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Development of Semi-Analytical Approach for Wing Structural Weight Estimation

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Abstract

This paper presents a further development of the wing structural weight estimation method —the semi-analytical approach— proposed in Torenbeek [1] by generalizing the geometrical-based analyses; therefore, the method is applicable to both the conventional straight-tapered wing planform and the highly cranked wing planform. The method analytically evaluates the weight of structural materials required to resist bending and shear of the primary structural box. The contributions of secondary structures such as control devices, and weight penalties due to component attachments are estimated on a basis of empirical method. The original method is revised in some details and some new procedures are introduced. The method is tested against the original method, a semi-analytical method and an empirical method. Overall the results well agree with the actual wing weight although there are some discrepancies. The present method is relatively simple, and can be easily coded and used as an effective design tool for sensitivity study in the initial design stage.

1. Introduction

At the initial aircraft design phase, accurate prediction of aircraft weight is essential, particularly for sensitivity analysis to achieve the optimum design configuration. The aircraft structural weight, directly under control of the designer, can contribute up to 35% of the gross weight and the weight of wing structure generally accounts for 10 to 15% of the gross weight. Sophisticated prediction methods, perhaps the most accurate, of wing structural weight mainly based on the experiences and data of previous designs are employed in the aircraft industry, but generally not available due to highly competition.

Much work has been performed to develop wing structural weight estimation techniques. The semi-analytical approach, based on theoretical analyses and empirical data, is widely adopted because the approach requires a limited number of key design parameters which are available in the early design stage. Although the approach is not as accurate as the theoretically derived weight method like the finite element analysis requiring more geometric details and design parameters which are not available or known in the initial design stage, the semi-analytical approach is relatively accurate, simple and particularly useful for the sensitivity study. In general, the semi-analytical approach analytically evaluates the weight of materials required to carry bending and shear of the primary structural box. The contributions of secondary structures such as control devices and weight penalties due to component attachments are estimated separately on a basis of empirical and statistical data. However, most of the existing semi-empirical methods are currently limited to the conventional straight-tapered wing planform; for example, those presented in Torenbeek [1], Howe [2] and Macci [3]. An estimation method for the cranked wing is presented in Howe [4]. Nevertheless, the proposed semi-analytical methods are mostly presented somewhat in closed-form formulas using typical wing geometry as the representative; therefore, application of these methods is restricted.

This paper presents a further development of the wing structural weight estimation method, based on the semi-analytical approach, developed by Torenbeek [1] by generalizing the geometrical-based analyses. Consequently, the method, which is more geometrical related, can be employed to predict the wing structural weight of both the conventional straight-tapered planform and the highly cranked wing planform applied to most tailless aircraft; and suitable for sensitivity study of wing design parameters at the initial aircraft design phase.

2. Methodology and Assumptions

The structural constitution of an aircraft wing can be generally divided into two parts: the primary box structure and secondary structure as shown in Fig. 1. The primary box structure is generally constructed from upper and lower stiffened skin panels, a front and a rear spar beam, ribs and a center section inside the fuselage. The secondary structure consists of fixed leading and trailing edge structure, high-lift devices and control surfaces. The primary box structure ultimately carries all aerodynamic and inertia loads acting directly or transferred from the secondary structure, which in turn cause shear, bending and torsion on the box structure.

The weight of skins and spar webs of the idealized structure box (e.g. without cutouts and attachments) is estimated by the material required to resist the spanwise bending and shear taking account of inertia relief effects. The idealized box weight is then corrected with additional penalty weights to make allowance for deviations from the idealized structure; for examples, non-tapered skins, joints, mountings, cutouts etc. The weight of the secondary structure is estimated separately based on empirical methods and then added to the primary box-structure weight providing the total wing structural weight.

Overall assumptions and simplified analyses are made as follows [1, 2]:

