On-line area-based computation method for first-order plus dead-time model system identification from step response

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Abstract

The online computation algorithm is virtual need for implementation in modern digital controller. In this paper the new on-line area based computation algorithm for first-order plus dead-time model system identification drive from linear monotonic process step response is presented. The estimated parameter from this algorithm is properly fitted with the ideal model. Although there are large amounts of measurement noise presented, the parameter estimation error is fairy small, according to the inherent characteristics of the area-based method. The effectiveness has been proven through a large number of simulation tests.

1. Introduction

The process characteristics must be known before the appropriate controller tuning. So the system identification is performed when the analytical model is not available or not reliable. The estimate model of process can be obtained by using the experimental data. There are several testing methods, such as step, pulse, pseudo-random binary sequence, sinusoidal or relay feedback tests [1]. Among the several testing methods, the step test is the most widely accepted as the standard tool for process control engineer. The step testing can be easily implemented on microprocessor-based controller such as programmable logic controllers (PLC), distributed control systems (DCS), even the single loop controller.

The monotonic process that normally face in area of chemical process, water and waste and HVAC is sufficiently described the relation between response Y(s) and input U(s) by the parametric model of first-order plus dead-time models (FOPDT).

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$$Y(s) = \frac{K}{Ts + 1} e^{-Ls} U(s) \qquad (1)$$

The model has three characteristic parameters to find. The static gain (K) is normally determined by the steady-state level of the process output. The other two parameters, Astrom and Hagglund [1] proposed the graphical method to find the time constant (T) and the dead time (L) of the process. But the accuracy of this method is depending on the drawing of the line tangent to the process reaction curve at the point of maximum rate of change. To eliminate this dependency on the tangent line, Smith [2] proposed that the values of the L and T be selected such that the model and actual response coincide at two points in the region of high rate of change. The two points recommended are (L + T/3) and (L + T). That is the time at which the process output reaches 28% of K and 63% of K.

However all of graphical-based methods have to share the same drawback. It is quite sensitive to large measurement noise. The area-based methods may have better estimation robustness [1]. In this method the static gain K is obtained as before, while L and T are measured by using the area A_o and the average residence time T_{ar} shown in Fig. 1

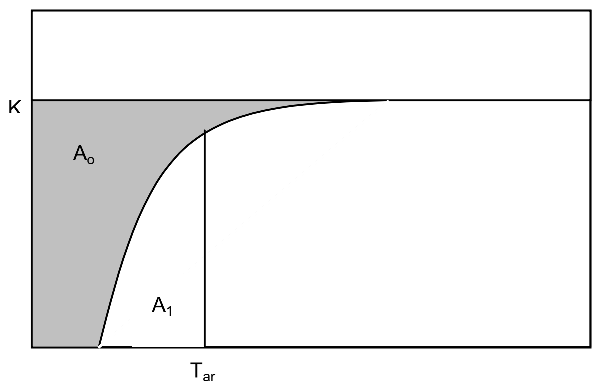

Fig. 1. Area method for a monotonic step response

The average residence time T_{ar} is computed from the area A_o in Fig. 1 as

$$T_{ar} = \frac{A_o}{K} = \frac{\int_0^{\infty} \left[y^{(\infty)} - y(t) \right] dt}{K} \qquad (2)$$

The area A_1 under the step response up to time T_{ar} is also measured. Then T and L can be estimated as

$$T = \frac{eA_1}{K} = \frac{e\int_0^{T_{ar}} y(t)dt}{K},$$
$$L = T_{ar} - T = \frac{A_0}{K} - \frac{eA_1}{K}. \qquad (3)$$

This method is attractive because of the inherent characteristics of area-based orient, that take the whole range of data to calculate the area under the process reaction curve. For this reason, this method is less sensitive to high-frequency noise than the previous graphical-based method, that takes only a few data point to estimate the model parameters.

