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Parameter estimation of the brake disk using in a floating caliper disk brake model with respect to low frequency squeal

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Abstract
Noise from a vehicle is always a concern for any automotive industry looking for passenger comfort. This also holds for the different types of brake noise, in particular for squeal, which is a source of discomfort both to passengers and passers-by. Intensive research on low frequency squeal (noise between 1-5 kHz) has been carried out. A model of the floating caliper disk brake has been recently proposed by the author with the aim to predict the onset of squeal. Flutter type instability resulting from non-conservative restoring forces is assumed to be the reason behind squeal in this model. The measurement confirmed that the frequency of squeal and the vibrating frequency of the brake disk during squeal are almost the same. The problem of validating the model against experimental observation lies in the fact that all the system parameters included in the model, especially the brake disk, are to be carefully chosen. In this work some parameters of the brake disk are estimated and a method is suggested to estimate these parameters in a systematic way. For that purpose, the values are to be accurately selected by modal analysis and help of system identification toolbox in MATLAB.
Keywords: squeal, brake disk, and modal analysis

1. Introduction
Disk brake squeal has been considered as a serious problem by automobile manufacturers for quite a long time. However it does not interfere in the operation of the brakes. Squeal is considered to be a high frequency noise having a broad frequency range between 1 to 16 kHz [1]. It is commonly accepted by engineers and scientists working in the field, that brake squeal in the disk brake is initiated by an instability due to the friction forces leading to self-excited vibrations. The self-excited brake system then oscillates, reaching a limit cycle. The reason for the onset of instability has been put forward on three possible grounds, namely, the change of the friction characteristic with the speed of the vehicle, change of the relative orientation of the disk and the brake pads leading to a modification of the friction force, and a flutter instability which is found even with the constant friction coefficient. Various schematic models have been constructed but most of these models have not been validated by physical experiments. It is of course well known that a negative slope in the friction characteristic leads to instability and self-excitation. On the other hand, the friction properties in a brake depend strongly on the temperature and wear (actually on the whole temperature and wear history) and also on the pressure, humidity, etc. It is known from experience with brake squeal, that there may be instability and self-excitation even in the absence of the negative slope. There are obviously other mechanisms for instability, even if the friction characteristic with negative slope may seem an obvious culprit. The author believes that the flutter instability which may even occur with constant coefficient of friction is a more realistic candidate in most cases.

Figure 1. A floating caliper disk brake.

In this paper the floating caliper disk brake model which can be used to reproduce the prominent features of squeal has been proposed by the author [2], [3]. The transverse vibrations play a significant role for squeal in a frequency range of 1-5 kHz and therefore due important is given to the vibration of the brake disk. Although the behavior of the model shows good qualitative agreement with those usually experienced in practice, quantitative agreement has not been established and the main difficulty lies in estimating the model parameters. The brake disk is considered as an annular flexible thin plate and modeled as a Kirchhoff plate [4]. Its corresponding parameters are estimated by the modal analysis [5] in similar boundary conditions applied to a real brake disk. The model of the brake disk is validated in the bases of impedance and natural frequency using system identification toolbox in MATLAB. A method of estimating the parameters is to minimize the difference between the model's output and the measured output. Furthermore, an experimental set-up was built in the

Darmstadt laboratory for measuring the vibrations in the disk brake system.

2. Mathematical-Mechanical model

2.1 The floating caliper disk brake model

A floating caliper disk brake, shown in figure 1, consists of a brake disk, housing, piston, yoke, and brake pads. During operation oil pressure applied to the cylinder of the housing drives the piston to secure contact between the outer brake pad and the brake disk. Owing to the floating nature of the housing the whole assembly moves transverse to the brake disk bringing the inner brake pad into firm contact. A schematic model of the disk brake assembly is shown in figure 2.

In the model, the brake disk is considered as an annular thin plate rotating with angular velocity Ω. The caliper is modeled by two rigid bodies (masses m and M) held together by a rotational spring. The friction material is modeled by distributed nonlinear elastic springs and linear dampers. The yoke is modeled as a linear system with discrete rigid bodies and rotational springs simulating the bending stiffness of different parts.

