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Design of Compliance Mechanisms Using Topology Optimisation

Sujin Bureerat, Adulyasak Boonapan, Tawatchai Kunakote and Jumlong Limtragool
Department of Mechanical Engineering
Faculty of Engineering, Khon Kaen University, Thailand, 40002
Email: sujbur@kku.ac.th

Abstract

This paper presents an alternative approach for synthesis of compliance mechanisms. The compliance mechanism is found to be useful particularly for the design of MEMS actuators. Rather than being constructed by joining rigid bodies with hinges or some other constraints, this mechanism can be manufactured using just one piece of an elastic plate. The synthesis of such a mechanism can be carried out by using topological design. The use of radial-basis function interpolation as filtered topological design variables is derived. The technique is termed Approximate Density Distribution (ADD). The optimiser is the Optimality Criteria Method (OCM). The ADD method and the sensitivity filtering technique are applied to solve two compliance mechanism synthesis problems. The results obtained using the present and traditional techniques are illustrated, compared and analysed.

Keywords: Compliance Mechanism, Topology Optimization, ADD, Optimality Criteria Method, MEMS

1. Introduction

Topology optimisation is a structural design process that is performed in the stage of conceptual design. Topological design of structures is traditionally carried out by employing finite element analysis and numerical optimisers. The design problem can be thought of as the art of using limited material whilst having optimum design objective. Starting with predefined design domain, the structure is discretised to have a great number of finite elements. Topological design variables are parameters that define a structural topology. The classical design variables are finite element densities or thicknesses. This means that elements with nearly zero densities lead to voids while the others represent material existence on the structure. The problem is normally classified as large-scale optimisation. The optimisers traditionally used to deal with this design problem are optimality criteria method, sequential linear programming and the methods of moving asymptotes. Evolutionary algorithms, although having the advantage with the use of 1-0 discrete design variables, appear to be ineffective for this task due to a large number of design variables [1].

According to design objectives, the topology design can be classified as, for example, structural compliance minimisation, dynamic stiffness (eigenvalue) maximisation and maximisation of buckling factor [2]. The design strategy can also be applied to synthesise the so-called compliance mechanisms. Much work has been made and contributed to this research field e.g. [3], [4], [5] and [6]. A compliance mechanism, sometimes called jointless mechanism, attains mobility through its flexibility rather than using hinges, gears or some other constraints as its rigid counterpart. The advantages in using such a mechanism are that, it needs fewer parts and steps to be manufactured and it needs no lubrication. The mechanism is useful particularly in MEMS as the use of hinges or bearings for micro-scale actuators is rather impossible. However, it is also disadvantageous in that the mechanism is likely to experience high fatigue and stress concentration. Also, it can be a less efficient mechanism due to the loss of elastic energy [3].

The paper presents the use of Approximate Density Distribution (ADD) [7] design variables for topology optimisation of compliance mechanisms. The ADD technique employs radial-basis function interpolation for approximating structural topologies instead of using element densities directly. The technique is adapted to be used with the OCM and based upon the Solid Isotropic Material with Penalisation (SIMP) concept [8]. Two compliance mechanism design problems are assigned to benchmark the present method. The results obtained from using the new technique and that from using the OCM with sensitivity filtering technique are illustrated compared and discussed. It is shown that the results from using the OCM with ADD technique are as effective as ones obtained from the OCM with the classical sensitivity filtering technique.

2. Topological Design

A typical topological design of structures can be posed as

$$\min_{\boldsymbol{\rho}} F(\boldsymbol{\rho}) \tag{1}$$

subject to

$$m(\rho) = r.m(1)$$
$$\mathbf{0} < \rho^0 \leq \rho \leq \mathbf{1}$$

where ρ is the vector of topological design variables having ρ^0 and $\mathbf{1}$ as it lower and upper bounds respectively
F is an objective function
m is structural mass
and $r \in (0,1)$ is the ratio of mass reduction compared to the initial mass.

