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Some analytical methods of plastic collapse of circular steel tube under quasi-static axial compression

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Abstract
Circular tubes many have been widely used as structural members in engineering applications. Therefore, its' collapse behavior has been studied for many decades, focusing on its energy absorption characteristics. This paper is also aimed to investigate the plastic collapse of circular tube subjected to axial compression. The study was carried out with experiments, analyticals and computer simulations. The experiment was conducted with a number of tubes having various D/t ratios. Theoretical analysis of the collapse was made using hinges line method. The problem was also investigated by computer simulation technique, using a commercial FE package (ABAQUS). Results from those three parts were compared.

Keywords: Plastic collapse, Circular Tube, Thin-walled, Collapse mechanism

Notation

R	initial (undeformed) tube radius
t	tube thickness
l	length of tube section
W_{ext}	work of external forces
W_{int}	work of internal forces
W_B	internal work in bending at the circumferential plastic hinge
W_s	internal work for stretching plastic hinge
α	angle of rotation of plastic hinge
δ	vertical displacement of tube
H	fold length
H_m	mean fold length
P	force applied to the tube
P_m	mean crushing load
M_o	fully plastic bending moment per unit length of tube
M_p	bending of plastic hinge
σ_o	yield stress of the material
P_y	yield load

1. Introduction

Circular tubs are widely used as structural members in offshore pipeline and platforms, land-based pipelines, support structure and energy absorbing devices. Many researches investigated the experiment and theoretical of circular tube subjected to static axial compression. Alexander [1] analysed the axisymmetric concertina mode (see Fig. 1(a)) of deformation by considering the formation of stationary hinges and assuming the tube length between the hinges as rigid. The region between the extreme hinges was assumed to buckle outward only. He obtained an expression for the fold length by minimizing the total energy due to the membrane strain and the plastic bending moments at the hinges. The investigation also proposed an equation for predicting the mean collapse load. Abramowicz and Jones [2] later modified Alexander's model by introducing curvature in the deformation fold length, Fig 1 (b). They used ultimate stress instead of yield stress to account the strain hardening. Only the calculation of the collapsed load was addressed in their analysis. Wierzbricki and Bhat [3] employed a moving hinge mechanism starting from each end of the fold length. Grzwbieta and Murray [4] proposed a method to determine the load history between a peak and a minimum during an oscillation of the load compression curve. They assumed that the two curve regions are separated by a straight region where each region is one-third of the fold leg length, see Fig.1(c). In all these studies the fold formation was assumed to be outside the mean radius of tube. Wiezbicki et.al. [5] studied the axisymmetric mode of deformation of round tubes by considering inward and outside folding radial displacement (Fig. 1(d)) according to the amount of internal folding. The loads – displacement history as well as the mean load were analysed. Singace et al. [6] analysed and developed Wierzbicki's model [5].They introduced the derivation and discussion of the eccentricity factor of tube folding in concertina mode. The analysis agreed with the experimental observations.

In general two modes of deformation may be observed when circular tubes collapse, which include axisymmetric (also called concertina modes), see [Fig.1 (a)] and non-symmetric (diamond mode). Andrew et al. [7] show that thick cylinders (small D/t ratio, D/t<80-90) buckle in the concertina mode of deformation, whereas thin cylinders (high D/t ratio) buckle in the diamond modes. The number of lobes increases with increasing of

D/t ratio. For a given tube, it was found that the absorbed energy is more important in the mode concertina than that in the diamond mode.

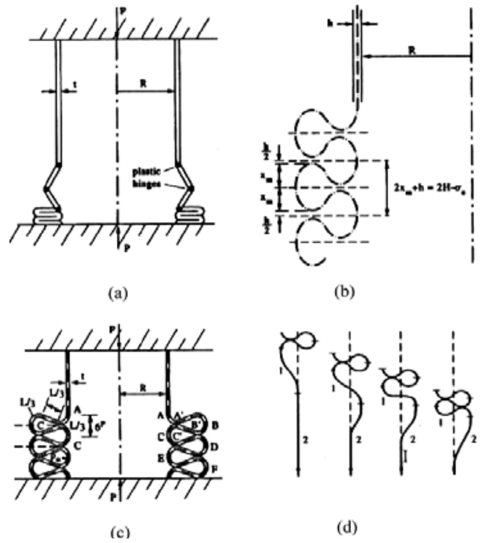


Figure 1. The plastic collapse mechanism of tube for concertina mode [1]-[4].

