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Finite element analysis of a three dimensional rubber block in frictional contact

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Abstract

The mechanics of a tire in contact with a frictional flat or curved surface is importance to the understanding of road/vehicle load transfer characteristics. Such contact takes place between the road and the tread blocks of tires. It is necessary to understand the responses of these blocks to contact forces. This work considers the tire tread model as rectangular block in contact with a frictional flat surface on the top face. The contact pressure is calculated for wide range of coefficient of friction and stick - slip phenomenon is predicted as a function of coefficient of friction.

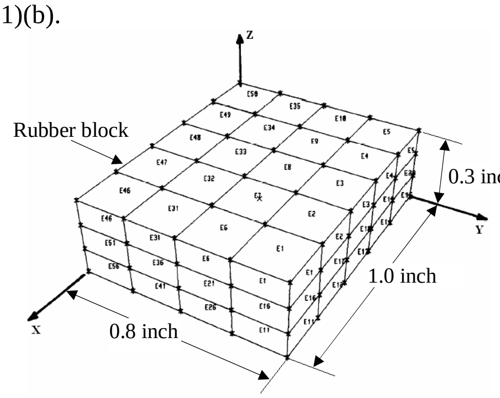
Keywords: Contact pressure, Friction plate, Rubber block, Stick-slip phenomenon

1. Introduction

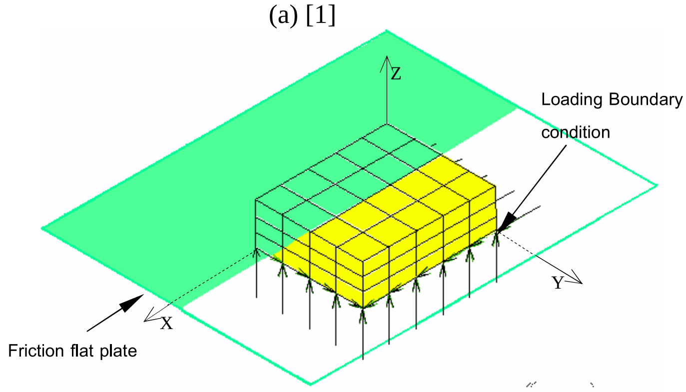
The mechanics of a tire in contact with a frictional flat or curved surface is importance to the understanding of road/vehicle load transfer characteristics. Such contact takes place between the road and the tread blocks of tires. It is necessary to understand the responses of these blocks to contact forces. The stick and the slip are important phenomenon which occurs at the contact region. The classification of stick and slip phenomenon is purposed for this work. we considers the simple model of tire tread as rectangular block in contact with a fixed frictional flat surface on the top face as shown in figure 1)(b). The loading is from the bottom face. The rubber is modeled as the Mooney-Rivlin type of material. The contact normal stress is calculated for a wide range of coefficients of friction and the stick-slip phenomenon is predicted as a function of coefficient of friction.

2. Finite element model

Finite element method is widely used in solving engineering problems. Contacting between tire and the road at static situation can be modeled by contacting between tread of tire and frictional flat plate. The dimensions of the tread block are 1.0, 0.8, and 0.3 inch in the X, Y and Z directions, respectively [see figure 1)(a)]. For this study the problem is divided in two parts, the first one is the deformable part (rubber block), and the second one is the frictional rigid flat plate as shown in figure 1)(b).



(a) [1]



(b)

Figure 1. Finite element model for rubber block

The significant phenomenons for this work are stick and slip status on the tire tread contact surface, so we consider only the surface element. Element type number 84 is taken to be used with 60 elements. It is three dimensional brick element, first-order isoparametric element (arbitrarily distorted brick) with mixed formulation [5]. The rubber block is compressed in the Z direction from the bottom surface nodes by the displacement loading curve as shown the figure 3). The boundary conditions are divided in two parts. For the first one, the rubber block is fixed in the Z direction on the top face and the

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second one, the bottom face of block is fixed in the X and Y directions as shown in the figure 1) (b).