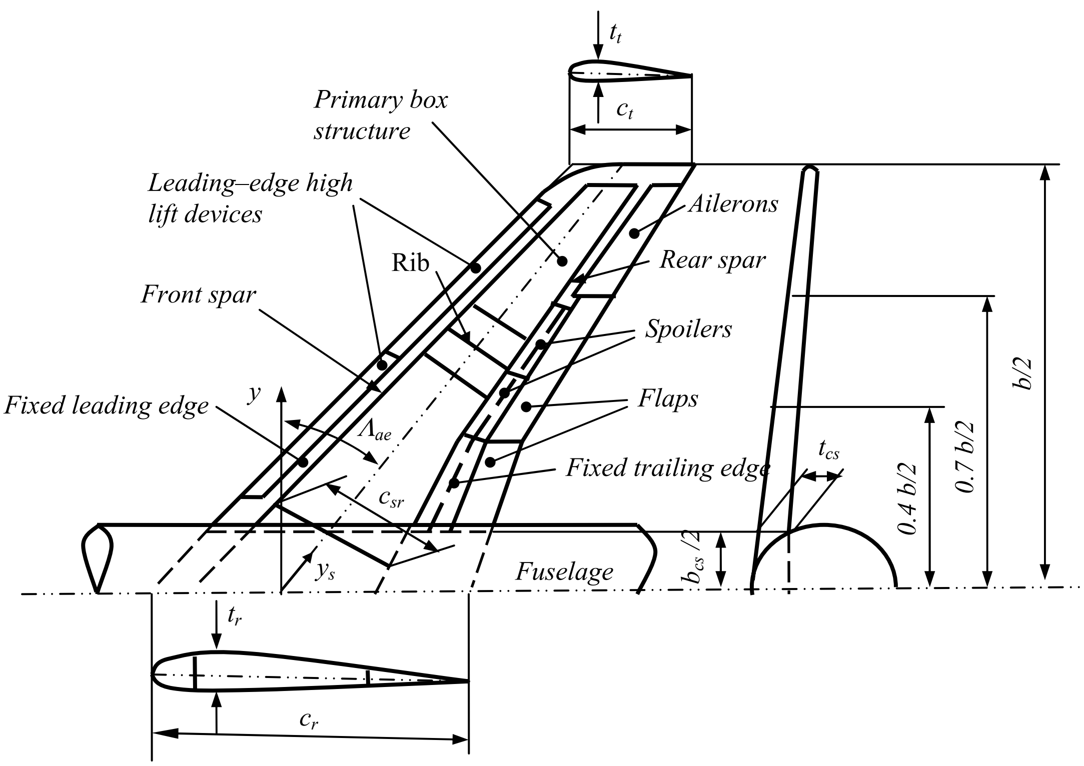

Fig. 1 Structure components of a conventional straight-tapered wing and nomenclature

- The bending is resisted only by the spar flanges/caps and the stiffened skin panels. The spar cap area and stiffener area are embedded in the cover skins forming an equivalent plate thickness. Therefore, the simple beam theory can be used.
- The shear load is taken only by the spar webs.
- Experience suggests that the predominant loads acting on the wing are the bending and shear loads. The torsional load, therefore, will not be taken into consideration explicitly. However, the box structure will be examined for adequate torsional stiffness and locally strengthened, particularly at the outer wing.
- The design load cases are considered as maneuvering and gust loads in symmetrical flight. This shows adequate accuracy for wing weight prediction with a simple analytical approach.
- The spanwise airload distribution of a semi-elliptical shape is assumed at the design cases.
- It is assumed that the each structure components is stressed to the allowable stress of the most critical failure criteria of the structure so that the minimum weight is obtained.

3. Idealized Primary Box Structure Weight

The weight of the idealized primary box is evaluated from the amount of material required to resist bending and shear due to aerodynamic lift and inertia loads. Therefore, the idealized box weight $W_{Box.idel}$ is defined as

$$W_{Box,idel} = W_{Lift} + \Delta W_{Iner} + W_{Rib} \tag{1}$$

$$W_{Box,idel} = W_{Lift} + \left(\Delta W_{Fuel} + \Delta W_{Str} + \Delta W_P\right)_{Iner} + W_{Rib} \tag{2}$$

where ΔW_{Lift} and ΔW_{Iner} are the material weight of the box skins and spar webs required to resist the aerodynamic lift and inertia loads respectively. W_{Rib} is the weight of the wing ribs. The ΔW_{Iner} consists of the inertia relief due to fuel ΔW_{Fuel}, wing structure ΔW_{Str} and concentrated loads ΔW_p such as powerplants and systems. Notice that the inertia weights provide bending and shear relief effects to the wing structure; therefore, they are computed as *negative values*.

The idealized box weight can be rearranged as follows:

$$\begin{aligned} W_{Box,idel} &= W_{Lift}\left(1 + \frac{\Delta W_{Fuel}}{W_{Lift}} + \frac{\Delta W_{Str}}{W_{Lift}} + \frac{\Delta W_P}{W_{Lift}}\right) + W_{Rib} \\ &= W_{Lift}\left(1 + \Delta r_f + \Delta r_s + \Delta r_p\right) + W_{Rib} \\ &= W_{Lift} r + W_{Rib} \end{aligned} \tag{3}$$

where Δr_f, Δr_s and Δr_p is the inertia relief factor due to fuel, structure and concentrated load, respectively; and r is the total relief factor on the wing box structure having a value of less than 1.

It should be noted that the lift and each inertia loads acting on the wing will produce both bending and shear; therefore, the material required to resist bending and shear must be evaluated for each of the loads.

3.1 Material Required to Resist Bending

At a spanwise station y, the cover skins of the primary box are assumed to take the entire bending load leading to compressive stress on the upper skin and tensile stress in the lower skin. The stress due to the bending is given by

$$\sigma(\mathrm{y_s}) = \pm \frac{M(\mathrm{y_s})t(\mathrm{y_s})}{2I_z(\mathrm{y_s})} \tag{4}$$

where the cross section perpendicular to the elastic axis is considered and the neutral axis is assumed at the center of the cross section. The bending moment of inertia $I_z(y_s)$ can be obtained by replacing the skin and stiffener sectional areas of each cover panel (upper and lower panels) with an equivalent panel area of a constant thickness at an effective distance; therefore,

$$I_z(\mathrm{y_s}) = 2A_{sk}(\mathrm{y_s})\eta_t\left(\frac{t(\mathrm{y_s})}{2}\right)^2 \tag{5}$$

