

Computation of Isothermal Swirling Flows in a Combustor Using Modified Dissipation Equation k - ϵ Models

Jarruwat Charoensuk¹ Wee Chabthanom^{2*}

¹ Lecturer, Department of Mechanical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Chalong Krung Road, Ladkrabang District, Bangkok 10520

² Graduated student, Department of Mechanical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Chalong Krung Road, Ladkrabang District, Bangkok 10520

*E-mail: wee_7@yahoo.com

Abstract

The commonly used standard k - ϵ model is not suitable for computing swirling flows due to the large velocity gradients. It is believed that one of the sources of accuracy limitations of the standard k - ϵ model is the dissipation equation. This study focuses on the assessment of the modified dissipation equation k - ϵ model, namely, Chen's k - ϵ model, and RNG k - ϵ model for the prediction of swirling flows in a combustor. The predicted results are compared with the results from standard k - ϵ model and the experimental results for three different swirl intensity numbers, 0, 0.3, and 0.5. The predicted axial velocities by all of the turbulent models are in good agreement with those from the experiment. However, for swirl number 0.5, the standard k - ϵ fails to predict the central recirculation, while the results using the modified k - ϵ models are relatively impressive. The performances of the two modified k - ϵ models are competitive, however the Chen's k - ϵ model can predict the recirculating flow slightly better than the RNG k - ϵ model. In addition, the computational times for all three models are in the same magnitude.

Keywords: Swirling Flows, Computational Fluid Dynamics, Modified k - ϵ Model

1. Introduction

Turbulent swirling flow is important because of their widespread use in industrial applications for example combustors (especially in gas turbine), ramjet engines, industrial furnaces and dust collectors. In combustors, it is established that recirculation zone helps to stabilize the flame near the burner. In general, the recirculation zones are formed in flows when an adverse force due to axial pressure gradient exceeds the inertia of the incoming jet of fluid particles and a stagnation point is formed. This can be brought about by the use of opposed jets flowing radially inward from the liner wall, the use of an air swirler, or introducing a bluff body into the main stream [1].

Despite the vast interest in this topic in the last decade, the underlying physical and chemical interactions in the combustors are not yet well understood. The experimental study for the design of the combustor is limited by its cost. So the Numerical study

is an interesting alternative choice. The major development in computer science makes it more popular. The critical parts of the numerical model are the choice of suitable turbulence closure model, differencing scheme and reaction model.

The most well-known turbulence model is the k - ϵ model, with conflicted opinions from a number of published research studies. The major drawback of the standard k - ϵ model is the fact that the eddy viscosity is identical for all the Reynolds stresses due to the isotropic assumption in boussinesq's relationship. Most research studies revealed that the k - ϵ has certain limitations in predicting the swirling flows because the streamline curvature in the recirculation zones cause large changes in the higher order quantities of the turbulent structure, thereby, losing the isotropic structure of turbulence and also results in additional turbulence generation terms [2,3]. An alternative to the problem is the use of higher order turbulence models, such as LES or RSM models. Both models have been demonstrated to be capable of reproducing the major features of the swirling recirculation flow [4]. In spite of a great increase in the computational complexity and time requirements in LES and RSM, the standard k - ϵ model remains a commonly used model in the prediction of turbulent reacting flows. The accuracy of the standard k - ϵ model is sacrificed compared to its advantages like simplicity and economy, especially in complex simulation like reacting flows which is involving heat transfer, mass transfer and chemical reaction. Brewster et al. [5] commented that k - ϵ turbulent model is adequate in many cases for modeling in gas turbine combustors. One of the main suspected sources of accuracy limitations for the standard version of the k - ϵ model is the ϵ -equation. This leads to the creation of the k - ϵ model with modified ϵ -equation such as Chen's k - ϵ model and RNG k - ϵ model. These variants of the k - ϵ model are only slightly more expensive than the standard version, as far as the computational time is concerned. [6].

The objective of the current study is to assess the capability of the modified ϵ -equation version of the standard k - ϵ model, namely, Chen's k - ϵ model, and RNG k - ϵ model in order to predict the cold flow in a dump combustor for three different swirl numbers. A

validation of the turbulent models has been carried out by comparing the results against the experimental data of Ahmed.