The lower bounds are set as small positive values so as to prevent singularity in a structural stiffness matrix. As stated earlier, the objective functions can be structural compliance, dynamic stiffness and buckling factor. Stress and some other structural constraints are excluded from the optimisation problem as they can cause some difficulties. The constraints, however, are later taken into account in the stage of preliminary and detailed designs. Numerical difficulties occurring when performing topological design are intermediate density leading to a stiffened plate rather than a structural topology, checkerboard from the instability of finite element analysis, and mesh dependency or a variety of optimum topologies of one design domain with various mesh resolutions, [9], [10].

Figure 1 displays a generic topology optimisation of a plate. The plate is subjected to external loads with the given boundary conditions. Design domain is where the material distribution or topology design variables deal with. Voids and unchanged regions can be predefined. For linear finite element analysis, structural compliance can be computed as

$$c(\boldsymbol{\rho}) = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} \rho_e \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e \qquad (2)$$

where c is structural compliance

$\mathbf{U}$ is the vector of structural displacements

$\mathbf{K}$ is a structural global stiffness matrix

$\mathbf{u}_e$ is element nodal displacements

$\mathbf{k}_e$ is an element stiffness matrix

and N is the number of structural elements.

By using (1) directly, the problems of intermediate density and checkerboard can be arisen. In the SIMP model, the penalty parameter p is introduced to prevent the former problem whereas the sensitivity filtering technique is presented to suppress the checkerboard patterns. As a result, the compliance objective function can be rewritten as [2]

$$c(\boldsymbol{\rho}) = \sum_{e=1}^{N} \rho_e^p \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e \qquad (3)$$

and its filtered derivative value is given as [8]

$$\frac{\partial \hat{c}}{\partial \rho_e} = \frac{1}{\rho_e \sum_{f=1}^{N} H_f} \sum_{f=1}^{N} H_f \rho_f \frac{\partial c}{\partial \rho_e} \qquad (4)$$

where $r_{\min}$ is a parameter to be defined

$H_f = r_{\min} - dist(e,f)$

and $\{ f \in N, dist(e,f) < r_{\min}, e = 1,\ldots,N \}$.

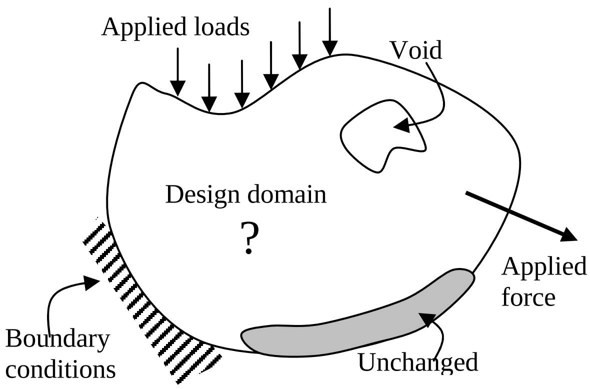

Figure 1 topology optimisation

For the problem of compliance mechanism synthesis, with the predefined input forces, the problem is concerned with optimising the displacements at some selected nodal points. Thus, the objective function value in dependent on desired output displacements. It can be somewhat expressed similar to the minimum compliance problem as

$$u_{out}(\boldsymbol{\rho}) = \boldsymbol{\lambda}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} \rho_e \boldsymbol{\lambda}_e^T \mathbf{k}_e \mathbf{u}_e \qquad (5)$$

where u_{out} is the desired output displacement

and $\boldsymbol{\lambda}$ is the vector of structural displacements due to adjoin loads (see [2]).

The elements of the vector $\boldsymbol{\lambda}$ determine whether the problem is maximisation or minimisation. The penalty parameter is included to the objective function as in (2) and filtered derivatives can be calculated in a similar fashion to (3). The optimisation problem is illustrated in figure 2. Also note that it is more beneficial to add a linear spring to the input and output nodes [8].