where $A_{sk}(y)$ is the equivalent panel cross-sectional area and η_t is the efficiency factor at the y spanwise station which can be obtained from a drawing of the wing cross section or may be estimated from [1]

$$\eta_t \approx \frac{1}{3}\left[1 + (t_{FS}/t)^2 + (t_{RS}/t)^2\right] - 0.025 \tag{6}$$

where t_{FS} and t_{RS} are the thickness at the front spar and rear spar, respectively, and t is the maximum thickness of the cross section perpendicular to the elastic axis. The efficiency factor can also be simply approximated as $\eta_t \approx 0.8$. Consequently,

$$A_{sk}(\mathrm{y_s}) = \frac{M(\mathrm{y_s})}{\eta_t t(\mathrm{y_s})\sigma(\mathrm{y_s})} \tag{7}$$

The total weight of skin materials required on both sides of the wing to resist bending is then obtained from

$$W_{sk} = 2\rho g \int_0^{b_s/2} \frac{M(\mathrm{y_s})}{\eta_t t(\mathrm{y_s})\sigma(\mathrm{y_s})} dy_s \tag{8}$$

Define the structural span fraction as

$$\eta_s = \frac{y_s}{b_s/2} \tag{9}$$

we then obtain

$$W_{sk} = 2\rho g \frac{b_s}{2} \int_0^1 \frac{M(\eta_s)}{\eta_t t(\eta_s)\sigma(\eta_s)} d\eta_s \tag{10}$$

where ρ is the material density. Notice that $A_{sk}(y_s)$ can be seen as the required cross-sectional area of a panel to resist bending which can be different between the upper and lower panels depending on the allowable stresses $\sigma(y_s)$; therefore, the integral term must be calculated for each of the upper and lower panels. By assuming that the efficiency factor and the allowable stress is constant in the spanwise direction, we obtain

$$W_{sk} = 2\frac{\rho g}{\eta_t \bar{\sigma}} \frac{b_s/2}{t(\eta_s=0)} \int_0^1 \frac{M(\eta_s)}{t(\eta_s)/t(\eta_s=0)} d\eta_s \tag{11}$$

where $t(\eta=0)$ is the thickness at the aircraft center line and $\bar{\sigma}$ is equal to the allowable tensile stress for the lower skin and the allowable compressive stress for the upper skin.

The thickness ratio $t(\eta)/t(\eta=0)$ represents the variation of local thickness along the span which generally results in complicated integration. To simplify the integral term, in this present method, the $t(\eta)/t(\eta=0)$ at the centroid of the solid trapezoidal prism $\bar{t}_{RC}$ representing a semi-span wing box is taken as the representative. However, it is found that many straight tapered wings have a pronounced decrease in the thickness-to-chord ratio t/c between wing root and the kink usually at 40 % semi-span from the centerline. In addition, in the previous development, the interconnection between fuselage and wing has been ignored. Therefore, to correct these effects, Torenbeek [1] suggested that the term $(b_s/2)/t(\eta=0)$ is replaced by the effective cantilever ratio R_C given as

$$R_C = \frac{b_s - b_{cs}}{2t_{cs}} \left(\frac{2}{3} + \frac{1}{3} \frac{(t/r)_r}{(t/r)_{0.4}} \right) \tag{12}$$

Consequently, the total weight of skin materials required to resist bending is then obtained from

$$W_{sk} = 2\frac{\rho g}{\eta_t \bar{\sigma}\, \bar{t}_{RC}} R_C \int_0^1 M(\eta_s)\, d\eta_s \tag{13}$$

or

$$W_{sk} = 2\frac{\rho g}{\eta_t \bar{\sigma}\, \bar{t}_{RC}} R_C I_M(\eta_s) \tag{14}$$

where the bending moment function is defined as

$$I_M(\eta_s) = \int_0^1 M(\eta_s)\, d\eta_s \tag{15}$$

3.2 Material Required to Resist Shear

At any spanwise station, the spar webs of the primary box are assumed to take the shear load $F(y_s)$ leading to shear stress $\tau(y_s)$ uniformly distributed over the webs. The area of web material required to resist shear load $A_{sw}(y_s)$ is

$$A_{sw}(\mathrm{y_s}) = F(\mathrm{y_s}) / \tau(\mathrm{y_s}) \tag{16}$$

The total weight of the web material required to resist the shear load is then obtained from

$$W_{sw} = 2\rho g \int_0^{b_s/2} A_{sw}(\mathrm{y_s})\, dy_s \tag{17}$$

By assuming a constant allowable shear stress along the span $\bar{\tau}$, we have

$$W_{sw} = 2\frac{\rho g}{\bar{\tau}} \frac{b_s}{2} \int_0^1 F(\eta_s)\, d\eta_s \tag{18}$$

or

$$W_{sw} = 2\frac{\rho g}{\bar{\tau}} \frac{b_s}{2} I_V(\eta_s) \tag{19}$$

where the shear function is defined as

$$I_V(\eta_s) = \int_0^1 F(\eta_s)\, d\eta_s \tag{20}$$

3.3 Lift Contribution on the Wing Wight

3.3.1 The Bending Moment due to Lift

At any spanwise wing station y from the aircraft centerline (Fig. 2), the local lift can be written as