In this paper an experimental and analytical study on the axial compression of circular steel tubes of varying D/t ratios under quasi – static loading are presented. This study focuses only on tubes deforming into concertina folds. The buckle was assumed to bend outward only. The elastic effect was neglected because the extensive plastic deformation and the plastic energy dissipated in the structure larger than three times the elastic energy of deformation [8]. The deformation process was simulated by using a commercial finite element code (ABAQUS). Results are then compared.

2. Experiments set up

In this study, there were eighteen tubes of mild steel (yield tensile stress: 260 MPa) of different diameter to thickness ratios. The nominal diameter to thickness ratios were 25.5 from 63.73 as shown in Table1. The length of each specimen was 150 mm. The end condition of the tubes was simply supported. The experiments were conducted on ESH Universal Testing Machine of 200 ton capacity. The specimen was crushed axially with speed of 10 mm/min. The force and corresponding displacement during tube crushing were recorded. The experimental setup is shown in Fig.2.

Table 1 Specimen dimension

Specimen label	Average diameter D(mm)	Thickness t(mm)	D/t
UB1	45.92	1.79	25.6
UB2	57.42	1.82	31.55
UB3	73.40	1.73	42.43
UB4	85.80	1.72	49.79
UB5	109.62	1.73	63.36
UB6	110.75	2.26	49

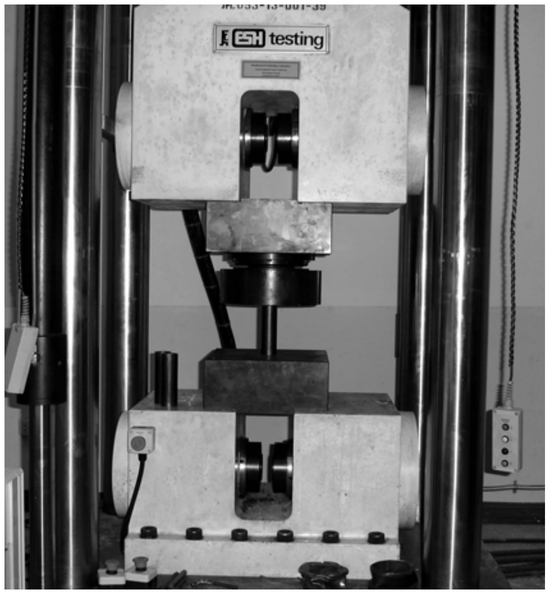


Figure 2. The test setup of circular tube by using the ESH Universal Testing Machine.

3. Analytical model

The folding mechanism of circular tubes was analyzed using hinge line method and followed the Alexander mode [1]. Mode of collapse mechanism was assumed axisymmetric (also called concertina type) folding. In this paper, the folding legs of tube were assumed to buckle outward only, as shown in fig 3.

The folding is facilitated by a kinematics mechanism with three circumferential plastic hinges. Plastic deformation is due to bending at the circumferential plastic hinges at A, B and C and hoop stretching of the regions between AB and BC, as show in fig. 3.

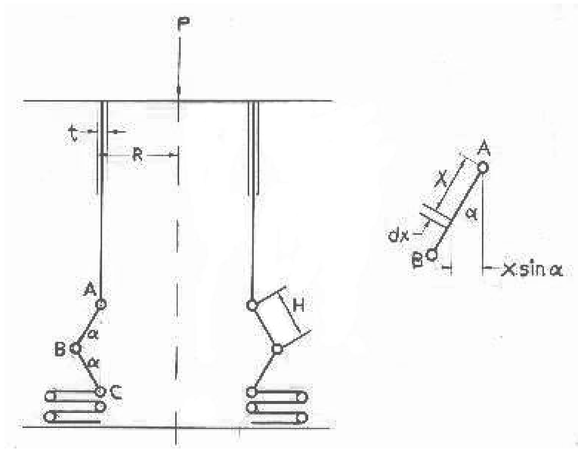


Figure 3. The axisymmetic folding of tube under axial compression [1].

