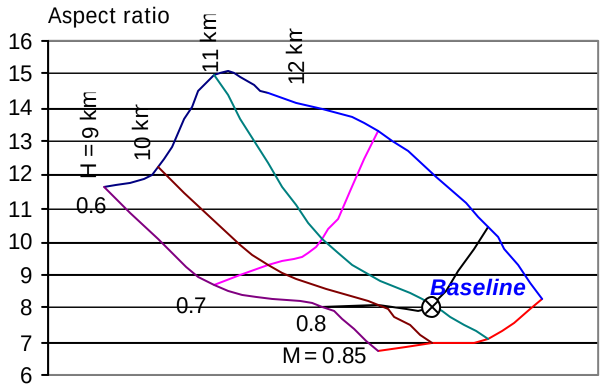


Figure 4 Variation of aspect ratio with cruise conditions

Figure 5 shows the variation of the required takeoff thrust with cruise altitude. For the altitude up to 11 km, the dynamic pressure, thus drag, reduces as the altitude increases, for a given cruise Mach number. Therefore, the thrust required to meet the specific excess power requirement (or maximum speed) is less even though the engine thrust available also reduces as the altitude increases. However, based on the engine thrust model in

Howe [5], at the altitude greater than 11 km the engine thrust reduces more rapidly, and the dynamic pressure, thus drag, reduces more slowly for a given cruise Mach number due to the constant speed of sound. Therefore, greater thrust is required.

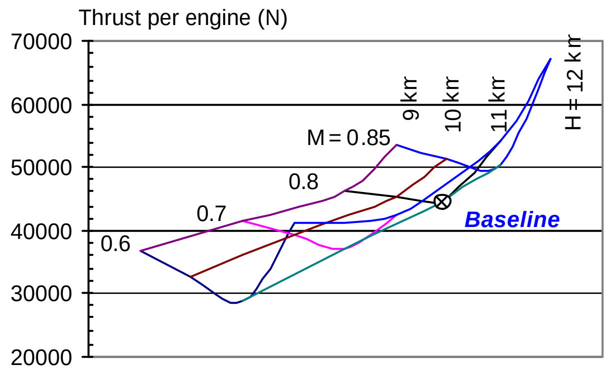


Figure 5 Variation of takeoff thrust with cruise conditions

The effect of the specified takeoff field length on the takeoff mass can be seen from Figure 6. It can be seen that increasing the available takeoff length to 2000 m results in a smaller engine required and, hence, takeoff mass reduction. However, further increase in takeoff length does not help to decrease the takeoff mass. This is due to the fact that the engine was sized from the 2nd segment climb constraint which does not relate to the takeoff length. It is interesting to note that designing the aircraft for the field length of 1800 m would provide access to 70% of major European airports, 1600 m accessible to 75 %, and 2000 m accessible to 60% [10].

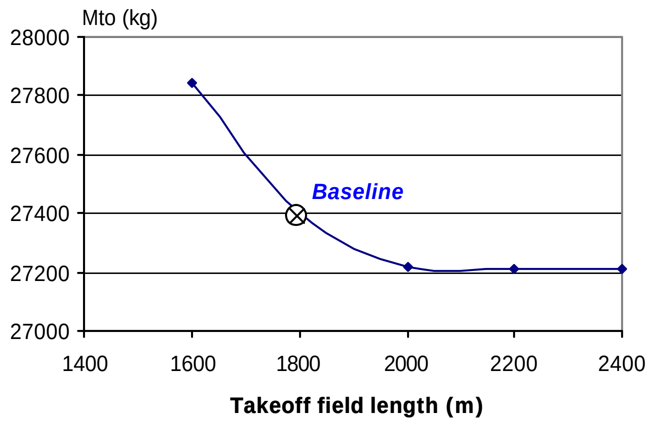


Figure 6 Effect of takeoff field length on the takeoff mass

The engine technology assessment is shown in Figure 7. With better engine technology through the increase of the engine thrust to weight ratio, the aircraft can be lighter and smaller.

The feasible design space of the baseline aircraft for a typical range of aircraft thrust to weight ratio (T/W) and wing loading (W/S) is shown in Figure 8. It can be seen that although it is not always necessary to select the highest feasible W/S and lowest T/W, the combination of high W/S and low T/W are normally required to obtain a lighter design.

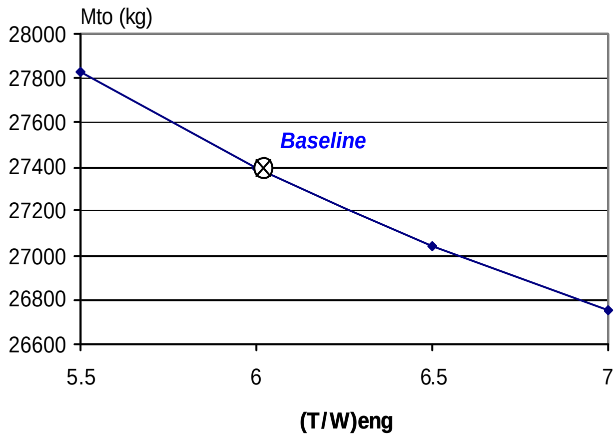


Figure 7 Effect of engine thrust to weigh ratio

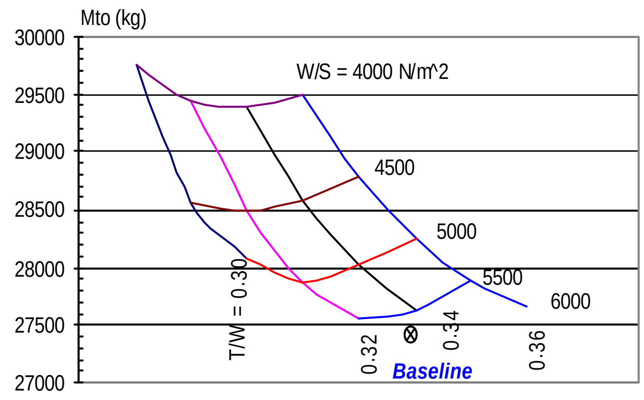


Figure 8 Effect of aircraft T/W and W/S on the takeoff mass of the baseline aircraft

4. Conclusions

More insight of the aircraft design problem is obtained through the use of a simple transport aircraft MVO program. Several tradeoff studies were performed to explore the feasible design space, and to study the interaction among the design disciplines, requirements and constraints. The ability of MVO to obtain the optimum design faster and more accurate would provide valuable data to the aircraft designers particularly during the early design stage in which the configuration feasibility study is performed. This would speed up the design process and reduce costly feedback later. Three wing mass models were evaluated, and the results showed consistency on the aircraft configuration although there was discrepancy on the wing mass prediction. It was found that the Howe's wing mass method would provide a more accurate prediction due to the introduction of the correction factor for aircraft type in the formula.

5. References

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