Nonetheless, this method still have some disadvantage. From the above equations (2), (3) it is obvious that estimation accuracy is mainly dependent on the area A_o. In order to achieve an accurate A_o, the testing process must be wait until the response completely enter the new steady state.

To reduce the testing time. Qiang Bi et al. [5] proposed an instrumental variable least-squares for the area-based method. Using a new set of linear regression equations, it is not necessary to wait for process response to settle at new steady state value. As a consequence, the testing time is drastically reduced.

However the on-line computation algorithm have not been presented. Therefore, this paper presents a novel on-line algorithm which can be implemented on a digital computer and a microprocessor-based controller.

In section 2, off-line computation algorithm based on the area method drived from the linear regression equation with the instrumental variable least square (IV-LS) is revised. Section 3 presents the new on-line computation algorithm based on off-line IV-LS. In section 4, the effectiveness and the numerical stability of estimated parameters are shown by simulation results. Conclusions are given in section 5.

2. Area-based off-line computation method

Assuming that an example process is in zero initial steady state. The step input u(t) with amplitude of h is applied at t=0. The output response y(t) is recorded untill the process reach the new steady state condition. For a monotonic process described by equation (1), the response y(t) after $t \geq L$ is expressed by

$$y(t) = hK(1 - e^{-(t-L)/T}) + w(t) \quad for \quad t \geq L \qquad (4)$$

where w(t) is measurement noise, assumed to be white noise. Equation (4) can be rewritten as

$$e^{-(t-L)/T} = 1 - \frac{y(t)}{hK} + \frac{w(t)}{hk} \quad for \ t \geq L \qquad (5)$$

Integrating y(t) in equation (4) from t = 0 to t = τ (τ ≥ L)

$$\int_0^{\tau} y(t)dt = hK(t + e^{-(t-L)/T})\Big|_L^{\tau} + \int_0^{\tau} w(t)dt$$

Since y(L) = 0, equation (5) can be written as

$$\int_0^{\tau} y(t)dt = hk\left[\tau - L - T\frac{y(\tau)}{hk}\right] + [Tw(t)]\Big|_L^{\tau} + \int_0^{\tau} w(t)dt \qquad (6)$$

Let

$$A(\tau) = \int_0^{\tau} y(t)dt, \quad \delta(t) = [Tw(t)]\Big|_L^{\tau} + \int_0^{\tau} w(t)dt$$

Then, equation (4) can be written as

$$A(\tau) = hK\left[\tau - L - T\frac{y(\tau)}{hK}\right] + \delta(t)$$

or in matrix form

$$\left[h\tau - h - y(\tau)\right]\begin{bmatrix} K \\ LK \\ T \end{bmatrix} = A(\tau) - \delta(t) \quad for \ \tau \geq L \qquad (7)$$

At each sampling of y(τ) where τ ≥ L, the system of linear equation is formed as

$$\Psi\Theta = \Gamma + \Delta \quad for \ \tau \geq L \qquad (8)$$

where Θ = [K LK T]^T,

$$\Psi = \begin{bmatrix} hmT_s & -h & -y\left[mT_s\right] \\ h(m+1)T_s & -h & -y\left[(m+1)T_s\right] \\ \vdots & \vdots & \vdots \\ h(m+n)T_s & -h & -y\left[(m+n)T_s\right] \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} A\left[mT_s\right] \\ A\left[(m+1)T_s\right] \\ \vdots \\ A\left[(m+n)T_s\right] \end{bmatrix}, \quad \Delta = \begin{bmatrix} -\delta\left[mT_s\right] \\ -\delta\left[(m+1)T_s\right] \\ \vdots \\ -\delta\left[(m+n)T_s\right] \end{bmatrix}$$

where T_s is the sampling interval and $mT_s \geq L$.