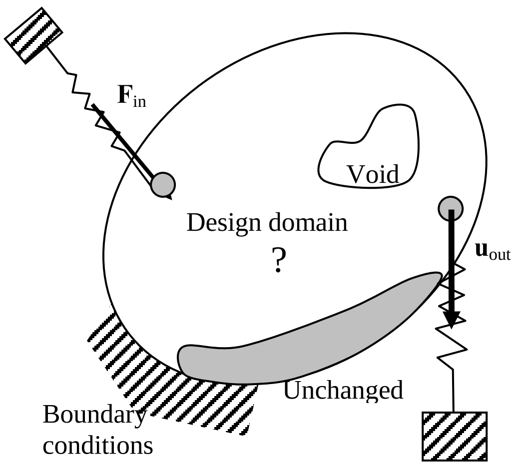

Figure 2 Compliance synthesis

3. Optimisers

ADD is a simple numerical technique exploiting interpolation for approximation of element densities from the known densities at some particular points. From a rectangular design domain being meshed into n elements as shown in figure 3, let $\mathbf{r}_j^0$ be the position vectors of m sampling points (plus sign) and $\mathbf{r}_k^v$ be the position vectors of the centre points of the n elements ('o' sign). With the idea of interpolation with radial-basis functions, the densities at the centre points of the elements, ρ, can be approximated from the given densities at the sampling points, ρ^{ADD}, by the relation:

$$\boldsymbol{\rho} = \mathbf{C}\mathbf{A}^{-1}\boldsymbol{\rho}^{ADD} = \mathbf{T}\boldsymbol{\rho}^{ADD} \qquad (6)$$

where $\mathbf{C} = [c_{ij}]_{n\times m} = [f(d(\mathbf{r}_k^v, \mathbf{r}_j^0))] = [f(d_{kj})]$

$$\mathbf{A} = [a_{ij}]_{m\times m} = [\,f(d(\mathbf{r}_i^0, \mathbf{r}_j^0))] = [\,f(d_{ij})]$$
$$f(d_{ij}) = d(\mathbf{r}_i, \mathbf{r}_j)$$
and $d(\mathbf{r}_i, \mathbf{r}_j) = \sqrt{(\mathbf{r}_i - \mathbf{r}_j)^T(\mathbf{r}_i - \mathbf{r}_j)}$.

For more details, see [7] and [11]. The number of ADD design variables is usually lower than that of the element centre points [7]. Figure 4 and 5 demonstrate how the mapping of densities from the ADD domain to ones at the finite element centre points works. The plot of sampling points and element centre points are given in figure 4 while the mapping of densities at the sampling points to become the finite element densities is shown in figure 5. It is shown that the actual structural configuration (on the finite element domain) is controlled by the density values from the ADD design domain. Due to the less resolution on ADD domain, the checkerboard and one-node connected hinge patterns which appear on the ADD domain are automatically prevented when mapped to the actual topology on the finite element domain. The ADD can be thought of as the filter of topological design variables.