Coulomb friction law, to be used for this work, can be characterized by:

$$\|\sigma_t\| < \mu\sigma_n \text{ (stick)} \tag{1}$$

and $$\sigma_t = -\mu\sigma_n \cdot \tilde{t} \text{ (slip)} \tag{2}$$

where σ_t is the tangential (friction) stress,

σ_n is the normal stress,

μ is the friction coefficient, and,

$\tilde{t}$ is the tangential vector in the direction of the relative velocity : $\tilde{t} = \frac{\tilde{v}_r}{\|\tilde{v}_r\|}$, in which $\tilde{v}_r$ is the relative sliding velocity.

Similarly, the Coulomb friction model can also be written in terms of the nodal forces instead of stress as follows:

$$\|f_t\| < \mu f_n \text{ (stick)} \tag{3}$$

and $$f_t = -\mu f_n \cdot t \text{ (slip)} \tag{4}$$

where f_t is the tangential (friction) force

f_n is the normal force

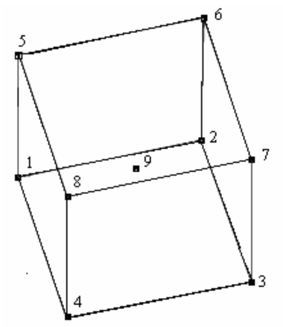


Figure 2. Elements 84 with 9 nodes

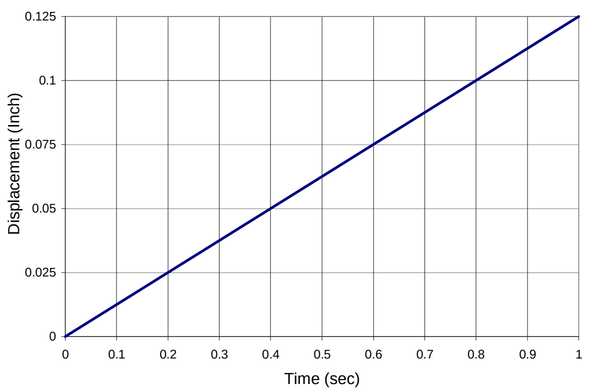


Figure3. Displacement loading curve

3. Material properties

In the present work the Mooney-Rivlin type of rubber material model has been modified for nearly incompressible materials. For such materials, the strain energy function W takes the following form:

$$W = W(I + II + III)$$

$$W = C_{10}(I-3) + C_{01}(II-3) + K(III-1)^2 \tag{5}$$

where I, II and III are the three invariants of the Green-Lagrange strain tensor.

C_{01}, C_{10} are Mooney-Rivlin constants, and,

K is the bulk modulus.

These invariants can be expressed in terms of the three principal stretch ratios X_1, X_2 and X_3 as follows:

$$I = X_1^2 + X_2^2 + X_3^2$$

$$II = X_1^2 X_2^2 + X_1^2 X_3^2 + X_2^2 X_3^2$$

$$III = X_1^2 X_2^2 X_3^2$$

For incompressible materials, we have $III = 1$. The material properties used for numerical work are $C_{10} = 100\ psi$, $C_{01} = 20\ psi$, and $K = 30{,}000\ psi$ [1].

4. Contact stress determination

Element type 84 to be used in this work is the three dimensional brick type element with nine-node, eight nodes in the corner of element and one is located at the center of the element as shown in figure 2). The contact stress at the contacting surface can be calculated by averaging the nodal normal stresses at the four corner of individual surface element face.

5. Finite element results

Due to symmetry, the finite element model used for the present work is considered only half of the surface elements as shown in figure 4). Two important points for this study are contact pressure distribution and the effect of friction on the contact status (stick and slip phenomenon) on the rubber block face.

For the frictionless condition, the contact stress distributions increases with increasing of the level of axial compressive strain are as shown in figure 5) (a) and (b). It can be seen that the contact pressure increases as one move toward the center of the contacting surface.