Nomenclature

g	Gravity acceleration
ρ	Density
k	Turbulence energy
μ	Viscosity
μ_t	Turbulent viscosity
S_{ij}	Mean strain tensor
ε	Turbulent dissipation
P	Pressure
δ_{ij}	Kronecker delta
Ω_{ij}	Vorticity tensor

2 Turbulent Models

2.1 Standard k- ε Model

The standard k- ε is a commonly used model in the prediction of turbulent flows due to its advantages like simplicity and economy. In this model, turbulent kinetic energy and the dissipation rate are obtained from the following transport equations

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial t} (\sqrt{g} \rho k) + \frac{\partial}{\partial x_j} \left(\rho \tilde{u}_j k - \frac{\mu_{eff}}{\sigma_k} \frac{\partial k}{\partial x_j} \right) = \quad (1)$$

$$\mu_t (P + P_B) - \rho \varepsilon - \frac{2}{3} \left(\mu_t \frac{\partial u_i}{\partial x_i} + \rho k \right) \frac{\partial u_i}{\partial x_i}$$

where

$$\mu_{eff} = \mu + \mu_t \quad (2)$$

$$P \equiv 2s_{ij} \frac{\partial u_i}{\partial x_j} \quad (3)$$

$$P_B \equiv -\frac{g_i}{\sigma_{h,t}} \frac{1}{\rho} \frac{\partial \rho}{\partial x_i} \quad (4)$$

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial t} (\sqrt{g} \rho \varepsilon) + \frac{\partial}{\partial x_j} \left(\rho \tilde{u}_j \varepsilon - \frac{\mu_{eff}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) = \quad (5)$$

$$C_{\varepsilon 1} \frac{\varepsilon}{k} \left[\mu_t (P + C_{\varepsilon 3} P_B) - \frac{2}{3} \left(\mu_t \frac{\partial u_i}{\partial x_i} + \rho k \right) \frac{\partial u_i}{\partial x_i} \right]$$

$$-C_{\varepsilon 2} \rho \frac{\varepsilon^2}{k} + C_{\varepsilon 4} \rho \varepsilon \frac{\partial u_i}{\partial x_i} + C_{\varepsilon 5} \frac{\varepsilon}{k} P_{NL}$$

The Boussinesq relationship is used in the k- ε models, which is given by:

$$-\overline{\rho u'_i u'_j} = 2\mu_t s_{ij} - \frac{2}{3} \left(\mu_t \frac{\partial u_k}{\partial x_k} + \rho k \right) \delta_{ij} \quad (6)$$

$$k \equiv \frac{\overline{u'_i u'_i}}{2} \quad (7)$$

The Turbulent viscosity is linked to k and ε via

$$\mu_t = f_\mu \frac{C_\mu \rho k^2}{\varepsilon} \quad (8)$$

The disadvantage of Boussinesq relationship is that it assumes μ_t is an isotropic scalar quantity, which is not strictly true.

2.2 Chen's k- ε Model

In the standard k- ε model the dissipation time scale, k/ε is the only turbulence time scale used in closing the ε equation. In Chen's k- ε model, the production time scale, k/P , is used in closing the ε equation as well as the dissipation time scale, which is claimed to allow the energy transfer mechanism of turbulence in respond to the mean strain rate more effectively. The transport equations of the Chen's k- ε turbulence model are as follows:

Turbulence energy

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial t} (\sqrt{g} \rho k) + \frac{\partial}{\partial x_j} \left(\rho \tilde{u}_j k - \frac{\mu_{eff}}{\sigma_k} \frac{\partial k}{\partial x_j} \right) = \quad (9)$$

$$\mu_t (P + P_B) - \rho \varepsilon - \frac{2}{3} \left(\mu_t \frac{\partial u_i}{\partial x_i} + \rho k \right) \frac{\partial u_i}{\partial x_i}$$