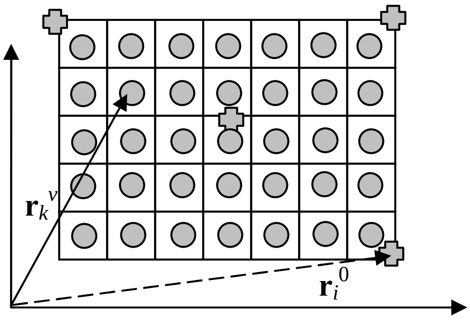

Figure 3 Sampling points and element centre points

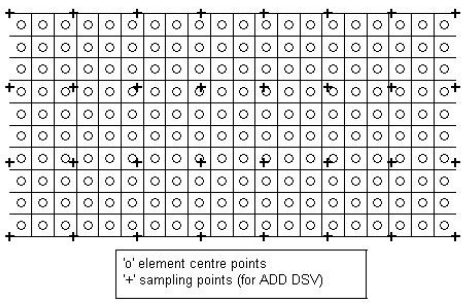

Figure 4 Sampling points & element centre points, example

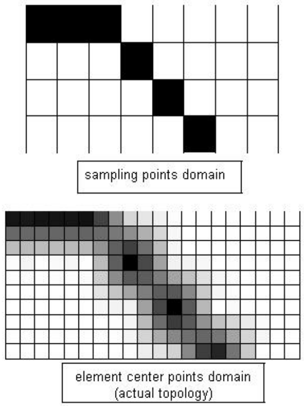

Figure 5 mapping from ADD domain to finite element domain

The topology optimiser used in this paper is the optimality criteria method presented in [8]. The algorithm is based upon optimality conditions where the resulting design variables are expected be either their corresponding lower or upper bounds. The design process starts with an initial solution and it is then updated iteratively until reaching the optimum. The updating scheme can be written as [8]

$$\rho_e^{new} = \begin{cases} \max(\rho_e^{\min}, \rho_e^{old} - l) \\ \text{if } \rho_e^{old} B_e^{\eta} \le \max(\rho_e^{\min}, \rho_e^{old} - l) \\ \rho_e^{old} B_e^{\eta} \text{ if } \max(\rho_e^{\min}, \rho_e^{old} - l) \\ < \rho_e^{old} B_e^{\eta} < \min(\rho_e^{\max}, \rho_e^{old} + l) \\ \min(\rho_e^{\max}, \rho_e^{old} + l) \\ \text{if } \rho_e^{old} B_e^{\eta} \ge \min(\rho_e^{\max}, \rho_e^{old} + l) \end{cases} \qquad (7)$$

where

$$B_e = \max\left(-\frac{\partial f}{\partial \rho_e}, 0\right) \Big/ v\frac{\partial m}{\partial \rho_e}$$

v is a Lagrange multiplier to be determined on each loop

$l \in (0,1)$ is a moving limit

η is a numerical damping ratio set to prevent premature convergence

and $\rho^{\min}$ and $\rho^{\max}$ are the lower and upper bounds of the topology design variables respectively.

When implementing the ADD technique, the lower and upper bounds cannot be the same as ones given in the problem (1) but they are computed as:

$$\rho^{\min} = \mathbf{T}^{\#}\rho^0 \qquad (8)$$
$$\rho^{\max} = \mathbf{T}^{\#}\mathbf{1}$$

where $\mathbf{T}^{\#}$ denotes the pseudo-inverse of $\mathbf{T}$.

Moreover, the gradient of a function f with respect to ρ^{ADD} can be obtained from [7]

$$\nabla f\big|_{\boldsymbol{\rho}^{ADD}} = \mathbf{T}^T \nabla f\big|_{\boldsymbol{\rho}} \qquad (9)$$

The procedure of OCM with the use of ADD is similar to that using the element thicknesses (or densities) as topological design variables. The difference is that the

OCM with ADD variables does not need sensitivity filtering technique or any additional numerical scheme to deal with checkerboard and one-node connected hinge problems as in the classical approach. The evaluation of function derivative can be carried out by using (9). The termination criterion for the OCM algorithm is that when the change of element densities is sufficiently small.

4. Numerical Experiment

Two compliance mechanism synthesis problems are set to verify the present approach. The first design problem is named OPT1 and the structural half-model is illustrated in figure 6. The input force is applied at the left-hand bottom corner and the desired (maximum) output displacement is at the right-hand bottom corner of the design domain as shown. The structure is made up of material with 200×10^9 N/m^2 Young modulus and 0.3 Poisson's ratio. The aspect ratio of the design domain is $L/H = 2$. The plate is discretised to have 80×40 elements while the number of ADD variables or sampling points is 75×36 equally distributed throughout the plate.

