The 19th Conference of Mechanical Engineering Network of Thailand
19-21 October 2005, Phuket, Thiland

Kinematic Evaluation of the High Class Kinematic Group Mechanisms

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Abstract
High class kinematic groups are commonly present as the consisting components of complex industrial mechanisms. Their geometric and kinematic description can not be done in a standard way. Due to the extensive use of computers, development of efficient analytical methods gains more importance. In this paper a new method for describing this class of mechanisms is proposed. The basic idea of this method is to decompose the high class kinematic group to second class kinematic groups. System of equations are obtained from the kinematic constrains in the linkage. The Banach's principle of contraction is used and position of the mechanism is obtained through fixed point iteration method. This method is very efficient and quickly gives accurate solutions concerning the position of mechanism.
Keywords: High class kinematic group, Position analysis

1. Introduction

In search for mechanism that can fulfill certain task it is interesting to examine those with kinematic groups of higher classes because they can offer multiple solutions. Existence of multiple solutions means that high class kinematic group and therefore, the whole mechanism, can be assembled in various configurations which can be further optimized.

As the position analysis is performed on the level of kinematic groups, the complexity of the problem depends on the structure of the mechanism being analyzed, namely on the class of the kinematic groups creating it.

For the second class groups of the different forms the analytical relations determining positions of the moving links have been derived in the explicit form [1].

Determination of the links position in the higher class groups is more complex, which results from the complex structure of such mechanism. Most of the methods used so far lead to a system of highly nonlinear equations which makes finding the explicate solution impossible. Numerical methods are commonly applied for solving this system of equations [2]. Application of these methods has many problems:

- For the method convergence, the starting values of variables must be close to exact solutions which require sketching of system configuration.
- Absence of convergence exists when system is very close to singular positions.
- Non-linear equations always have multiple solutions, number of which is unknown.

Method presented in this paper can easily overcome mentioned problems. The basic idea of this method is to decompose the high class kinematic group to second class kinematic groups (dyads). System of equations are obtained from the kinematic constrains in the linkage. The Banach's principle of contraction is used and position of the mechanism is obtained through fixed point iteration method [3], [4].

2. Kinematic analysis of the complex mechanism

Well known Chebyshev rowing mechanism is presented in Figure 1.

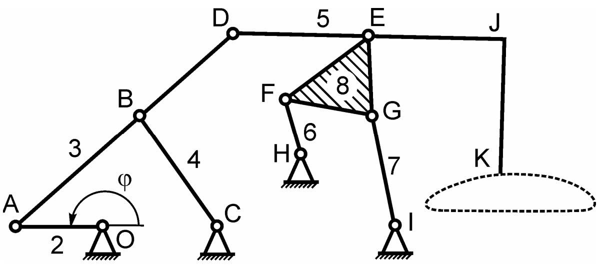

Figure 1. Complex mechanism

Position (and later velocity and acceleration) of the tip K has to be determined. Position angle φ, as well as angular velocity and acceleration of the input link 2 are known. Since velocity and acceleration analysis yield to linear equation problem, prime concern is going to be mechanism position analysis, namely determination of its initial position. With known initial position mechanism motion simulation is easily solved as subsequent array of finite displacement problems.

Mechanism shown in Figure 1. is a complex mechanism consisting of three kinematic groups (Figure 2.): the first class kinematic group - input link 2, the second class kinematic group – links 3 and 4 and the third class kinematic group – links 5, 6, 7 and 8.

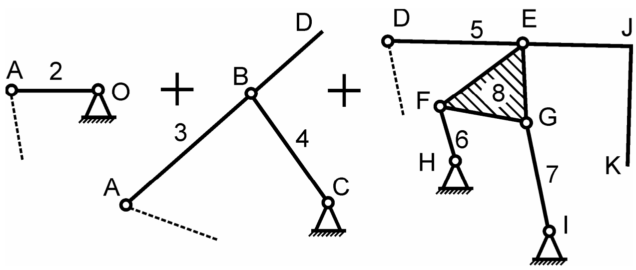

Figure 2. Mechanism constitutive kinematic groups

Kinematic analysis will be performed in exactly the same order as the mechanism assembly.