Let the geometry of the fold at any instant is defined by the angle α. Axial shortening (of an initial height 2H) at the instant define by δ, is

$$\delta = 2H(1-\cos\alpha) \qquad (1)$$

Axial velocity of A is

$$V = \dot{\delta} = 2H \sin \alpha \dot{\alpha} \quad (2)$$

Global energy balance of the load structure can be established by equating the external rate of work ($\dot{W}_{ext}$) with internal energy dissipation ($\dot{W}_{\text{int}}$), hence

$$\dot{W}_{ext} = \dot{W}_{\text{int}} \quad (3)$$

The rate of work of external forces $\dot{W}_{ext}$ can be expressed as:

$$\dot{W}_{ext} = P\dot{\delta} = P_m.2H \quad (4)$$

Where P_m is the mean crushing load and if we assume that the expression for P_m contains the fold length $2H$. The rate of internal work of tube ($\dot{W}_{\text{int}}$) is comprised of two parts. $\dot{W}_B$ is the rate of work for bending at the circumferential plastic hinges at A, B and C and $\dot{W}_s$ is the rate of work for stretching in AB and BC, so that;

$$\dot{W}_{\text{int}} = \dot{W}_B + \dot{W}_s \quad (5)$$

Plastic bending at A, B and C: the rate at which energy is dissipated at the three hinges is

$$\dot{W}_B = (M_{PA} + 2M_{PB} + M_{PC})\dot{\alpha} \quad (6)$$

Denoting the fully plastic unit bending moment by $M_0 = \frac{\sigma_0 t^2}{4}$, note that $M_{PA} = M_0(2\pi R_A)$ etc. Noting also that $R_A = R_c = R$ and that $R_B = R + H \sin \alpha$. Equation (6) can be rewritten as

$$\dot{W}_B = 2\int_0^{\pi/2} 2\pi R M_0 d\alpha + 2\int_0^{\pi/2} 2\pi M_0 (R + H \sin \alpha) d\alpha$$

$$= 4\pi M_0 (\pi R + H) \quad (7)$$

Stretching in AB and BC: the expression of various sections is differential. A small length dx at a distance x from A (fig. 3) is at a radius $R + x \sin \alpha$ and has a strain $\varepsilon = \frac{x \sin \alpha}{R}$. Rate of work done for the plastic straining of the element dx at a deformation rate $\dot{\varepsilon}$ is

$$d\dot{W}_s = \sigma_0 \dot{\varepsilon}(dVolume) = \sigma_0 \dot{\varepsilon}(2\pi Rtdx) \quad (8)$$

Note that the incremental strain compare with α is

$$\dot{\varepsilon} = \frac{x \cos \alpha \dot{\alpha}}{R}$$

The total rate of work for stretching of AB and BC in this position is

$$\dot{W}_s = 2\int_0^H 2\pi Rt(dx)\sigma_0 . \frac{x \cos \alpha \dot{\alpha}}{R} \quad (9)$$

The total energy required for plastic expansion of ABC (x changes from 0 to H and α changes from 0 to $\pi/2$) is obtained by integration as

$$W_s = 8\pi M_0 (\frac{H^2}{t}) \quad (10)$$

Using equations (2) through (5), it can be seen that

$$P_m = 2\pi M_0 \left(\frac{\pi R}{H} + 1 + \frac{2H}{t} \right) \quad (11)$$

Mathematically, we get H from $\frac{dP_m}{dH} = 0$. This operation on equation (6), produces $H = H_m$

$$H_m^2 = \frac{\pi Rt}{2} \quad (12)$$

Using equations (5), (6) and (9), an expression for P is obtained as

$$P = \frac{4\pi M_0}{H \sin \alpha} \left(R + \frac{H}{2} + \frac{H^2}{t} \cos \alpha \right) \quad (13)$$

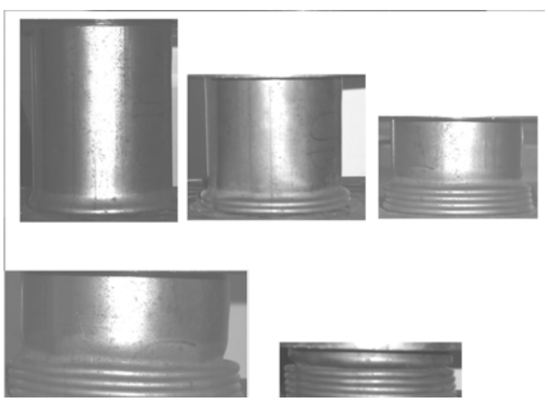
The author will use equation (13) to analyze the force – displacement response and compare with the results of the experiment and computer simulation.