Turbulence dissipation rate

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial t} (\sqrt{g} \rho \varepsilon) + \frac{\partial}{\partial x_j} \left(\rho \tilde{u}_j \varepsilon - \frac{\mu_{eff}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) = \quad (10)$$

$$C_{\varepsilon 1} \frac{\varepsilon}{k} \left[\mu_t (P + C_{\varepsilon 3} P_B) - \frac{2}{3} \left(\mu_t \frac{\partial u_i}{\partial x_i} + \rho k \right) \frac{\partial u_i}{\partial x_i} \right]$$

$$-C_{\varepsilon 2} \rho \frac{\varepsilon^2}{k} + C_{\varepsilon 4} \rho \varepsilon \frac{\partial u_i}{\partial x_i} + C_{\varepsilon 5} \frac{\mu_t^2}{\rho} \frac{P^2}{k}$$

2.3 RNG k- ε Model

This model is a variation of k- ε standard model via the Renormalization group theory. Some protagonists claim that it is more fundamental than the standard approach. The transport equations of the RNG k- ε turbulence model are as follows:

Turbulence energy

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial t} (\sqrt{g} \rho k) + \frac{\partial}{\partial x_j} \left(\rho \tilde{u}_j k - \frac{\mu_{eff}}{\sigma_k} \frac{\partial k}{\partial x_j} \right) = \quad (11)$$

$$\mu_t (P + P_B) - \rho \varepsilon - \frac{2}{3} \left(\mu_t \frac{\partial u_i}{\partial x_i} + \rho k \right) \frac{\partial u_i}{\partial x_i}$$

Turbulence dissipation rate

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial t} (\sqrt{g} \rho \varepsilon) + \frac{\partial}{\partial x_j} \left(\rho \tilde{u}_j \varepsilon - \frac{\mu_{eff}}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) = \quad (12)$$

$$C_{\varepsilon 1} \frac{\varepsilon}{k} \left[\mu_t (P + C_{\varepsilon 3} P_B) - \frac{2}{3} \left(\mu_t \frac{\partial u_i}{\partial x_i} + \rho k \right) \frac{\partial u_i}{\partial x_i} \right]$$

$$-C_{\varepsilon 2} \rho \frac{\varepsilon^2}{k} + C_{\varepsilon 4} \rho \varepsilon \frac{\partial u_i}{\partial x_i} + \frac{C_\mu \eta^3 \left(1 - \frac{\eta}{\eta_0} \right)}{1 + \beta \eta^3} \frac{\rho \varepsilon^3}{k}$$

where

$$\eta \equiv S \frac{k}{\varepsilon} \quad (13)$$

$$S \equiv \left(2s_{ij} s_{ij} \right)^{1/2} \quad (14)$$

A comparison of equations of RNG k- ε model with their standard model counterpart reveals that the difference is the additional, last term in the dissipation

model that arises from the RNG analysis and represents the effect of mean flow distortion of ϵ .

3. Application

3.1 The Isothermal Dump Combustor Experiment by Ahmed et al.[7,8]

The geometry of the sudden expansion combustor is as shown in fig.1. The inlet pipe has a diameter of 101.6 mm. The combustor chamber has a diameter of 152.4 mm and a total length of 1850 mm (See Table 2 for flow specification). The experimental works are conducted for swirl intensity number 0, 0.3, and 0.5. The swirl number is defined as [9]:

$$S = \frac{\int_0^R UWr^2 dr}{R \int_0^R U^2 r dr} \quad (15)$$

Table 1 Values assigned to model coefficients

	Standard	Chen's	RNG
C_μ	0.09	0.09	0.085
σ_k	1.0	0.75	0.719
σ_μ	1.22	1.15	0.719
σ_h	0.9	0.9	0.9
σ_m	0.9	0.9	0.9
$C_{\epsilon 1}$	1.44	1.15	1.42
$C_{\epsilon 2}$	1.92	1.9	1.68
$C_{\epsilon 3}$	0.0,1.0	0.0,1.0	0.0,1.0
$C_{\epsilon 4}$	-0.33	-0.33	-0.387
$C_{\epsilon 5}$		0.25	
κ	0.42	0.4153	0.4
E	9.0	9.0	9.0
η_0			4.38
β			0.012

Measurements of the mean velocities are available at 12 different x/H cross-sections for 25 y/H sections ranging from the centerline to $D/2$.