First, position of point A on the first class kinematic group will be calculated:

$$x_A = x_O + \overline{OA}\cos\varphi_2 \qquad y_A = y_O + \overline{OA}\sin\varphi_2 \tag{1}$$

Next, second class kinematic group (links 3 and 4) is added to link 2 at the point A thus forming fourbar mechanism OABC (Figure 3.)

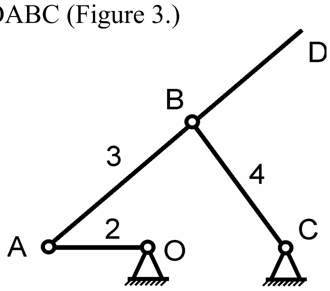

Figure 3. Fourbar mechanism

Analytical relations determining positions of the moving links for the second class group can be derived in the explicit form. After the calculations position of points B and D will be determined.

At the end, the third class kinematic group is connected to the fourbar mechanism at the point D. Now, with known position of D, H and I, position analysis is performed using fixed point iteration method and all relevant kinematic parameters are calculated .

Before solving the particular example, theoretical concepts for kinematic analysis of second and third class kinematic groups will be presented.

2.1 Kinematic analysis of the second class kinematic group

General form of the dyad consisting of two links connected by rotational joint is presented in Figure 4. External joints A_1 and A_2 are of the rotational type so this type of dyad is called RRR type. Positions of the external joints A_1 and A_2 are known, while position of the middle joint A_3 has to be determined.

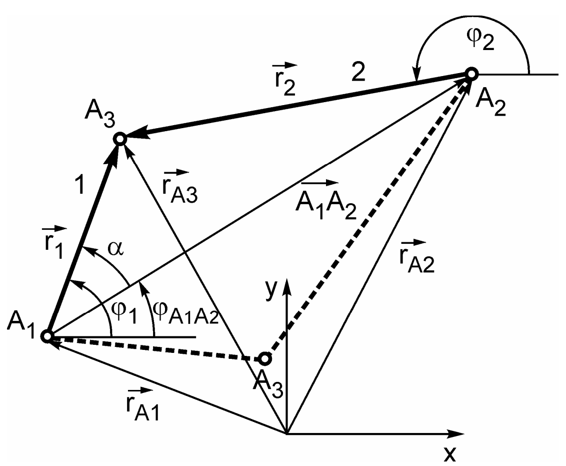

Figure 4. Dyad (type RRR)

Vector equation describing dyad is:

$$\vec{r}_{A1} + \vec{r}_1 = \vec{r}_{A2} + \vec{r}_2 = \vec{r}_{A3} \tag{2}$$

New vector is introduced:

$$\overrightarrow{A_1A_2} = \vec{r}_{A2} - \vec{r}_{A1} \tag{3}$$

where:

$$\overline{A_1A_2}^2 = (x_{A2} - x_{A1})^2 + (y_{A2} - y_{A1})^2 \tag{4}$$

$$\varphi_{A1A2} = arctan\left(\frac{y_{A2} - y_{A1}}{x_{A2} - x_{A1}}\right) \tag{5}$$

Now angle α can be obtained:

$$\alpha = \pm arccos\left(\frac{r_1^2 + \overline{A_1A_2}^2 - r_2^2}{2r_1\overline{A_1A_2}}\right) \tag{6}$$

Position angle of link 1 is:

$$\varphi_1 = \varphi_{A1A2} \pm \alpha \tag{7}$$

Sign $\pm$ represents two possible configurations of dyad assembly (Figure 4. - full and dotted line). Now, position of joint A_3 can be obtained as:

$$x_{A3} = x_{A1} + r_1\cos\varphi_1 \qquad y_{A3} = y_{A1} + r_1\sin\varphi_1 \tag{8}$$

Position angle of link 2 is:

$$\varphi_2 = arctan\left(\frac{y_{A3} - y_{A2}}{x_{A3} - x_{A2}}\right) \tag{9}$$

2.2 Kinematic analysis of the third class kinematic group

General case of the third class kinematic group with vectors describing its position is presented in Figure 5.