The half-model of the plate for the second synthesis problem, termed OPT2, is shown in figure 7. The plate is made up of the same material as used in OPT1. The aspect ratio is $L/H = 2$. The number of finite elements used for structural analysis is 3128 elements while the number of ADD design variables is 2994. The input force is applied in the horizontal direction while the required output displacement is in the vertical direction.

The 4-node membrane finite element formulation is employed for structural analysis. Each synthesis problem is solved by two design strategies that are the original approach OCM with sensitivity filtering technique and the OCM with the present ADD design variables. They are called OCM1 and OCM2 respectively. The predefined parameters for the OCMs are set as: $r_{\min}$ for OCM1 is $0.1H$, $p = 3$, $\eta = 1/3$ and $m = 0.2$.

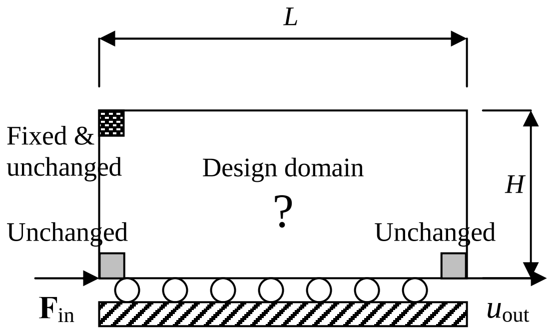

Figure 6 Half-model of OPT1

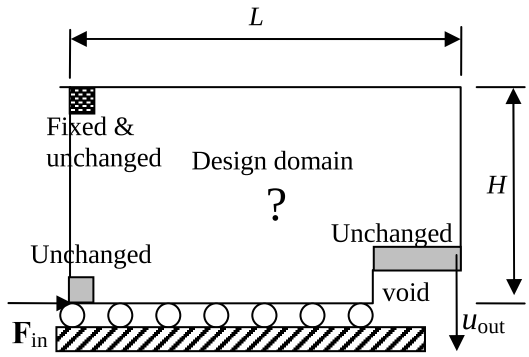

Figure 7 Half-model of OPT2

5. Design Results

The optimum results of OPT1 obtained from using OCM1 with 70%, 60%, 50% and 40% of mass reduction are illustrated in figure 8, 9, 10 and 11 respectively. The optimum results of OPT1 obtained from using OCM2 with 70%, 60%, 50% and 40% of mass reduction are shown in figure 12, 13, 14 and 15 respectively. Note that the figures show the plot of full topologies. All the resulting compliance mechanisms have quite similar assemblies but different configurations. In the case of using OCM2 with 70% mass reduction, the search procedure was terminated before reaching the optimum. Figure 16 shows the comparison of the optimum objective function values of the mechanisms. Note that, for simplicity in comparison and illustration, all the function values are normalised to be in the range of (0, 1). It is shown that OCM1 is superior to OCM2 for 40% and 70% mass reduction cases while OCM2 has the edge for the mass reduction ratios of 50% and 60%.

Figure 8 Optimum topology of OPT1 from OCM1 with 70% mass reduction

Figure 9 Optimum topology of OPT1 from OCM1 with 60% mass reduction

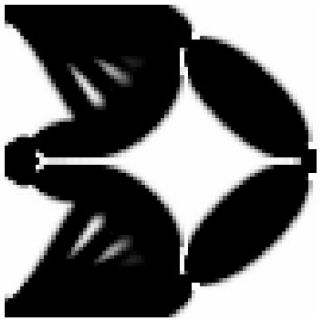

Figure 10 Optimum topology of OPT1 from OCM1 with 50% mass reduction

Figure 11 Optimum topology of OPT1 from OCM1 with 40% mass reduction

Figure 12 Optimum topology of OPT1 from OCM2 with 70% mass reduction

Figure 13 Optimum topology of OPT1 from OCM2 with 60% mass reduction

Figure 14 Optimum topology of OPT1 from OCM2 with 50% mass reduction

Figure 15 Optimum topology of OPT1 from OCM2 with 40% mass reduction

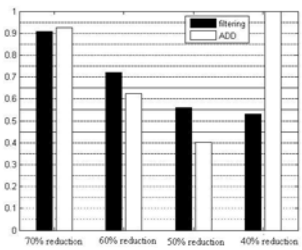

Figure 16 Comparison of normalised optimum objective values: OPT1

The optimum topologies of OPT2 obtained from using OCM1 with mass reduction ratios of 70%, 60%, 50% and 40% are displayed in figure 17, 18, 19 and 20 respectively. The optimum mechanisms of OPT2 obtained from using OCM2 with 70%, 60%, 50% and