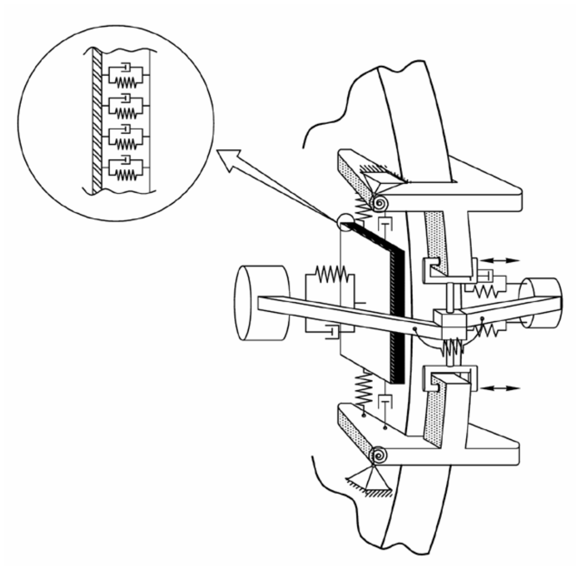

Figure 2. A schematic model.

2.2 The brake disk model

The brake disk, shallow-hat ventilated disk shown in figure 3, consists of the disk-element which is in contact with the brake pads during operation. The hat-element provides the geometric offset necessary for mounting the brake disk to the vehicle. Thickness, inner diameter and outer diameter of the brake disk, depth and thickness of the hat, numbers and spacings of the cooling vanes, and the mounting studs are some of the geometric parameters that set the brake disk's natural frequency spectrum and vibrating modes.

The stationary brake disk is modeled as a Kirchhoff plate with inner radius *a* and outer radius *b*. To consider a transverse vibration of the annular thin plate, thus the equation of motion [4] is obtained as

$$(\rho h)_{eq}\frac{\partial^2 w}{\partial t^2}+D\nabla^4 w=q(r,\theta,t)\,, \tag{1}$$

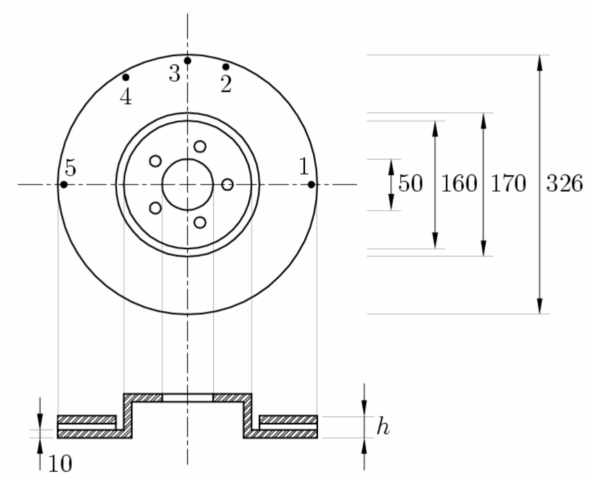

Figure 3. Brake disk.

where

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\,, \tag{2}$$

the in-plane stresses are neglected, $(\rho h)_{eq}$ is the equivalent mass density of the plate, and D is the flexural rigidity of the plate. Note that $(\rho h)_{eq}$ and D are independent parameters to be identified and the relation $D=\frac{Eh^3}{12(1-\nu^2)}$ cannot be used here. The equation of motion is discretized using the Galerkin method, where the solution is approximated in the form

$$w(r,\theta,t)=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}R_{m,n}(r)[\cos(m\theta)A_{m,n}(t)+\sin(m\theta)B_{m,n}(t)], \tag{3}$$

where $R_{m,n}(r)$ are the radial mode shapes of a non-rotating plate in terms of number of nodal diameters (m) and nodal circles (n). These shape functions are determined by solving the free vibration of the annular plate ($q(r,\theta,t)=0$) with the ansatz