The parameters in equation (8) can be estimate by the least squares method as

$$\hat{\Theta} = \left(\Psi^T\Psi\right)^{-1}\Psi^T\Gamma \tag{9}$$

Therefore, the K, T and L in equation (1) can be directly computed from equation (9). In the ideal situation with the absence of noise, equation (9) yield the true parameters. In the presence of a zero-mean uncorrelated noise (white noise) equation (9) still give good parameter estimation characteristics. However, if $\delta(\tau)$ is a zero-mean correlated noise instead of white noise, equation (9) is biased. To achive higher accurate parameter estimation, the instrumental variable least-square method is introduced. The following conditions are applied to the instrumental matrix Z.

First, the inverse of $\lim_{n\to\infty}(1/n)Z^T\Psi$ exists, and Second, $\lim_{n\to\infty}(1/n)Z^T\Delta = R_{Z\Delta} = 0$, i.e.,Z and Δ are uncorrelated. There are so many solutions of Z. In this case, Z is chosen as

$$Z = \begin{bmatrix} mT_s & -1 & \frac{1}{mT_s} \\ (m+1)T_s & -1 & \frac{1}{(m+1)T_s} \\ \vdots & \vdots & \vdots \\ (m+n)T_s & -1 & \frac{1}{(m+n)T_s} \end{bmatrix}$$

According to this instrumental matrix Z, the estimator of parameter in equation (8) is given by

$$\hat{\Theta} = \left(Z^T\Psi\right)^{-1}Z^T\Gamma \tag{10}$$

3. Area-based on-line computation method

3.1. Area based on-line least squares computation algorithm

With no loss of generality, T_s can be assumed as one unit of time. The on-line computation algorithm for equation (9) is as fellow [3].

$$\hat{\theta}(n+1) = \hat{\theta}(n) + \gamma(n+1)P(n)\psi(n+1)[a(n+1) - \psi^T(n+1)\hat{\theta}(n)] \tag{11}$$

$$P(n+1) = P(n) - \gamma(n+1)P(n)\psi(n+1)\psi^T(n+1)P(n) \tag{12}$$

$$\gamma(n+1) = \frac{1}{\left[1 + \psi^T(n+1)P(n)\psi(n+1)\right]} \tag{13}$$

Note that, lower case letter is for vector representation. Where. $\hat{\theta}(n+1)_{3x1}$ and $\hat{\theta}(n)_{3x1}$ is the estimated parameter column vector [K LK T]^T at time (n+1) and at time (n), $\psi(n+1)_{3x1}$ is the observation vector [h(m+n) -h -y[m+n]]^T at time (n+1). $a(n+1)_{1x1}$ is the area under the output response at time (n+1), and $P(n+1)_{3x3} = \left(\psi(n+1)^T\psi(n+1)\right)^{-1}{}_{3x3}$ It can be seen that *P(n)* is a direct measure of the error covariance at each n. To avoid the difficulty of matrix inversion ,the matrix inversion lemma is applied [3]. Resulting in equation (12). $\gamma(n+1)_{1x1}$ As shown in equation (11) is a scalar weight of the fitting error for the correction of next estimation.

3.2. Area-based on-line instrumental variable least squares computation algorithm

The instrumental variable method is useful not only for removing bias in the parameter estimation when the residual in the system equation is autocorrelated but also yields consistent estimation.

An on-line computation algorithm for equation (10) is as fellow [3].

$$\hat{\theta}_{IV}(n+1) = \hat{\theta}_{IV}(n) + \gamma(n+1)P(n)\bar{z}(n+1)\cdot[a(n+1) - \psi^T(n+1)\hat{\theta}_{IV}(n)] \tag{14}$$

$$P(n+1) = P(n) - \gamma(n+1)P(n)\bar{z}(n+1)\psi^T(n+1)P(n) \tag{15}$$

$$\gamma(n+1) = \frac{1}{\left[1 + \psi^T(n+1)P(n)\bar{z}(n+1)\right]} \tag{16}$$

where,

$\hat{\theta}_{IV}(n+1)_{3x1}$ and $\hat{\theta}_{IV}(n)_{3x1}$ are the estimated parameter column vector $[K \quad LK \quad T]^T$ estimated by the IV-LS method at time (n+1) and at time (n). $\bar{z}(n+1)_{3x1}$ is the instrumental vector

$$\left[(m+n+1) \quad -1 \quad \frac{1}{(m+n+1)} \right]^T$$

at time (n+1). Other variables can be found from equation (9).