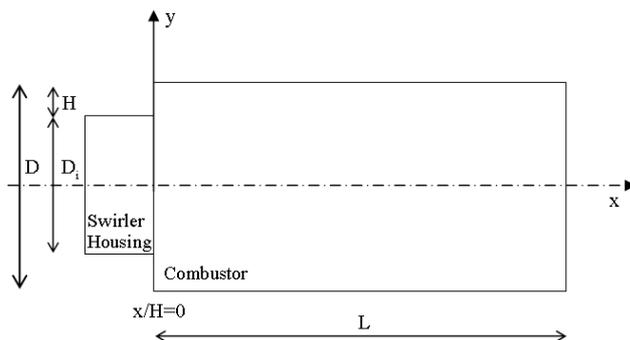


Figure 1 Geometry of the Combustor

The corner-type recirculation zone occurs due to the sudden expansion caused by the difference in diameter. The corner recirculation zone length decreases from about 8H in the non-swirling case to about 4.3H for swirl intensity number 0.3 and about 3.2H for swirl intensity number 0.5. Moreover, for swirl intensity

number 0.5, the vortex breakdown occurs and the central recirculation flow due to this breakdown extends to about 4.4H downstream of the step.

Table2 Flow Data

Parameter	Magnitude
Test section characteristics	
Inlet pipe diameter, D_i	101.6 mm
Combustor diameter, D	152.4 mm
Combustor Length, L	1850 mm
Height of the step, H	25.4 mm
Inlet fluid properties (air)	
Swirl number, S	0.0, 0.3, 0.5
Centerline velocity, U_{ref}	19.2 m/s
Inlet Reynolds number, Re	1.25×10^5

3.2 Mesh Generation

The computation domain for the simulations starts at $x/H=0.38$ which is the point that the experimental data are available. The data are used for the velocity inlet condition. The mesh in the combustor could be created as a structured mesh in cylindrical coordinated. This would have an undesirable effect of creating cells near the centerline with acute angle. The narrowness of the cells would increase as the mesh is refined. In order to avoid the problem mentioned above, the O-mesh (Butterfly mesh) is used in this study.

4. Results and Discussion

The simulations are performed using upwind differencing scheme for momentum, velocity, and turbulent energy for three different swirl intensity numbers. The SIMPLE (Semi-Implicit Method for Pressure Linked Equations) is used for pressure-velocity coupling. The comparisons between the predicted results, with the measured data are presented in Figs. 2 through 4.

The predicted radial profile of axial velocity based on the flow calculations using the standard $k-\epsilon$ model, Chen's $k-\epsilon$ model, and RNG $k-\epsilon$ model are compared with the measurements in Fig. 2 for swirl intensity number 0. The general features of the measured axial velocity profiles are reasonably well predicted by all the turbulent models. The length of the corner recirculation simulated by the standard $k-\epsilon$ model is about 4.5, which is significantly under-predicted, compared with the measured result (8H). The Chen's $k-\epsilon$ and RNG $k-\epsilon$ can predict the length of corner recirculation correctly, 8H and 7.5H, respectively.

Fig.3 shows the prediction of axial velocity for swirl intensity number 0.3. The predictions with all the turbulent models are in generally good agreement with the measurement. However, the axial velocities in the core region are significantly over-predicted for all models. It also shows that the peak axial velocities predicted are slightly under-predicted. The flatter axial velocity profile in the core region has been attributed to the deficiency in the ϵ -transport equation. The addition of the production time scale in Chen's $k-\epsilon$ model and the

additional term in the dissipation model that arises from the RNG analysis in RNG $k-\epsilon$ model can slightly improve the results. The predictions of corner recirculation are reasonable for all models (3H for standard $k-\epsilon$ model, 4H for Chen's and RNG $k-\epsilon$ models, compared with 4.3H for the experimental result).