It is of the RR-RR-RR type - central link 4 is connected by rotational joints with binary links 1, 2, and 3, which are furthermore, connected to the rest of the mechanism by the rotational joints also. Points A, B and C are called internal, and D, E and F external ones. Positions of all external points are known while positions of the internal points A, B and C have to be determined.

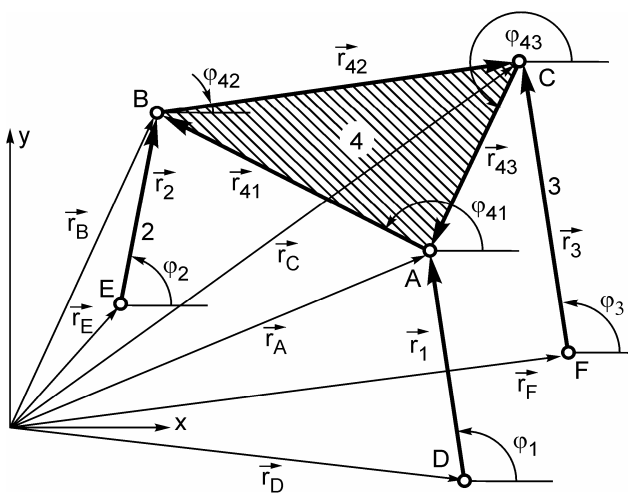

Figure 5. Vectors describing position of the third class kinematic group

In order to perform position analysis the third class kinematic group will be decomposed into three second class kinematic groups which will be analyzed using procedure described in 2.1.

STEP 1 – start, assuming $\vec{r}_1$ ($\vec{r}_1^{\,0}$), φ_1 ($\varphi_1^{\,0}$), $\vec{r}_A$ ($\vec{r}_A^{\,0}$)

First, value for angle φ_1 is assumed. Position of point A can be then calculated as:

$$\vec{r}_A = \vec{r}_D + \vec{r}_1 \tag{10}$$

STEP 2 – forming first dyad, obtaining $\vec{r}_B^{\,0}$

Now, a dyad of type RRR (links BE and BA) is formed:

$$\vec{r}_A + \vec{r}_{41} = \vec{r}_E + \vec{r}_2 = \vec{r}_B \tag{11}$$

Using equations (2)-(9) position of point B is obtained. Since angle φ_2 has two completely distinctive solutions there will be also two solutions for both φ_{41} and $\vec{r}_B$. One set (φ_2, φ_{41}, $\vec{r}_B$) has to be chosen in order to continue.

STEP 3– forming second dyad, obtaining $\vec{r}_C^{\,0}$

Links CF and CB form a RRR dyad:

$$\vec{r}_B + \vec{r}_{42} = \vec{r}_F + \vec{r}_3 = \vec{r}_C \tag{12}$$

Using equations (2)-(9) position of point C is obtained. Again, the angle φ_3 has two distinctive solutions leading to two solutions for both φ_{42} and $\vec{r}_C$. One set (φ_3, φ_{42}, $\vec{r}_C$) has to be chosen in order to continue.

STEP 4 – forming third dyad, obtaining $\vec{r}_1^{\,1}$, $\varphi_1^{\,1}$, $\vec{r}_A^{\,1}$

Now, a dyad of type RRR (links AC and AD) is formed:

$$\vec{r}_C + \vec{r}_{43} = \vec{r}_D + \vec{r}_1 = \vec{r}_A \tag{13}$$

Using equations (2)-(9) position of point A is obtained. One solution for φ_1 i.e. φ_{43} and $\vec{r}_A$ has to be chosen. With chosen φ_1, new value $\vec{r}_1^{\,1}$ for vector $\vec{r}_1$ is obtained. Difference between new and starting value is an error vector:

$$\Delta\vec{r}_1^{\,1} = \vec{r}_1^{\,0} - \vec{r}_1^{\,1} \tag{14}$$

This end the first iteration (Figure 6.).