It is obvious that the different choices of Z yield the different efficiency of estimation. Nevertheless, in any case of Z, the efficiency is always inferior to the generalized least squares method [2]. However, for the simple implementation on a microprocessor- based controller, the IV-LS is chosen.

4. Implementation and simulation testing

4.1. Listening period

The equation (7) is effective where $\tau \geq L$[5]. This means that the computation must begin after $\tau = L$. That can be know by monitoring the measured signal and calculating noise band B_n at the zero initial steady state. Before the controller issues the step signal to actuator. After that the step signal is fed to actuator. When $y(\tau)$ satisfies the condition $abs(y(\tau)) > 2B_n$, the computation is begun.

4.2. Initial value of $\hat{\theta}$

It is recommended by Hsia[3], that $\hat{\theta}(0)$ is arbitrarily and $P(0) = \alpha I$ where α is a very large positive scalar and I is an identity matrix. This condition will force the $\hat{\theta}$ to coincide with the same result from the off-line method with the degree of freedom equal to the unknown parameter. This initial value gives the best first guess. And to improve the guessing value of initial condition, the equation (7) and the time that controller starts to log-on the measured signal L_{guess}, can be used as the initial conditions. Then the initial values are

$[K_{(arbitrarily)} \quad L_{guess}K_{(arbitrarily)} \quad T_{(arbitrarily)}]^T$

4.3 Asymptotic properties of recursive identification methods

The asymptotic properties of recursive on-line algorithms are the same as those of corresponding off-line method [4]. This means that as long as the conditions in equation (9) and (10) are hold, the on-line algorithms will give the same result as the off-line method, when the iteration is approach to infinity.

4.4 Filtering

It is a common practice to band-pass filter the signal before introducing them into the algorithm to get rid of static levels and high frequency disturbances [1].

4.5 Simulation test

Supposing that the parameters of the first-order plus dead-time models in equation (1) are K = 4.2, L = 60, T = 360. Or in matrix form $[4.2 \ (60)4.2 \ 360]^T$, the ideal model is

$$Y(s) = \frac{4.2}{360s+1} e^{-60s} U(s)$$

The noise is modeled as a white noise with variance one scaled by 0.2, which makes the noise-to-signal ratio (NSR) of 30% approximately.The outcome of noise generator is ploted in Figure 2.

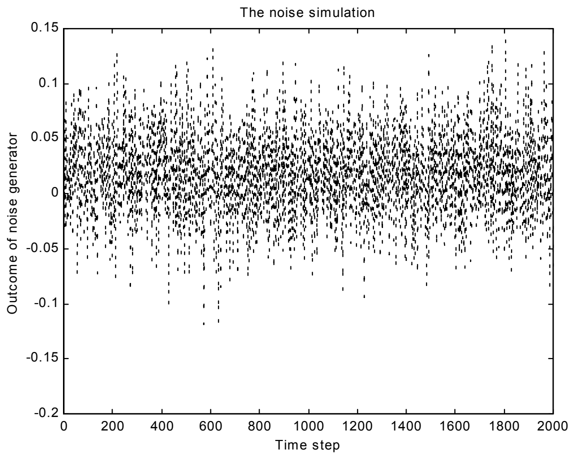

Fig. 2. Noise outcome from noise generator.

A noisy response is generated by superimposeing the noise to noise-free response from the ideal model. As shown below in Figure 3. This noisy response is the input to test the developed algorithm.