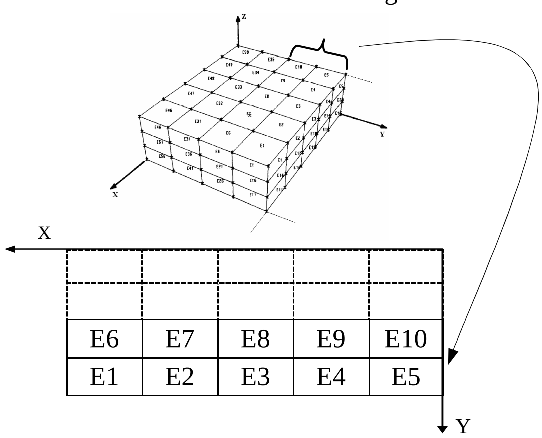
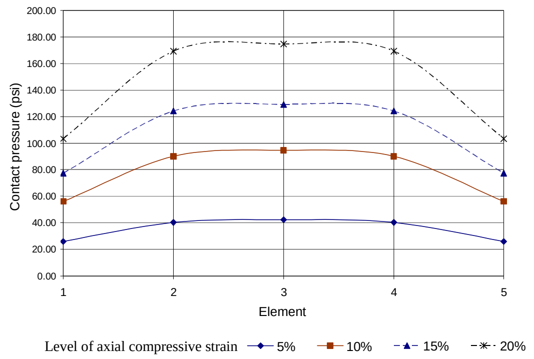
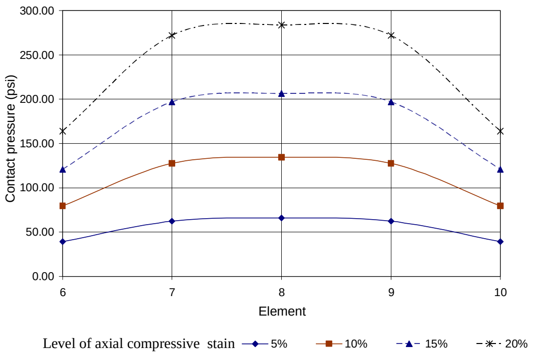