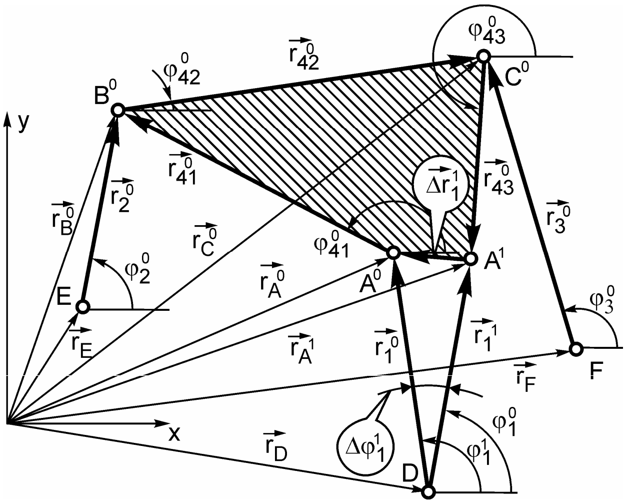

Figure 6. First iteration

Second iteration is initiated with vector $\vec{r}_1^{\,1}$, in order to obtain $\vec{r}_1^{\,2}$ ($\varphi_1^{\,2}$, $\vec{r}_A^{\,2}$). The succeeding iterations are done in the previously shown way. The procedure is interrupted when the following condition is met:

$$\left|\Delta\vec{r}_1^{\,n}\right| < \varepsilon \tag{15}$$

Using this method, error vector is significantly reduced in each iteration - vector $\vec{r}_1$ converges towards its accurate value. In most cases only few iterations were necessary for the condition (15) to be met.

All three dyads, which this third class kinematic group is consisted of, have two solutions (assembly configurations) (Eqs. (11)-(13)), so there are, in total, eight different vector contours that can describe the mechanism. Vector contour formed in the presented way has a single solution for each kinematic parameter, so problem of the number of possible solutions, i.e. possible mechanism configurations, comes down to determination of the number of different vector contours – in this case there exist eight of them. Real solutions are determined by examining, one by one, all of the eight contours.

Through Eqs. (10)-(14), indirectly, following function is formed:

$$\varphi_1 = f(\varphi_1) \tag{16}$$

Before implementing fixed point iteration method to solve Eq. (16) for φ_1, necessary conditions for method convergence have to be investigated. Analytical form of $f(\varphi_1)$ can be extremly complex, so combination of numerical and graphical methods are used and solution is found visually (Figure 7.).

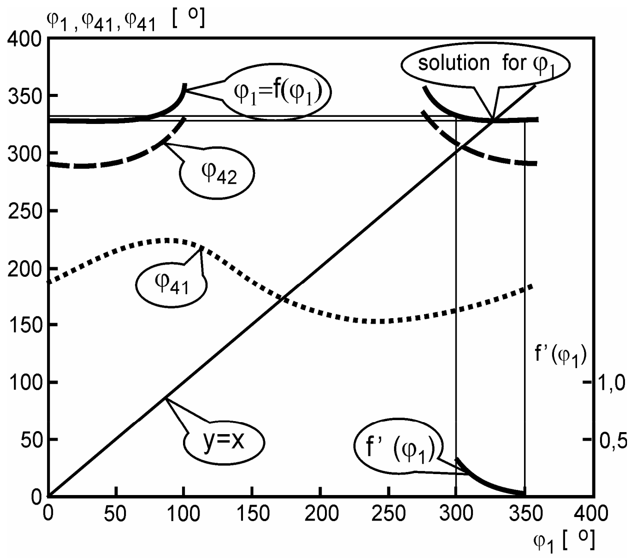

Figure 7. Procedure for checking fixed point iteration method convergence necessary conditions

Function $f(\varphi_1)$ is continuous in [300°,350°], for $300° < \varphi_1 < 350° \Rightarrow 300° < f(\varphi_1) < 350°$, $\max\left|f'(\varphi_1)\right| < 1$ in the interval so there exist a unique solution $\varphi_1 = \alpha$ of (16) in [300°,350°], and for any initial guess in [300°,350°] iteration procedure will converge to α. (Figure 7. corresponds to Solution 2.4 of the example.)

Now, fixed point iteration procedure is invoked, and solution for φ_1 and thereby for all other kinematic parameters of the third class kinematic group, can be calculated with prescribed accuracy.

After repeating the complete procedure – once for each vector contour all possible real solutions for φ_1 and, thus, all possible third class assembly configuration are obtained.