$$dL(y') = q_\infty c_l c \, dy' \tag{21}$$

where q_∞ is the dynamic pressure, c_l is the section lift coefficient and c is the local cord. As

$$L = nW \cong q_\infty C_L \bar{c} b \tag{22}$$

where n is the load factor, W is the airplane weight and C_L is the wing lift coefficient. $\bar{c}$ is the geometric mean chord and b is the wing span. The lift distribution can be defined by means of the generalized circulation function $\gamma' = c_l c / (C_L \bar{c})$, and with $\eta' = y'/(b/2)$, we obtain

$$dL(y') = \frac{nW}{2} \gamma' \, d\eta' \tag{23}$$

The bending moment due to lift at a spanwise station y (Fig. 2), resulting from the lift contribution outboard of the station, can be evaluated as

$$M_{BL}(y_s) = \int_y^{b/2} \frac{y' - y}{\cos \Lambda_{ea}} dL(y') \tag{24}$$

or

$$M_{BL}(y_s) = \frac{nW}{2} \frac{b_s}{2} \int_\eta^1 \gamma' (\eta' - \eta) \, d\eta' \tag{25}$$

where Λ_{ea} is the angle of the elastic axis which is approximately equal to the mid-chord sweep $\Lambda_{1/2}$, and the structural span b_s is defined as

$$b_s = \frac{b}{\cos \Lambda_{ea}} \tag{26}$$

The location of the center of pressure y_{cp} on a semi-span wing can be obtained by setting

$$M_{BL}(y_s = 0) = \frac{nW}{2} \frac{b_s}{2} \int_0^1 \gamma' \eta' \, d\eta' = \frac{L}{2} \frac{y_{cp}}{\cos \Lambda_{ea}} \tag{27}$$

Therefore,

$$\eta_{cp} = \frac{y_{cp}}{b/2} = \int_0^1 \gamma' \eta' \, d\eta' \tag{28}$$

We then have

$$M_{BL}(y_s) = \frac{nW}{2} \frac{b_s}{2} \eta_{cp} I_1(\eta) \tag{29}$$

where

$$I_1(\eta) = \frac{\int_\eta^1 \gamma' (\eta' - \eta) \, d\eta'}{\int_0^1 \gamma' \eta' \, d\eta'} \tag{30}$$

For the straight-tapered wing, the $I_1(\eta)$ is rather independent on the lift distribution and can be approximated very well by [1]

$$I_1(\eta) = (1 - \eta)^{3 - 2\lambda + \lambda^2} \tag{31}$$

The center of pressure η_{cp} on the straight-tapered wing planform can be obtained from [1]

$$\eta_{cp} = \frac{2}{3\pi} + \frac{1 + 2\lambda}{6(1 + \lambda)} \qquad \text{when } \Lambda_{1/4} = 0 \tag{32}$$

The effect of wing sweepback can be taken into account by adding 0.00035 per degree of the quarter-chord line sweep $\Lambda_{1/4}$.

It can be seen that, for the simple straight-tapered wing planform, $I_1(\eta)$ and the center of pressure η_{cp} can be evaluated from Eqs. (31) and (32). However, for a general wing planform e.g. the cranked wing, a more general approach is required.

The lift distribution and the center of pressure on the actual wing planform can be computed by a simple theory such as lifting-line theory or by more sophisticated methods such as vortex lattice and panel methods. To simplify and reduce computational time, the concept of the equivalent wing planform given in ESDU [5] can be employed. The equivalent wing concept can be applied in principle to any wing with multi-cranked or curved leading and trailing edges to determine the overall aerodynamic characteristics of the wing. The approach is to define an "equivalent" straight-tapered wing planform which can adequately represent the actual wing planform so that the available data and prediction methods of overall aerodynamic characteristics of the straight-tapered wing planform can be employed to the equivalent planform. Based on the equivalent wing parameters of the actual wing planform, the bending moment due to lift can then be obtained from Eqs. (29), (31) and (32).

From a spanwise station η_s up to an outboard station η_{up}, and according to Eqs. (15) and (31), we have the moment function as

$$I_{M,BL}(\eta_s) = \frac{nW}{2}\frac{b_s}{2}\eta_{cp}\int_{\eta_s}^{\eta_{up}} I_1(\eta)d\eta_s \tag{33}$$

and, after the integration, we obtain

$$I_{M,BL}(\eta_s) = \frac{nW}{2}\frac{b_s}{2}\eta_{cp}\left(-\frac{(1-\eta_{up})^{(4-2\lambda+\lambda^2)} - (1-\eta_s)^{(4-2\lambda+\lambda^2)}}{4-2\lambda+\lambda^2}\right) \tag{34}$$

Therefore, the total weight of skin material required to resist bending due to lift can be obtained from Eq. (14) as

$$W_{BL} = 2\frac{\rho g}{\eta_t \bar{t}_{RC}} R_C\left(\frac{I_{M,BL}(\eta_s)}{\bar{\sigma}_t} + \frac{I_{M,BL}(\eta_s)}{\bar{\sigma}_c}\right) \tag{35}$$