Fig.4 shows the axial velocity profile for swirl intensity number 0.5. It can be clearly seen that the standard $k-\epsilon$ model fails to predict the central recirculation. The predictions with Chen's and RNG $k-\epsilon$ model are in generally good agreement with the measurements. Both models yield similar results. However, a closer examination reveals that differences between the predictions of two turbulent models are evident in the central recirculation. Chen's $k-\epsilon$ model can predict the length of the central recirculation better than the RNG $k-\epsilon$ model (4H and 3.5H, respectively, compared with 4.4H for the experimental result).

The computational times are in the same magnitude for all models. For swirl number 0, the computational times of standard, Chen's, and RNG $k-\epsilon$ models are 675 s, 700 s, and 760 s respectively. The computational time of Chen's $k-\epsilon$ for swirl number 0.3 is 943 s, which is slightly less than the standard $k-\epsilon$ model and RNG $k-\epsilon$ model (1044 s, 1070 s, respectively). For swirl number 0.5, the computational time of both modified $k-\epsilon$ models are less than the standard $k-\epsilon$ model (908 s for Chen's $k-\epsilon$ model, 930 s for RNG $k-\epsilon$ model compared with 1034 s for standard $k-\epsilon$ model).

5. Conclusion

An isothermal flow in dump combustor is modeled using standard $k-\epsilon$ model, Chen's $k-\epsilon$ model, and RNG $k-\epsilon$ model for swirl intensity number 0, 0.3, and 0.5. The predicted results of axial velocity are compared with measurements. The conclusions from the investigation are as follows:

5.1 For swirl number 0.0, the predicted axial velocities by all three turbulent models are in generally good agreement with the experimental study.

5.2 For swirl number 0.3, the predicted axial velocities by all three turbulent models are in good agreement with the experimental study, except at the points near the core region of the combustor.

5.3 For swirl number 0.5, the standard $k-\epsilon$ fails to predict the central recirculation, while the predictions of the central recirculation using other models are impressive.

5.4 Comparing RNG $k-\epsilon$ model with Chen's $k-\epsilon$ model, the accuracy of central recirculation and corner recirculation length of the latter seems to be slightly superior.

5.5 The computational times for all models are in the same magnitude.

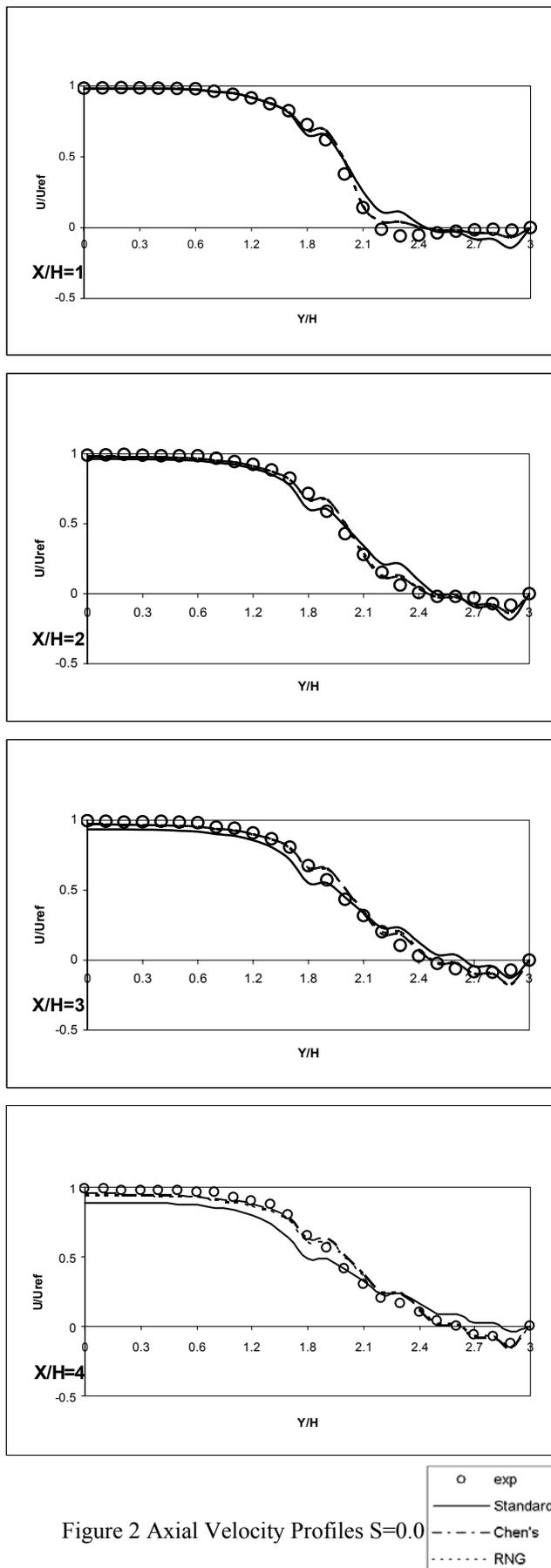
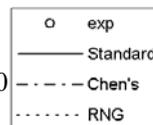