3. Example

Data for the complex mechanism (Figure 1.) are:

$O(0,0)$	$\overline{DE} = 130\ mm$	$\overline{EG} = 75\ mm$
$C(110,0)$	$\overline{EF} = 100\ mm$	$H(190,70)$
$\overline{AB} = 160\ mm$	$\overline{FH} = 55\ mm$	$I(280,0)$
$\overline{BC} = 130 mm$	$\overline{FG} = 85\ mm$	$\overline{EJ} = 130\ mm$
$\overline{BD} = 120 mm$	$\overline{GI} = 110\ mm$	$\overline{JK} = 130\ mm$
$\angle EJK = 90°$	$\varphi = 180°$	

First, position of point A is calculated (Eq. (1)):

$x_A = 0 \qquad y_A = -85\ mm$

Next, using Eqs. (2) – (9), where points A, B and C are considered as A_1, A_2 and A_3 respectively, two possible solutions for point B, i.e. two assembly configurations of the four bar mechanism OABC are obtained (Figure 8.).

Position of point D is calculated as it is going to be an input for the next step – position analysis of the third class kinematic group:

$$\begin{aligned} x_D &= x_A + \overline{AD} \cos \varphi_1 \\ y_D &= y_A + \overline{AD} \cos \varphi_1 \end{aligned} \tag{16}$$

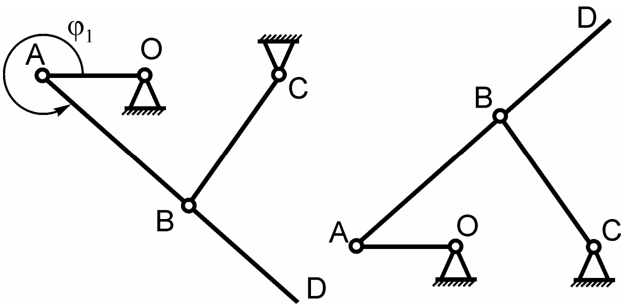

Figure 8. Possible assembly configurations of the four bar mechanism OABC

Solution 1:	Solution 2:
$\angle \overrightarrow{AB} = \varphi_1 = 318.49°$	$\angle \overrightarrow{AB} = \varphi_1 = 41.51°$
$\angle \overrightarrow{CB} = 234.66°$	$\angle \overrightarrow{CB} = 125.34°$
$B(34.81, 106.05)\ mm$	$B(34.81, -106.05)\ mm$
$D(124.66, 185.58)\ mm$	$D(124.66, -185.58)\ mm$

Position of points D (from the first phase) and H and I (connections to the ground link) are known. Now, following the procedure described in 2.2. position analysis of the third class kinematic group is performed. Positions of internal joints E, F and G as well as all links position angles are determined.

With angle φ_2 determined position of the tip point K is easily obtained as:

$$\begin{aligned} x_K &= \overline{DJ} \cos \varphi_2 + \overline{JK} \cos(\varphi_2 + 270°) \\ y_K &= \overline{DJ} \sin \varphi_2 + \overline{JK} \sin(\varphi_2 + 270°) \end{aligned} \tag{17}$$

Procedure has to be executed twice – first time for four bar mechanism Solution 1. and next for four bar mechanism Solution 2.

For the first case (Figure 9.) there exist two real solutions, while for the second case there are four of them (Figure 10.).

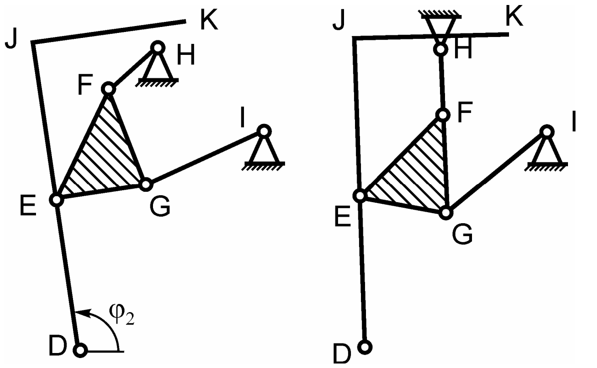

Figure 9. Possible assembly configurations for the third class kinematic group (four bar mechanism Solution 1.)