$$w(r,\theta,t)=R_{m,n}(r)\sin(m\theta+\gamma_m)\sin\omega_{m,n}t\,. \tag{4}$$

Subsequently, substituting (4) into (1) leads to

$$[(\frac{\partial^2}{\partial r_2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{m^2}{r^2})^2-\beta_{m,n}^4]R_{m,n}(r)=0\,, \tag{5}$$

where $\beta^4=\frac{\omega^2(\rho h)_{eq}}{D}$. To calculate the radial mode shape $R_{m,n}(r)$, equation (5) is rearranged as

$$[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-(\frac{m^2}{r^2}-\beta_{m,n}^2)][\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-(\frac{m^2}{r^2}+\beta_{m,n}^2)]R_{m,n}(r)=0, \tag{6}$$

and its solution will be

$$R_{m,n}(r) = \hat{R}_{m,n}(r) + \overline{R}_{m,n}(r)\,, \tag{7}$$

with

$$\begin{aligned}&\frac{d^2\hat{R}_{m,n}}{dr^2} + \frac{1}{r}\frac{d\hat{R}_{m,n}}{dr} - (\frac{m^2}{r^2} - \beta_{m,n}^2)\hat{R}_{m,n} = 0\\&\frac{d^2\overline{R}_{m,n}}{dr^2} + \frac{1}{r}\frac{d\overline{R}_{m,n}}{dr} - (\frac{m^2}{r^2} + \beta_{m,n}^2)\overline{R}_{m,n} = 0.\end{aligned} \tag{8}$$

The differential equations in (8) are of the Bessel type and their general solutions are

$$\begin{aligned}&\hat{R}_{m,n}(r) = C_{1m,n}J_m(m,\beta_{m,n}r) + C_{2m,n}Y_m(m,\beta_{m,n}r)\\&\overline{R}_{m,n}(r) = C_{3m,n}I_m(m,\beta_{m,n}r) + C_{4m,n}K_m(m,\beta_{m,n}r),\end{aligned} \tag{9}$$

where J_m, Y_m are the m^{th}-order Bessel functions of first and second kind, respectively. I_m, K_m are the m^{th}-order modified Bessel functions. The constants $C_{1m,n}$, $C_{2m,n}$, $C_{3m,n}$, and $C_{4m,n}$ are found by applying the boundary conditions. These boundary conditions [4] are

$$\text{(clamped inside)}\quad w(a,\theta,t) = 0 \tag{10}$$

$$\frac{\partial w}{\partial r}(a,\theta,t) = 0 \tag{11}$$

$$\text{(free outside)}\quad M_r(b,\theta,t) = 0 \tag{12}$$

$$V_r(b,\theta,t) = 0 \tag{13}$$

where

$$M_r = -D[\frac{\partial^2 w}{\partial r^2} + \nu(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2})]\,, \tag{14}$$

and

$$V_r = -D[\frac{\partial}{\partial r}(\nabla^2 w) + (1-\nu)\frac{1}{r}\frac{\partial}{\partial \theta}(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta})], \tag{15}$$

are the bending moment and the shear force (per unit length), respectively.