Figure 2 Axial Velocity Profiles S=0.0



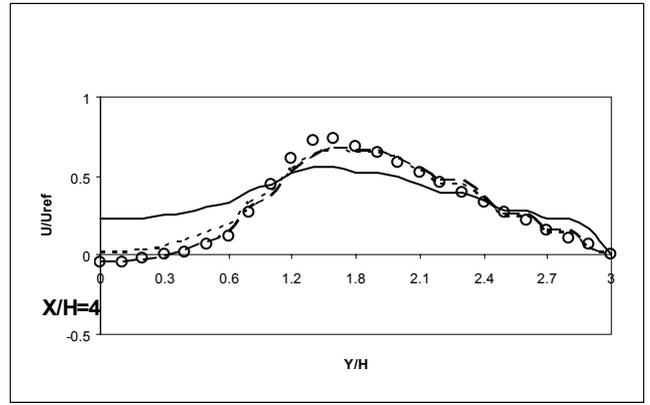
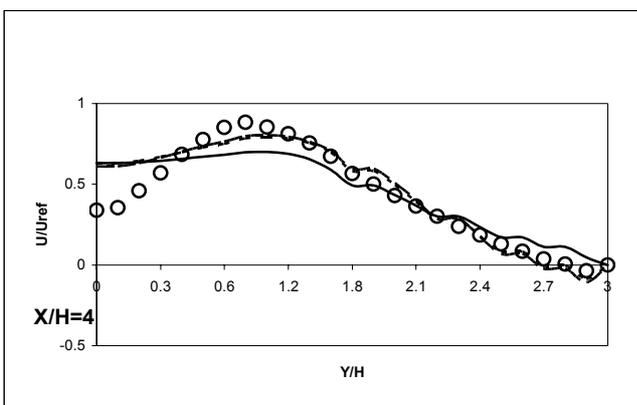
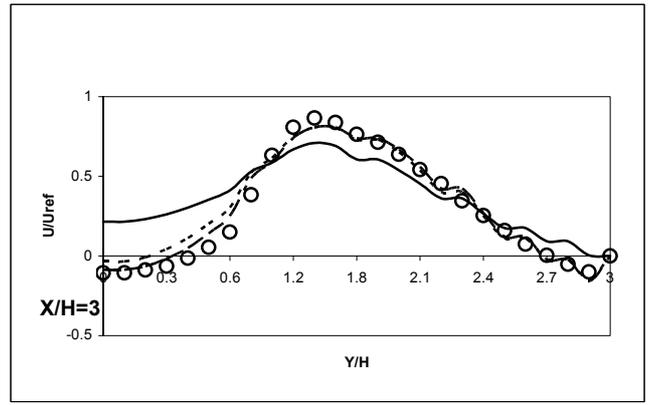
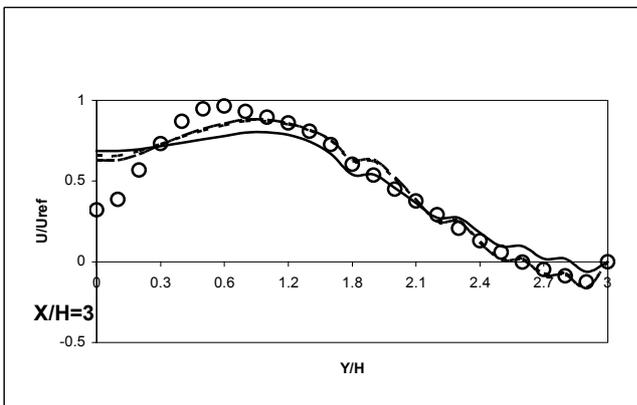
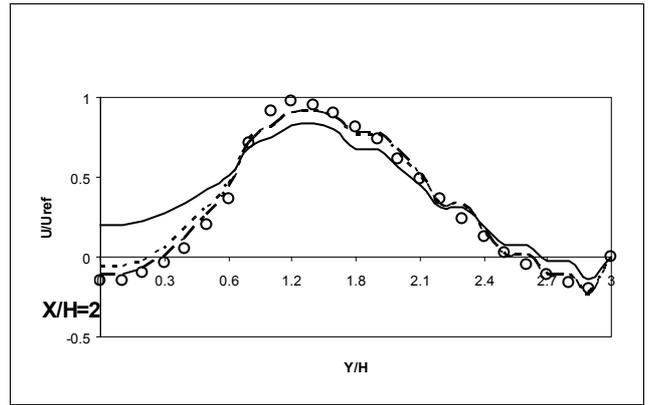
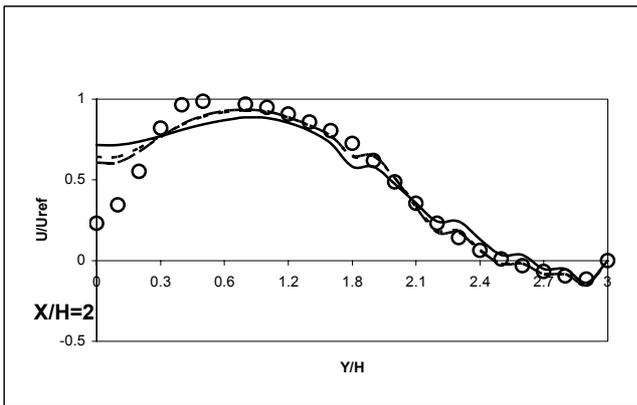