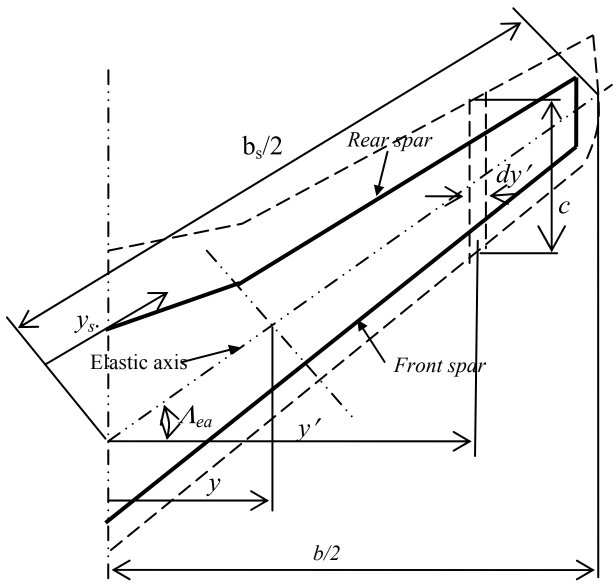

Fig. 2 Nomenclature for calculating the bending and shear due to lift

3.3.2 Shear Force due to Lift

The shear force due to lift at any spanwise station is due to the local lift contribution outboard of that station. Therefore; the shear force due to airload is given by

$$F_{SL}(y_s) = \int_{y}^{b/2} dL(y') \tag{36}$$

or

$$F_{SL}(y_s) = \frac{nW}{2} I_3(\eta) \tag{37}$$

where

$$I_3(\eta) = \int_{\eta}^{1} \gamma' d\eta' \tag{38}$$

According to Eq. (18), the total material required to resist the wing lift W_{SL} is

$$W_{SL} = nW\frac{\rho g}{\bar{\tau}}\frac{b_s}{2}\int_0^1 I_3(\eta)d\eta_s \tag{39}$$

It is found that the location of the center of pressure is also equal to the integral term. Therefore, the total material required to resist the wing lift can be obtained from

$$W_{SL} = nW\eta_{cp}\frac{\rho g}{\bar{\tau}}\frac{b_s}{2} \tag{40}$$

3.4 Inertia Relief due to Fuel Load

In this current method, the fuel load distribution is assumed to be trapezoidal shape representing the solid trapezoidal fuel tank as shown in Fig. 3.

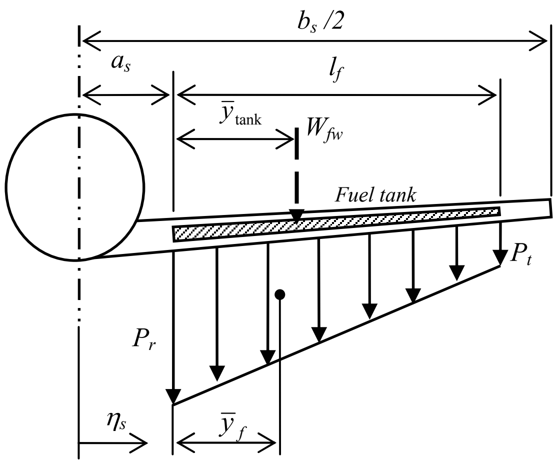

Fig. 3 Fuel load distribution

As

$$W_{fw} = \frac{nW_F}{2} \tag{41}$$

where W_F is the total fuel weight, we obtain

$$\frac{P_t}{P_r} = \frac{1-3k_f}{3k_f - 2} \quad ; k_f > 1/3 \tag{42}$$

where

$$k_f = \frac{\bar{y}_{tank}}{l_f} \tag{43}$$

and

$$P_t = \frac{2W_{fw}(3k_f - 1)}{l_f} \tag{44}$$

Define the span fractions as follows:

$$\eta_s = \frac{y_s}{b_s/2};\ \eta_{as} = \frac{a_s}{b_s/2};\ \overline{\eta} = \frac{\overline{y}_f}{b_s/2};\ \eta_{up} = \frac{a_s + l_f}{b_s/2} \qquad (45)$$

The shear and bending moment functions due to fuel load at any spanwise station can be calculated as the followings:

For $0 \le y_s \le a_s$,

$$I_{V,F1}(\eta_s) = W_{fw}\eta_{as} \qquad (46)$$

$$I_{M,F1}(\eta_s) = -\frac{W_{fw}\eta_{as}^2}{2}\frac{b_s}{2} + W_{fw}(\eta_{as} + \overline{\eta})\eta_{as}\frac{b_s}{2} \qquad (47)$$