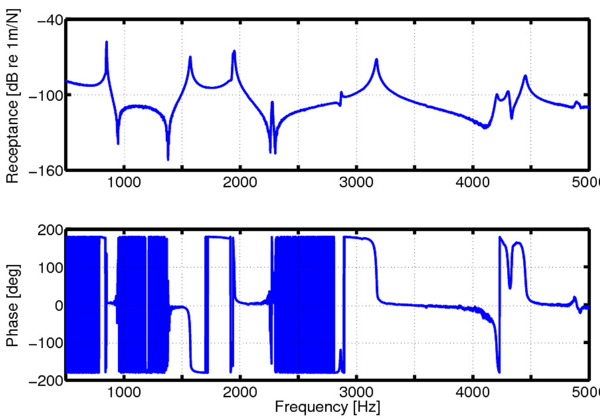

Figure 4. Receptance FRF and phase plot of the free disk

3. Experiments

The frequency response function (FRF) of the transverse vibrations of the brake disk is obtained by an experimental modal analysis performed on both the free disk and the mounted disk. The transverse vibrations of the brake disk are excited by an impact hammer and measured by uni-axial accelerometers at different points. In figure 3, the point 1 is hit by an impact hammer and the vibrating responses are measured at various points 2, 3, 4, and 5 where the angles depend on the interested vibrating modes. Figures 4 and 5 show the receptance FRF plots of the brake disk which can be obtained by the integration over time twice of the measured acceleration at point 2 where the receptance is the ratio of output transverse displacement and input applied force. To compare between both figures 4 and 5, some nodal circle vibrating modes are not excited when the disk is mounted and the natural frequencies tend to increase due to the stiffer brake disk. The normalized discrete-time Fourier transform has been used such that a unit sinusoid in the time domain corresponds to a unit amplitude in the frequency domain.

4. Identification of brake disk parameters

The brake disk is actually mounted with the suspension system. To estimate corresponding parameters between the brake disk and the annular thin plate, the transverse vibration modes in the frequency range 500-5000 Hz have been considered. The mass density $(\rho h)_{eq}$ and the flexural rigidity (D) of the annular thin plate are estimated using the system identification toolbox in Matlab. The equation of motion with impact excitation at the point, $r_0 = 0.155$ m and $\theta_0 = 0$, is

$$(\rho h)_{eq}\frac{\partial^2 w}{\partial t^2} + D\nabla^4 w = q(r_0,\theta_0,t_0)\,, \tag{16}$$

with

$$q(r_0,\theta_0,t_0) = \frac{F}{r_0}\delta(r-r_0)\delta(\theta-\theta_0)\delta(t-t_0)\,, \tag{17}$$

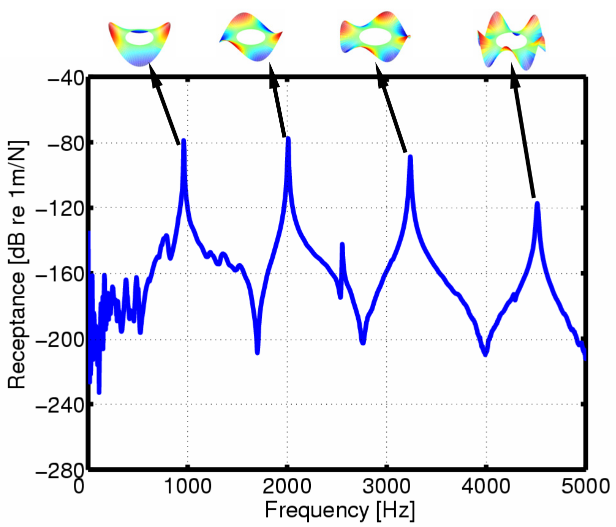

Figure 5. Receptance FRF plot of the mounted disk.

where F is a constant force. Only the transverse vibrating modes with one nodal circle are used to estimate the parameters of the annular thin plate. From the experiment, the natural frequencies of the (2,1), (3,1), (4,1), and (5,1) modes are 959.4, 2009.5, 3237.7, and 4512.8 Hz, respectively. Figure 6 shows the vibrating mode (3,1) of the brake disk. After discretizing (16), the linear equations of motion of the annular thin plate can be written in state-space matrix form as,

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}u,\end{aligned} \tag{18}$$

where $\mathbf{x} = [\dot{A}_{2,1}(t), \dot{A}_{3,1}(t), \dot{A}_{4,1}(t), \dot{A}_{5,1}(t), A_{2,1}(t), A_{3,1}(t)$ $A_{4,1}(t), A_{5,1}(t)]$ and u is the input and a function of F.

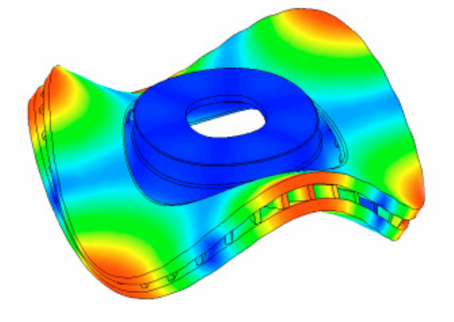

Figure 6. Icon mode of a vibrating brake disk, m=3 and n=1.