4. Computer simulation

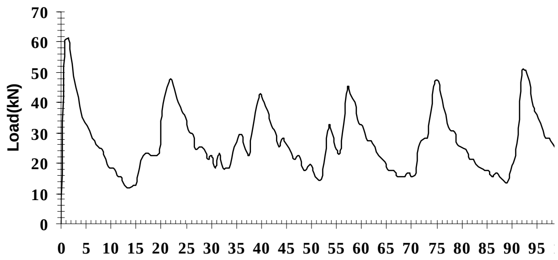
The collapsed behavior of circular tubes was also numerically analyzed by a commercial finite element package (ABAQUS). The tube model was constructed with a number of 4 node shell elements. All geometry parameters were same as experimental specimens. The tube was crushed by two rigid platens in axial direction. The material was assumed homogeneous, isotropic, constant thickness and perfectly elastic-plastic. Coefficient of friction between platens and tubes surfaces was assume as 0.3 in order to prevent sliding at the end. The self contact of the inner and outer surfaces of the shell were assumed frictionless. Their deformation shapes were recorded at different stages of compression and load-displacement response were plotted and compared with the experiments and analyticals.

5. Results and Discussions

This section presents the results of collapse behaviors of circular tubes for axisymmetric concertina mode.



(a)

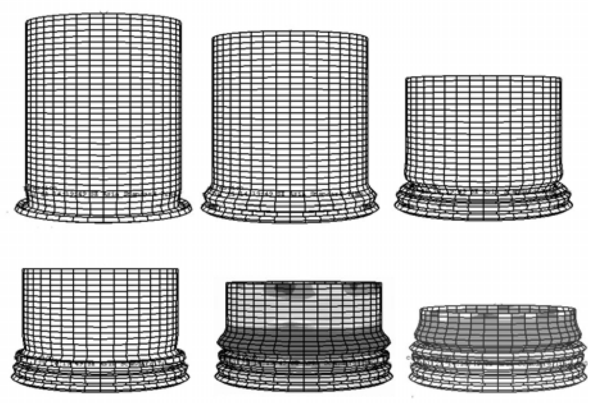


(b)

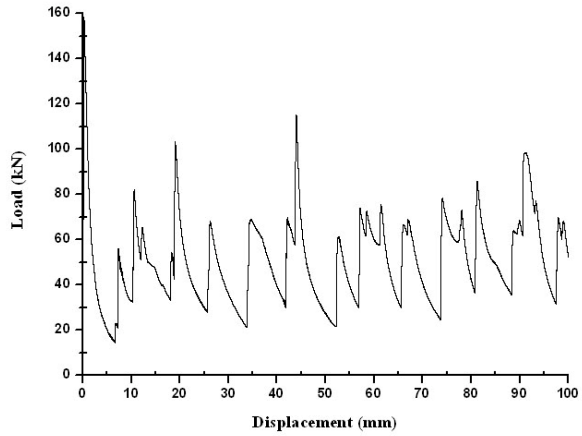
Figure 4. (a) Deformation process of a typical tube under axial compression, (b) load – displacement response of specimen No.UB5 at D/t =63.36

Fig.4 (a) demonstrates the deformation process of a typical tube (No. UB5) of mean diameter (D)= 109.62 mm, thickness(t)=1.73 mm and initial height = 149.89 mm. Its' compression load – displacement response is shown in fig.4 (b). The deformed pattern of tube is called the axisymetric concertina mode. The collapse starts when a radial folding begins at bottom end surface. In the elastic region, the load increases continuously. The next stage, plastic deformation is dominant with increase of compression load and the folding is continuously increased. Since this region is reached the maximum compression load, the folding continues at decreasing load due to the lower rate of resistive moment of strain hardening. The final stage, the compression load start increasing again, due to the work - hardening and the load is reached the second maxima. The further folding is continuously progressed, which the number of experimental folding is corresponded with the number of loop of load – displacement response as shown in fig. 4(b). The final folding stage is composed of five wrinkles at a downward distant 100 mm and theirs folding are the five loops of load–displacement response simultaneously.