40% mass reductions are displayed in figure 21, 22, 23 and 24 respectively. For the case of using OCM2 with 70% mass reduction, the search procedure was stuck at a local optimum as happened in OPT1 design case. Similar to OPT1, all the compliance mechanisms have similar assemblies but different topologies. The comparison of the optimum objective function values of the mechanisms are given in figure 25. It is shown, as obtained from OPT1, that OCM1 is superior to OCM2 for the mass reduction ratios of 40% and 70% while OCM2 is better with the use of 50% and 60% of mass reduction.

Figure 17 Optimum topology of OPT2 from OCM1 with 70% mass reduction

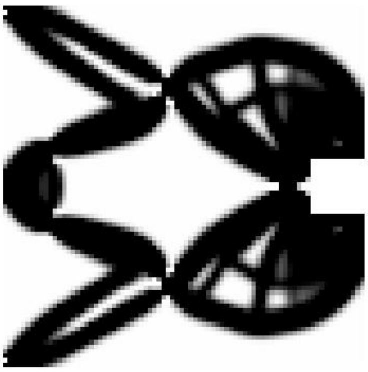

Figure 18 Optimum topology of OPT2 from OCM1 with 60% mass reduction

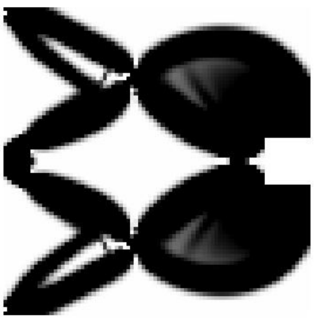

Figure 19 Optimum topology of OPT2 from OCM1 with 50% mass reduction

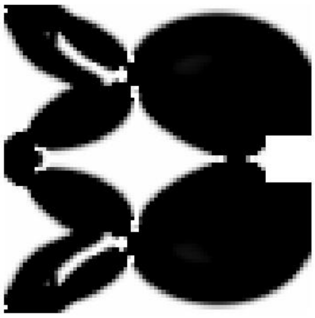

Figure 20 Optimum topology of OPT2 from OCM1 with 40% mass reduction

Figure 21 Optimum topology of OPT2 from OCM2 with 70% mass reduction

Figure 22 Optimum topology of OPT2 from OCM2 with 60% mass reduction

Figure 23 Optimum topology of OPT2 from OCM2 with 50% mass reduction

Figure 24 Optimum topology of OPT2 from OCM2 with 40% mass reduction

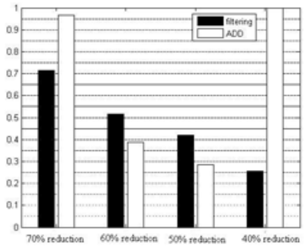

Figure 25 Comparison of normalised optimum objective values: OPT2

6. Conclusions and Discussions

Compliance mechanisms can be synthesised by using topology optimisation. The application of the ADD technique to compliance mechanism synthesis is as powerful as the classical approach using sensitivity filtering technique. The use of ADD is advantageous if the mass reduction ratios are 50% and 60% whereas the filtering technique is superior with the mass reduction ratios of 40% and 70%. The lower mass reduction ratios lead to the lower objective values except for the case of using OCM2 with 40% mass reduction. The mechanisms can be refined by performing shape and sizing optimisation so that stress, fatigue and dynamic constraints are met.

Acknowledgements

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