For $a_s \le y_s \le a_s + l_f$,

$$\begin{aligned} I_{V,F2}(\eta_s) &= \frac{(P_r - P_t)(\eta_{up}^3 - \eta_{as}^3)}{6l_f}\left(\frac{b_s}{2}\right)^2 + \\ &+ \frac{-\frac{\left(\frac{(P_r - P_t)\left(l_f + \eta_{as}\frac{b_s}{2}\right)}{l_f} + 2P_t\right)}{2}\frac{b_s}{2} - \frac{(P_r - P_t)\left(l_f + \eta_{as}\frac{b_s}{2}\right)}{2l_f}\frac{b_s}{2}}{2}(\eta_{up}^2 - \eta_{as}^2) \\ &+ \frac{\frac{(P_r - P_t)\left(l_f + \eta_{as}\frac{b_s}{2}\right)}{l_f} + 2P_t}{2}\left(l_f + \eta_{as}\frac{b_s}{2}\right)(\eta_{up} - \eta_{as}) \end{aligned} \qquad (48)$$

$$\begin{aligned} I_{M,F2}(\eta_s) &= -\frac{(P_r - P_t)(\eta_{up}^4 - \eta_{as}^4)}{24l_f}\left(\frac{b_s}{2}\right)^3 \\ &+ \frac{\frac{P_t}{2}\left(\frac{b_s}{2}\right)^2 + \frac{(P_r - P_t)\left(l_f + \eta_{as}\frac{b_s}{2}\right)}{2l_f}\left(\frac{b_s}{2}\right)^2}{3}(\eta_{up}^3 - \eta_{as}^3) \\ &+ \frac{-P_t\left(l_f + \eta_{as}\frac{b_s}{2}\right)\left(\frac{b_s}{2}\right) - \frac{(P_r - P_t)\left(l_f + \eta_{as}\frac{b_s}{2}\right)^2}{2l_f}\left(\frac{b_s}{2}\right)}{2}(\eta_{up}^2 - \eta_{as}^2) \\ &+ \frac{P_t\left(l_f + \eta_{as}\frac{b_s}{2}\right)^2(\eta_{up} - \eta_{as})}{2} + \frac{(P_r - P_t)\left(l_f + \eta_{as}\frac{b_s}{2}\right)^3(\eta_{up} - \eta_{as})}{6l_f} \end{aligned} \qquad (49)$$

The relief weight due to fuel load ΔW_{Fuel} can then be obtained from Eqs. (14) and (19).

3.5 Inertia Relief due to Wing Structure Weight

As the wing weight is the outcome of the present method, determination of inertia relief effect due to wing structural weight is fundamentally iterative requiring an initial guesstimated value of the wing weight. The typical wing weight is in a range of 10-15 percent of the gross weight.

In this present method, it is assumed the wing structural weight is distributed in proportion to the square of the local chord (Fig. 4) as suggested in Howe [2]. Therefore, we obtain

$$W_{sw} = \frac{nW_{wing}}{2} \qquad (50)$$

$$P = k_w\left((c_t - c_r)\eta_s + c_r\right)^2 \qquad (51)$$

$$k_w = \frac{3W_{sw}}{c_t^2 + c_t c_r + c_r^2} \qquad (52)$$

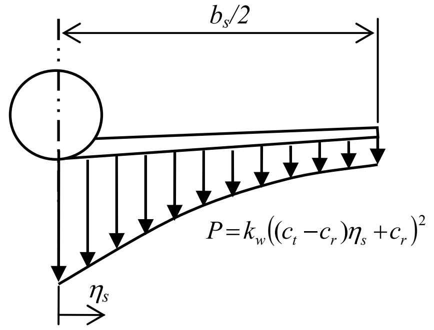

Fig. 4 Wing structural weight distribution

From a spanwise station η_s up to the outboard station η_{up}, the shear and bending moment functions due to structural weight are

$$\begin{aligned} I_{V,ST}(\eta_s) = k_w\Bigg[&-\frac{(c_t - c_r)^2(\eta_{up}^4 - \eta_s^4)}{12} - \frac{c_r(c_t - c_r)(\eta_{up}^3 - \eta_s^3)}{3} - \frac{c_r^2(\eta_{up}^2 - \eta_s^2)}{2} \\ &+ \frac{(c_t - c_r)^2\eta_{up}^3(\eta_{up} - \eta_s)}{3} + c_r(c_t - c_r)\eta_{up}^2(\eta_{up} - \eta_s) \\ &+ c_r^2\eta_{up}(\eta_{up} - \eta_s)\Bigg] \end{aligned} \qquad (53)$$

$$\begin{aligned} I_{M,ST}(\eta_s) = k_w\Bigg[&-\frac{(c_t - c_r)^2(\eta_{up}^5 - \eta_s^5)}{20} - \frac{c_r(c_t - c_r)(\eta_{up}^4 - \eta_s^4)}{6} \\ &- \frac{c_r^2(\eta_{up}^3 - \eta_s^3)}{6} + \frac{(c_t - c_r)^2\eta_{up}^4(\eta_{up} - \eta_s)}{4} + \frac{2c_r(c_t - c_r)\eta_{up}^3(\eta_{up} - \eta_s)}{3} \\ &+ \frac{c_r^2\eta_{up}^2(\eta_{up} - \eta_s)}{2}\Bigg] \end{aligned} \qquad (54)$$

The relief weight due to wing structure ΔW_{Str} can then be obtained from Eqs. (14) and (19).