An example result from finite element simulation is shown in fig. 5.



(a)



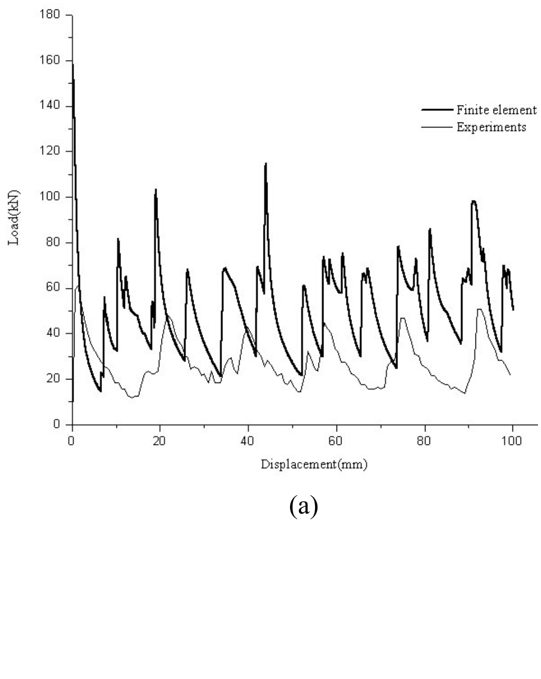
(b)

Figure 5. Results from finite element simulation (a) computed profile at different stages of collapse, (b) Finite element model load – displacement response of specimen No.UB5

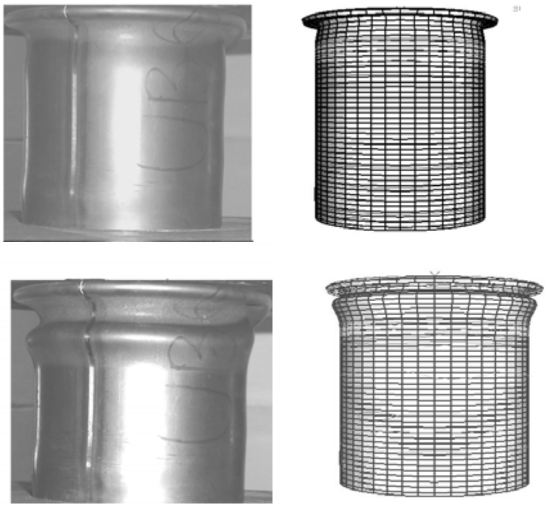
Fig. 5(a) demonstrates the axisymmetric concertina mode of tube number UB.5 received from finite element simulation. The finite elements collapse process is similar to the experimental observation in Fig. 4(a). Another comparison between finite element and experimental observations is also presented in fig. 6(b) for a specimen number UB.6. This figure provides clear picture of initiation and progressive collapse in the first and second stages of ring formation.

It is observed that the collapse begins by creating an expansion ring near either top or bottom end of tube. After the first ring completed, the following ring is formed. The process repeats in this manner continuously until the collapse terminated.

Fig. 6(a) shows the comparison between load-displacement obtained from finite element simulation and experiment of specimen number UB.5. The discussion on this graph will be present later.



(a)



(b)

Figure 6 Comparison of experiment and finite element simulation (a) the load – displacement response for circular steel tube, No. UB5, (b) the deformation profile for differential stage of specimen No. UB6 having D/t = 49

The theoretical analysis of the plastic collapse of circular tube under axial compression load is given from equation (13). After substituting all parameter, the load – displacement curve can be obtained as shown in Fig. 7 as an example