Figure 4. The surface element consideration on the rubber block

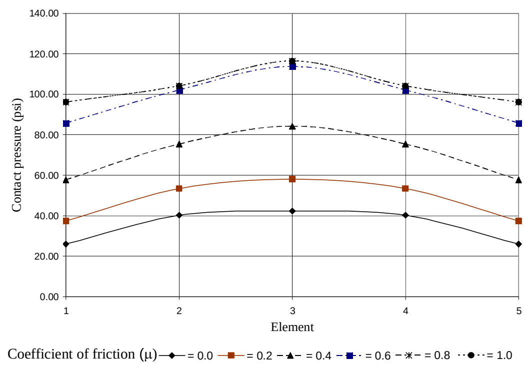


(a) For element 1 to 5

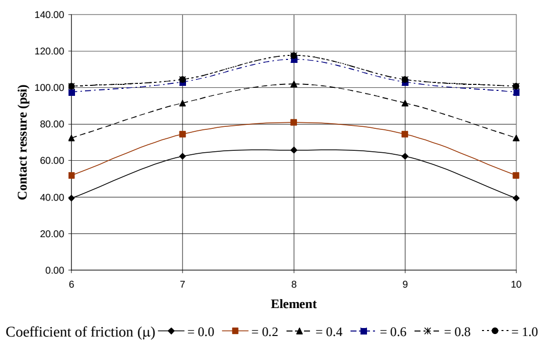


(b) For element 6 to 10

Figure 5. Frictionless contact pressure at various compressive strain

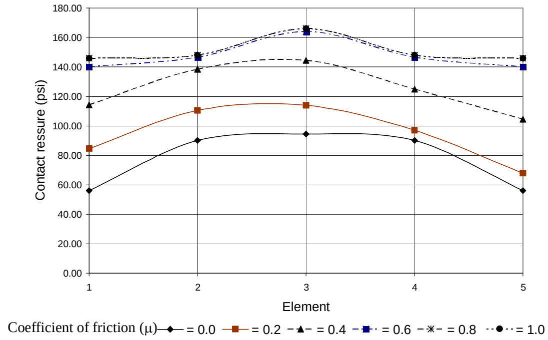


(a) For element 1 to 5

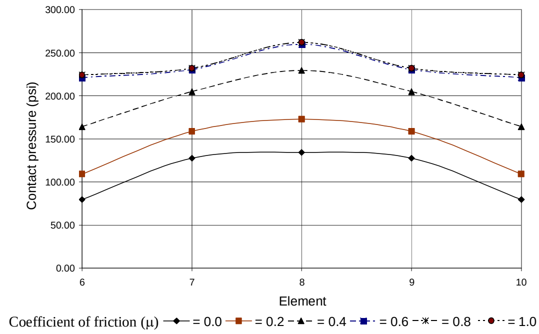


(b) For element 6 to 10

Figure 6. Contact pressure at various coefficients of friction at 5% compression



(a) For element 1 to 5



(b) For element 6 to 10

Figure 7. Contact pressure at various coefficients of friction at 10% compression

The effects of friction on contact pressure at axial compressive strain of 5% and 10% are as shown in figures 6) and 7), respectively. The contact pressure increases with increasing of friction coefficient up to a value of $\mu = 0.6$. The magnitudes of the contact pressures remain unchanged with $\mu \geq 0.6$ for the present loading condition. This implies that, at this coefficient of friction and above, the contact is maintained without any slippage during the course of increasing compressive strain.

The contact forces induced by contacting of rubber block and friction plate are normal and tangential friction forces, which can be used to describe the stick-slip phenomenon in the contacting region. From the simulation results of the problem by using finite element program, MSC MARC 2005, it can be analyzed the detail of stick and slip phenomenon by using equation (3) and (4), respectively. For this present work, the condition of stick in equation (3), we defined the new symbol for the left hand side of equation as F_t which is the part of tangential friction forces and the right hand side as $F_{t\,Cal.}$ which is the part calculated from the contact normal forces. So the new condition for stick can be changed to as in equation (5).

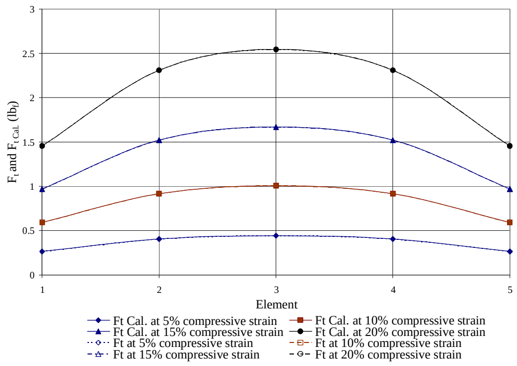
$$F_t < F_{t\,Cal.} \quad \text{(stick)} \qquad (5)$$

Similarly, the condition of slip can be defined by comparing F_t and $F_{t\,Cal.}$ as shown in equation (6).

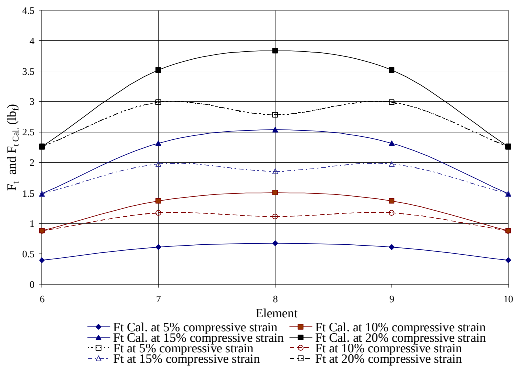
$$F_t = F_{t\,Cal.} \quad \text{(slip)} \qquad (6)$$