Solution 1.1:	Solution 1.2:
$\angle \overrightarrow{DE} = \varphi_2 = 98.32°$	$\angle \overrightarrow{DE} = \varphi_2 = 90.10°$
$\angle \overrightarrow{IG} = 204.69°$	$\angle \overrightarrow{IG} = 219.46°$
$\angle \overrightarrow{HF} = 221.97°$	$\angle \overrightarrow{HF} = 272.29°$
$E(105.87, -56.95)\ mm$	$E(121.44, -55.62)\ mm$
$F(149.11, 33.21)\ mm$	$F(192.20, 15.04)\ mm$
$G(180.06, -45.95)\ mm$	$G(195.071, -69.91)\ mm$

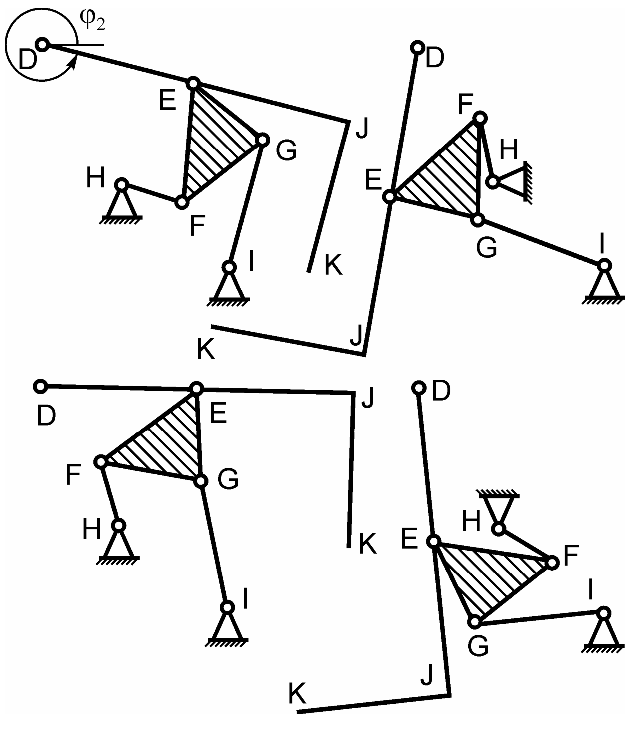

Figure 10. Possible assembly configurations for the third class kinematic group (four bar mechanism Solution 2.)

Solution 2.1:	Solution 2.2:
$\angle\overrightarrow{DE} = \varphi_2 = 345.57°$	$\angle\overrightarrow{DE} = \varphi_2 = 82.33°$
$\angle\overrightarrow{IG} = 74.74°$	$\angle\overrightarrow{IG} = 159.30°$
$\angle\overrightarrow{HF} = 342.28°$	$\angle\overrightarrow{HF} = 101.68°$
$E(250.56, 153.18)mm$	$E(104.36, 57.16)mm$
$F(242.41, 53.32)mm$	$F(178.86, 123.86)mm$
$G(308.96, 106.12)mm$	$G(177.10, 38.88)mm$

Solution 2.3:	Solution 2.4:
$\angle\overrightarrow{DE} = \varphi_2 = 358.75°$	$\angle\overrightarrow{DE} = \varphi_2 = 275.97°$
$\angle\overrightarrow{IG} = 101.40°$	$\angle\overrightarrow{IG} = 185.92°$
$\angle\overrightarrow{HF} = 106.27°$	$\angle\overrightarrow{HF} = 328.89°$
$E(254.63, 182.74)mm$	$E(138.18, 56.29)mm$
$F(174.58, 122.79)mm$	$F(237.09, 41.59)mm$
$G(258.26, 107.83)mm$	$G(170.59, -11.35)mm$

Complete mechanism configurations are obtained as combinations of four bar mechanism solutions and the corresponding third class group solutions (Figure 11.):

Solution 1. + Solution 1.1 = Complete mechanism 1.
Solution 1. + Solution 1.1 = Complete mechanism 2.
Solution 2. + Solution 2.1 = Complete mechanism 3.
Solution 2. + Solution 2.1 = Complete mechanism 4.
Solution 2. + Solution 2.2 = Complete mechanism 5.
Solution 2. + Solution 2.3 = Complete mechanism 6.