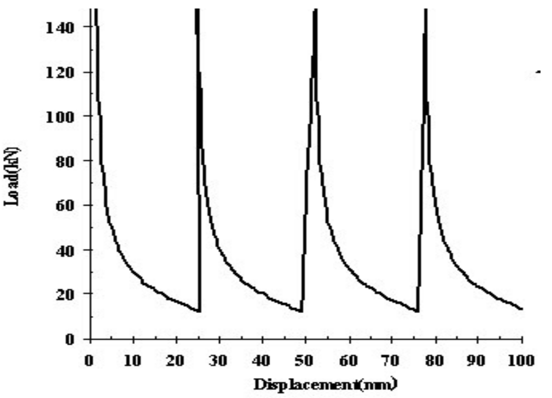


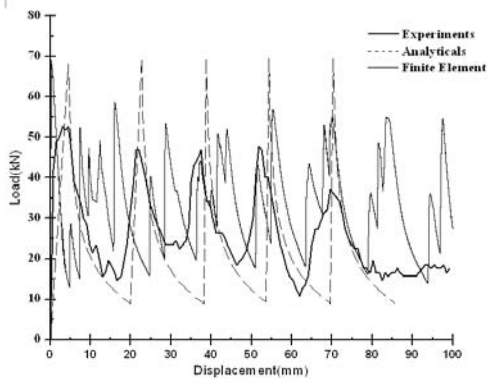
Figure 7. The theoretical analysis of load – displacement response of specimen No. UB5 having D/t = 63.36.

. At the onset of the buckle, when $\alpha = 0$, equation (13) gives the value of load as infinity. This is not true since the expression was derived based on plastic region only. Therefore this is equation cannot apply in the elastic zone. However there was a suggestion to use to approximate for the load in elastic zone as.

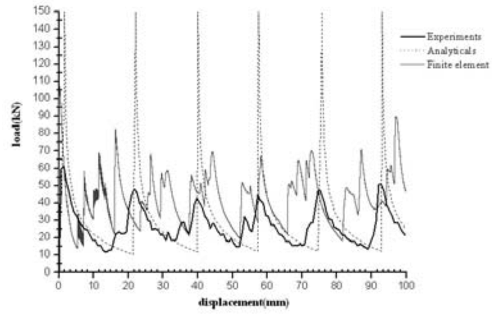
$$Py = \sigma_0 (2\pi Rt) \qquad (14)$$

So, the further analytical load-displacement curves will use equation (13) together with equation (14).

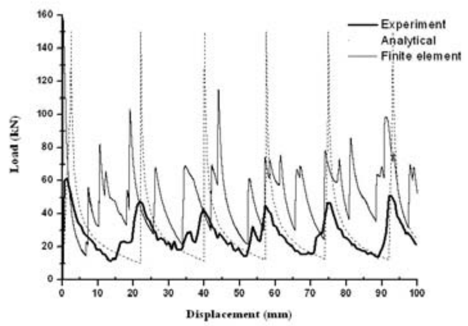
The results of the experiments and analytical are compared with the finite element simulation for different tubes, as shown in fig 8 (a, b, c, d)



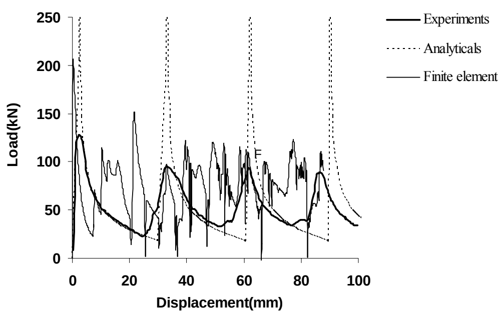
(a) Specimen No.UB1



(b) Specimen No. UB3



(c) Specimen No. UB5



(d) Specimen No. UB6

Figure 8. Comparison of experimental, analytical and simulated load – displacement response.

It can be seen from fig. 8 that the load – displacement curves of collapsed tubes fluctuate in a number of peaks. Each peak the formation of an axisymmetric ring. The curves obtained from finite element simulation are fluctuated in more loops and in higher magnitude compared to experiment. The analytical result gives curves with the same number and same position of peaks to the experiment. However, in general, the load-displacement curves from experiment, analytical and finite element simulation fluctuate in the same trend, especially specimens number UB1 and UB3. The discrepancy between each method may be attributed into the effect of friction between platens and tubes and the non-uniformity of thickness.

6. Conclusions

The axial collapsed behavior of circular steel tube for differential D/t was studied. The result of load – displacement response and deformation shape from finite element simulation and analytical agree with experimental results quite well. It may be concluded that the simple analytical model can be used to approximate the collapse mechanism of concertina mode. This model can be helpful in the early design stage with a promised accuracy.

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