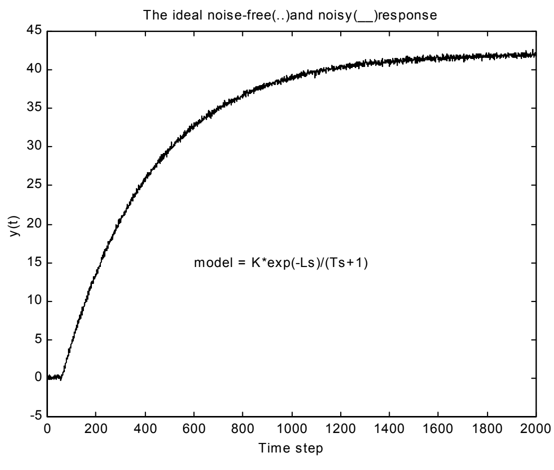

Fig. 3. Noisy response for testing

The area under the noisy response is shown in Figure 4. It should be noted that, the result of on-line integration of noisy response is nearly equal to the analytical calculation from the ideal model.

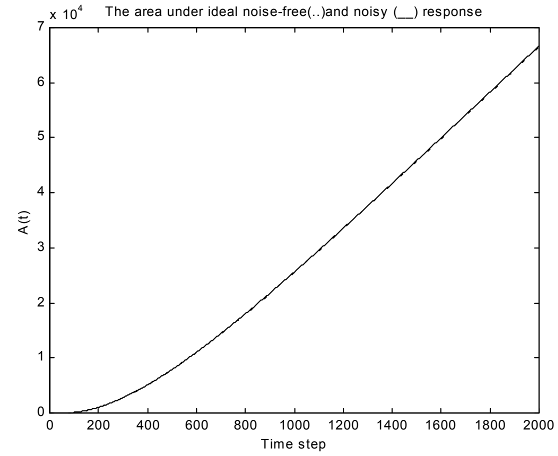

Fig.4.The area under response

The estimator for (K) static process gain , (KL) static process gain multiply by dead-time and (T) time-constant, can be found at the steady state of estimator value as shown in the Figure 5, 6, 7 respectively.