The stick and slip conditions can be classified by comparison of the F_t curve and the $F_{t\,Cal.}$ curve, which contact satisfy by equation (5), that contact condition is stick, and if any contact satisfy by equation (6) that contact condition is slip. According to the finite element results, the slip condition can be found by comparison of the F_t curve and the $F_{t\,Cal.}$ curve, that is coincides for the same axial compressive strain as in equation (6). For the stick condition, the F_t curve must be lined in lower level than the $F_{t\,Cal.}$ curve as in equation (5).

At the friction coefficient μ = 0.2, [figure 8) (a)] the F_t curve coincides the $F_{t\,Cal.}$ curve, that can be classified the contact condition is only slip, and the figure 8) (b) it also slip for 5% compressive stain, but when the compressive strain increase to 10%, 15%, and 20%, the contact condition is stick, it occur at the internal region of rubber block. We called this phenomenon as partially stick. It is the same for the friction coefficient μ = 0.4. When the friction coefficient equal to 0.6, 0.8, and 1.0, the contact conditions is only stick, that we called the perfectly stick, as shown in figure 9) (a) and (b).



(a) For element 1 to 5

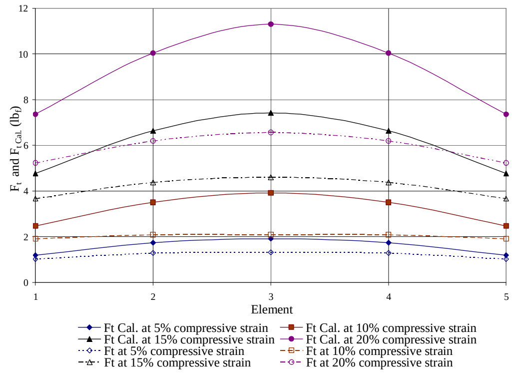


(b) For element 6 to 10

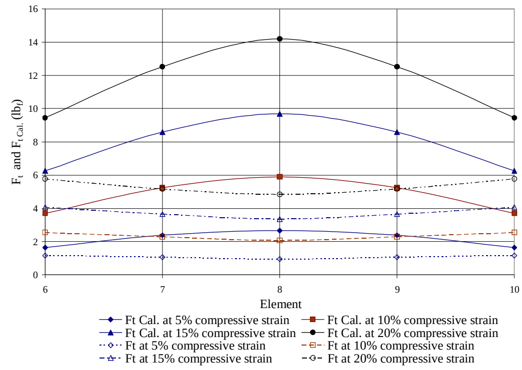
Figure 8. The comparison of contact friction forces with respect to the calculated Coulumb contact friction force with μ = 0.2

6. Conclusions

Finite element analysis of a rubber block in contact with frictional flat surface under axial compressive load has been analyzed. The three dimensional rubber block is modeled as Mooney-Rivlin type of rubber material. The FEM model consists of 60 brick element type number 84 with 9 nodes using Herrman formulation.



(a) For element 1 to 5



(b) For element 6 to 10

Figure 9. The comparison of contact friction forces with respect to the calculated Coulumb contact friction force with μ = 0.6

Results from the FEM simulations can be summarized as follow :

6.1 The contact pressure gradient increases with increasing level of the compressive strain.

6.2 The coefficient of friction is the cause of changing of the stick and slip phenomenon. The state of slip phenomenon can be changed to the state of stick by increasing of the coefficient of friction.

6.3 At the friction coefficient of μ = 0.6 is the starting point of perfect stick.

6.4 The stick and slip conditions can be classified by comparison of the F_t curve and the $F_{t\,Cal.}$ curve, using of equations (5) and (6).

6.5 The stick phenomenon occurs at the inner part of contact region and the slip occurs at the outer part of contact region at various friction coefficients except at μ = 0.0.

7. Acknowledgments

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8. References

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