3.6 Inertia Relief due to Concentrated Load

The bending and shear functions due to a concentrated load as shown in Fig. 5 are obtained from the simple beam analysis as

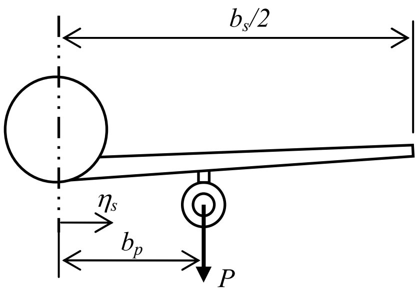

Fig. 5 Concentrated load acting on the wing

For $0 \le y_s \le b_p$,

$$\eta_p = \frac{b_p}{b_s/2} \tag{55}$$

$$I_{V,P}(\eta_s) = P\eta_p \tag{56}$$

$$I_{M,P}(\eta_s) = \frac{P\eta_p^2}{2}\frac{b_s}{2} \tag{57}$$

The relief weight due to concentrated load ΔW_P can then be obtained from Eqs. (14) and (19).

3.7 Torsion Load Contributions

It is suggested in Torenbeek [1] that the effect of torsional loads on static deformation can be taken into account by

- For straight unswept wings, decrease the maximum tension stress by 10% and increase the shear web weight by 20%.
- For taking account of sweepback, use the mid-chord sweepback angle $\Lambda_{1/2}$ instead of the sweep angle of elastic axis Λ_{ea}, and eliminate the sweepback correction on the center of pressure of the rigid wing.

To take account of aeroelastic effects, an empirical weight penalty is proposed as

$$\Delta W_{ae} = 0.05\frac{\rho g}{G} q_D \frac{(b\cos\Lambda_{le})^3(1-\sin\Lambda_{le})}{(t/c)_{ref}^2(1-M_D\cos^2\Lambda_{1/2})^{1/2}} \tag{58}$$

where $\rho g/G$ is equal to 10^{-6} per meter for Al-alloy, q_D and M_D denote the dynamic pressure and Mach number at the design diving speed, respectively, and Λ_{le} is the wing leading-edge sweep angle. The reference thickness-to-chord ratio $(t/c)_{ref}$ is considered at 70 percent of the semi-span wing. Typical wing weight penalty according to Eq. (99) is about 2 to 5 percent of the wing weight.

3.8. Weight of the Wing Ribs

Based on statistical data, the rib weight W_{rib} can be approximated [1] by

$$W_{Rib} = 0.5\times10^{-3}\rho g\, S\left(1+\frac{t_r+t_t}{2}\right) \tag{59}$$

Howe [2] suggested the rib weight estimation as

$$W_{Rib} = \frac{30.41\, S\, et_r^{0.5}}{1+\lambda}\left((1+\lambda+\lambda^2)+1.1t_r(1+\lambda+\lambda^2+\lambda^3)\right) \tag{60}$$

4. Allowable Stresses and Material Properties

The majority of present wings have been constructed of conventional aluminium alloy which the specific weight ρg may be taken as 28×10^3 N/m^3. The allowable stresses used in practical depend substantially on the type of material and type of loads carried.

The lower skin panels are mainly subjected to cyclic tensile stress and susceptible to fatigue failure. As a result, the aluminium alloy Al 2024-T3 which has very good fatigue resistance is normally employed. To cope with combined tensile and shear loading, it is suggested that the average allowable tensile stress $\bar{\sigma}_t \le 350\times10^6$ N/m^2 to be used in the lower skin panel [1, 2].

The upper skin panels are subjected to high compression stress and prone to buckling failure. To increase allowable buckling stress, the aluminium alloy Al 7075 is a typically choice for upper wing panels. It is recommended that the average allowable compression stress due to buckling limitation is [1]

$$\bar{\sigma}_c \le 0.8\times\left[400\times10^6(W_{TO}/10^6)^{1/4}\right] \quad (\text{N/m}^2) \tag{61}$$

Notice that the factor 0.8 is allowed for the reduction of panel loading outboard of the wing.

The allowable shear stress is typically about a half of the allowable buckling stress. In combination with the torsion-induced factor leading to 20% weight increase in the spar webs, the average allowable shear stress can be approximated by [1, 3]

$$\frac{\bar{\tau}}{\bar{\sigma}_c} \approx 0.42 \tag{62}$$

5. Corrections to the Idealized Box Weight

The idealized box weight is then corrected with additional penalty weights ΔW_{Pen} to make allowance for deviations from the idealized structure; for examples, non-tapered skins, joints, engine and landing gear mountings, cutouts, wing-to-fuselage connection etc. These effects cannot be considered analytically and they are relied on the empirical and statistical data of many existing aircraft. In this current approach, correction or penalty weights follow those given in Torenbeek [1].

6. Secondary Structure Weight

The secondary structure typically contributing 25 to 30 percent of the wing weight consists of the following groups:

- Fixed leading and trailing edges
- High-lift devices
- Control surfaces

In this current method, the weight estimations of secondary structures W_{Sec} also follow those presented in Torenbeek [1].

7. The Total Wing Weight

The total wing structural weight W_{wing} is the summation of the idealized wing box, penalty and secondary structure weights as follows:

$$W_{wing} = W_{Box,idel} + \Delta W_{ae} + \Delta W_{Pen} + W_{Sec} \tag{63}$$

or

$$W_{wing} = W_{Prim} + W_{Sec} \tag{64}$$

8. Applications and Discussions

The present method of wing structural weight estimation is tested against the original method [1], the semi-analytical 'F' method presented in Howe [2] and the empirical 'C_1' method given [2]. The Boeing 747-100 wing is used as the test case and the wing data are taken from Torenbeek [1]. The results are summarized in Table 1.