The impact excitation force applied transversely to the undeformed disk surface from the experiment is used as the input and the output measured at the same point of the input is the transverse acceleration. For simplicity of using the identification toolbox, the measured acceleration signals are integrated over time and the measured velocity signals are obtained to use as new output. Before using this integrated signal as the output, mean values or linear trends from the integration are removed by the software. It is common practice to use different output signals between model estimation and model validation. A method of estimating the parameters is the prediction error approach based on root mean-square error, where simply the parameters of the models are chosen such that the difference between the model's output and the measured output is minimized. A comparison of impulse response between experiment and estimated model is shown in figure 7.

The average correlation between the two signals over 10 ms is $R_d = 0.73$. The mass density and the flexural rigidity of the annular thin plate were obtained as $(\rho h)_{eq} = 126\ \mathrm{kg/m^2}$ and $D = 90\ \mathrm{kNm}$, respectively. They are the parameters of an ideal Kirchhoff plate dynamically equivalent to the brake disk withih the frequency range under consideration. The following table 1 compares the estimated and measured natural frequencies of vibrating modes.

Table 1 Comparison of measured and estimated modal properties of the brake disk.

No. of nodal diameters	2	3	4	5
Measured freq. [Hz]	959	2009	3237	4512
Estimated freq. [Hz]	958	2001	3497	5242
Difference [%]	0.0	-0.3	8.0	16.1

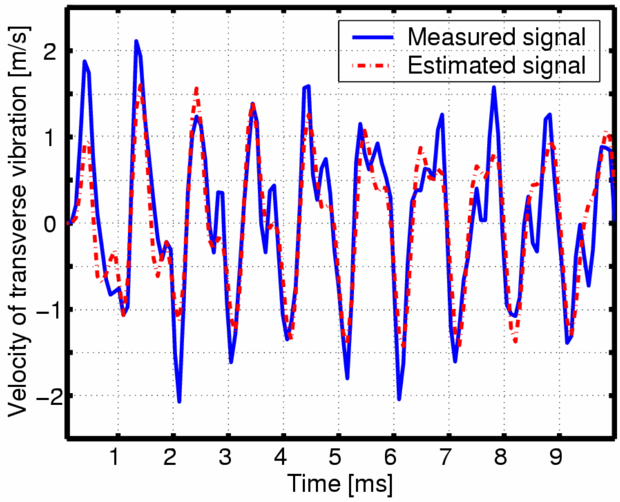

Figure 7. Impulse response from experimental and estimated models in term of velocity of the transverse vibration.

In the system identification toolbox the frequency response, transient response, noise spectrum, model residual, and zeros and poles are available. From all comparisons, one can observe that the estimated model matches well with the experimental model especially for the (2,1) and (3,1) modes.

The results show that with the relatively simple Kirchhoff plate model it was possible to correctly capture not only these natural frequencies and mode shapes but also the disk's complex dynamic impedance in this frequency range by appropriately choosing one set of parameters for $(\rho h)_{eq}$ and (D) only. This clearly shows that the simple Kirchhoff plate model can be a good description for the dynamic behavior of the brake disk with regard to squeal. Of course for the higher plate modes, shear stiffness and rotational inertia become important, so that a Reissner-Mindlin theory must be used for the plate.

5. Conclusion

The floating caliper disk brake model has been verified by the author in order to predict the onset of squeal in a frequency range 1-5 kHz. Some parameters have significant effect on squeal and need to be accurately selected. The transverse vibrations of the brake disk play an important role during squeal. The disk is modeled as a flexible thin plate and its parameters need to be validated in the frequency range of interest. The modal analysis has been used to obtain an impulse response which is later on used to compare with the estimated model using system identification toolbox in Matlab. The results show a good agreement between model and experiment in terms of impedance and natural frequency.

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