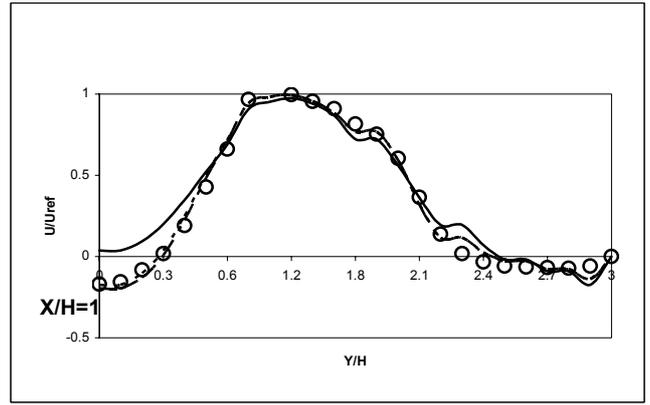
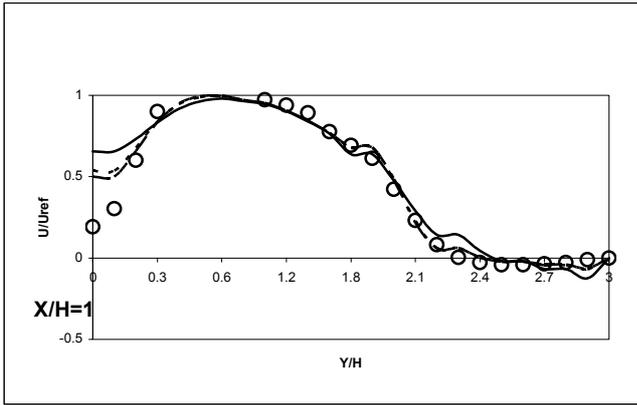


Figure 3 Axial Velocity Profiles $S=0.3$

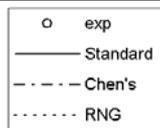
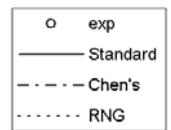


Figure 4 Axial Velocity Profiles $S=0.5$



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