It can be seen from Table 1 that overall the results well agree with the actual wing weight, i.e., within an accuracy of $\pm$7 % although there are some discrepancies. The fuel relief factor Δr_f of the original method seems to be incorrect. Based on a simple check, it is found that the fuel relief factor should be in the range of -0.2 to -0.25. Therefore, if Δr_f is taken as -0.22, the total wing weight will be about -9 % error. There is considerable discrepancy in the rib weight estimation (all rib weight estimations are based on empirical data). This requires further investigation although the conservative prediction of the F method is recommended. As anticipated, the empirical approach, C_1 method, also provides an accurate prediction by a simple closed-form formula due to its nature of formulating. Although all of the methods presented here provide almost the same degree of accuracy, it should be noted that the present method is more generalized and, therefore, will predict the wing structural weight more closely related to the actual wing design. Consequently, sensitivity study can be done in greater detail and accuracy.

The present method can be applied to the cranked wing by applying the method to the inner wing and outer wing separately (Fig. 6) enabling the influence of wing geometry on structure and aerodynamics of the cranked wing to be investigated. Due to the lack of actual data for an aircraft with cranked wing design, it is not possible at this stage to validate the present method. However, based on geometrical-based analyses and validation against the conventional straight-tapered wing, the method should provide relatively accurate prediction of the structural weight and indicate the correct trend of the weight growth of the cranked wing design.

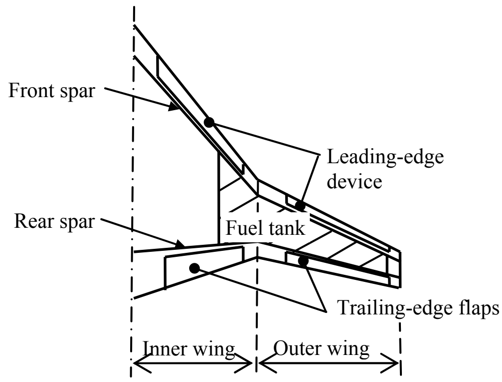

Fig. 6 Cranked wing planform

9. Conclusions

A development of the semi-analytical approach for wing structural weight estimation is presented. The present method follows mostly the method developed by Torenbeek [1]; however, the geometrical-based analyses are generalized. As a result, the method can be employed to predict the wing structural weight of the conventional straight-tapered planform and the highly cranked wing planform. The emphasis is on the development of the weight analysis of the primary box

Table 1 Comparison of wing weight estimation breakdown

Wing Weight Breakdown	Semi-Analytical Approach			Empirical Approach	Actual Wing Weight (B747-100)
	Original Method [1]	F Method [2]	Present Method	C_1 Method [2]	
Skin and spar web weights for resisting airload, W_{Lift} (kN)	271.459	–	275.054	–	–
Fuel relief factor, Δr_f	-0.0974	-0.258	-0.223	–	–
Structure weight relief factor, Δr_s	-0.096	-0.117	-0.084	–	–
Engine relief factor, Δr_p	-0.095	-0.10	-0.095	–	–
Total relief factor, r	0.7116	0.525	0.598	–	–
Rib weight, W_{Rib} (kN)	16.275	46.457	16.275	–	–
Idealized box-structure weight, $W_{Box,idel}$ (kN)	209.45	188.948	180.824	–	–
Weight required for torsional stiffness, ΔW_{ae} (kN)	13.010	-	13.010	–	–
Penalty weights, ΔW_{Pen} (kN)	36.211	44.217	36.211	–	–
Primary box weight, W_{Prim} (kN)	258.836	-	230.045	–	–
Secondary structure weight, W_{Sec} (kN)	132.717	127.915	136.274	–	–
Total wing weight, $\boldsymbol{W_{wing}}$ (kN)	391.533	407.537	366.320	368.312	384.4
% Error	+1.86	+6.02	-4.70	-4.19	0

structure since the box structure contributes up to 70 percent of the total wing weight and it has well defined structural and aerodynamic roles. The present method provides an accurate prediction of the wing structural weight, less than 5 % error, and the results obtained well agrees with the other estimation methods.

The present method is relatively simple, and can be easily coded and used as an effective design tool for a wing design at the initial design phase. The original method is revised and clarified in some details, and some new procedures are introduced as follows:

- Equivalent wing planform concept can be employed to take account of aerodynamic lift of the cranked planform which influences the bending moment due to lift on the wing.
- Linear distribution of the fuel load is assumed and replaced the statistical data or point load methods for calculating the fuel load relief; as a result, inertia relief due to fuel load can be calculated for any section of the wing.
- The wing structural weight distribution is assumed in proportion to the square of the local chord, and the structural weigh relief is obtained analytically. Therefore, inertia relief due to wing structure weight can be obtained for any section of the wing.
- The weight relief due to a concentrated load is derived explicitly.

Following these further developments, this present method enables the sensitivity study of wing design parameters of the conventional straight-tapered wing and the cranked wing to be investigated in greater details.

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