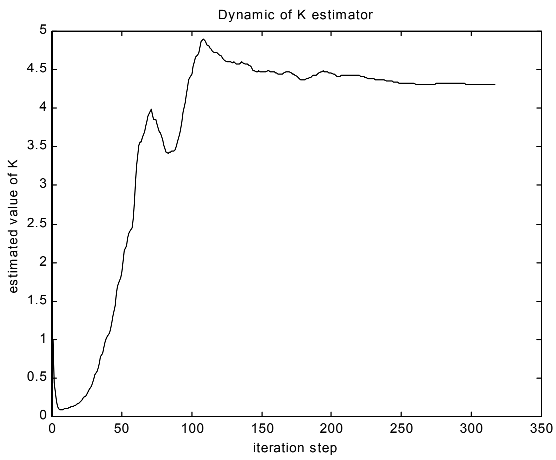

Fig.5. Dynamic of K estimator

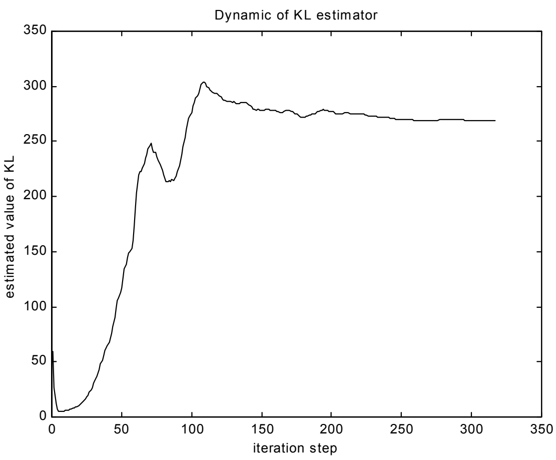

Fig.6. Dynamic of KL estimator

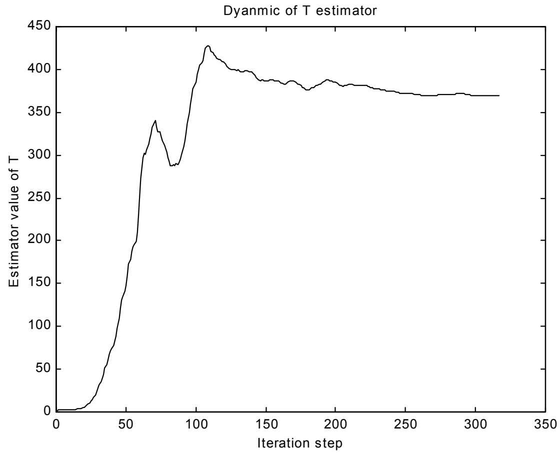

Fig.7. Dynamic of T estimator

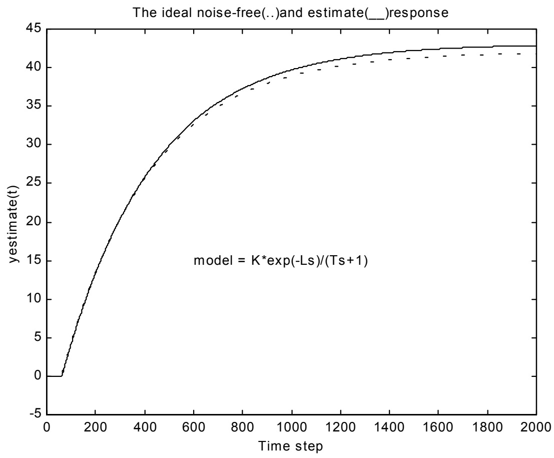

Fig.8. Comparison of the estimated with the ideal model

By using the estimated parametor K = 4.3095, T = 369.5156, L = 62.3717. It can be shown that the responses of estimated model is almostly identical with the ideal model as shown are in Figure 8.

This very good estimated parametor is found by using only 317 iterations that is 75% less than the graphical method.

4.6. Algorithm reliability

To prove the reliability and the consistency of the algorithm, a great number of test is performed. After running more than 1000 test. For this system, the statistical characteristics of the computed result are shown in tables 1.

Table 1.Test run statistical characteristics.

For $[4.2\ \ 60(4.2)\ \ \ 360\ ]^T$.

Paramerters\Stat.Cha.	Mean	SD.
Iteration	282.3510	80.1066
Static gain (K)	4.3114	0.1922
Dead time (L)	62.53	0.3032
Time constant (T)	368.2306	19.5726

The next two different FOPDT systems are tested.

Table 2.Test run statistical characteristics.

For $[2.2 \quad 120(2.2) \quad 1000]^T$.

Paramerters\Stat.Cha.	Mean	SD.
Iteration	524.3500	151.1365
Static gain (K)	2.1292	0.2455
Dead time (L)	119.3606	1.7775
Time constant (T)	953.4169	123.4733

Table 3.Test run statistical characteristics.

For $[6.2 \quad 80(6.2) \quad 700]^T$.

Paramerters\Stat.Cha.	Mean	SD.
Iteration	346.262	95.9435
Static gain (K)	6.2373	0.3425
Dead time (L)	82.5620	0.3646
Time constant (T)	700.6962	42.9921

All test are converge to final steady state. The estimated value are good fix with the ideal model and with remarkable consistency throughout all test.

5. Conclusions

A new on-line IV-LS computation algorithm for FOPDT was developed. The main advantage of the appraoch are reduced the step testing time and on-line computation at the acceptable reliability. Because of the non-complex calculation, the method is available for a embeded hardware. Moreover, the initial condition were clearly expressed. The consistency and reliability were demonstrated by a great number of re-run simulations.

6. References

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