At the end, it can be concluded that there exist, in total, six possible solutions for the complete mechanism position analysis i.e. there are six possible mechanism assembly configurations.

4. Conclusion

Kinematic analysis, mainly the position analysis of the mechanisms with high class kinematic groups presents a serious problem. The aim of the position analysis is to determine all possible configuration of the mechanism, for a given position of its input link. When the position is determined, then the analysis of velocities and accelerations becomes very simple.

In this paper a method for describing this class of mechanisms is proposed. Mechanism is decomposed to its constitutive parts (kinematic groups). Kinematic analysis is then performed for each kinematic group separately, strictly following the order of the mechanism assembly. Complete mechanism configurations are obtained as combinations of the corresponding solutions.

For the second class kinematic groups of the different forms the analytical relations determining positions of the moving links have been derived in the explicit form.

Determination of the links position in the higher class groups is more complex, and methods used so far encountered several serious problems.

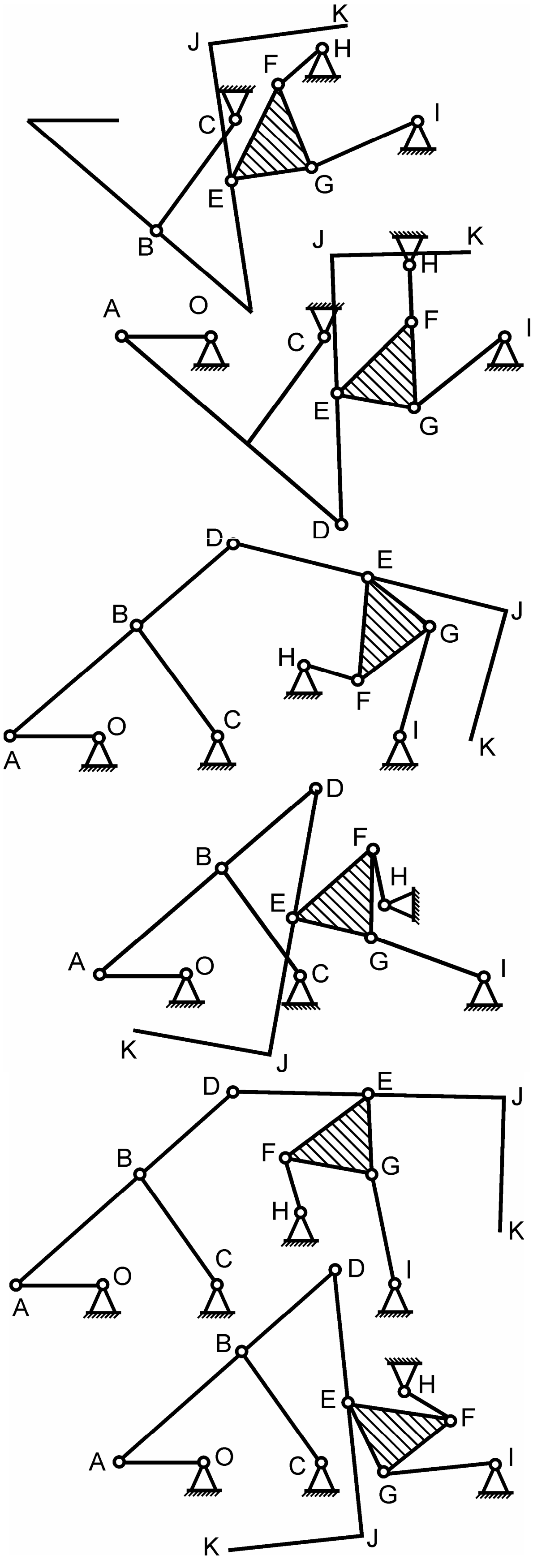

Figure 11. All possible assembly configurations of the complete mechanism

Method for position analysis presented in this paper is general and very efficient and easily overcomes problems met when using typical numerical methods.

Its basic idea is to decompose the high class kinematic group to second class kinematic groups. System of equations are obtained systematically from the kinematic constrains in the linkage. The Banach's principle of contraction is used and position of the mechanism is obtained through fixed point iteration method. Accurate solutions concerning the position of mechanism are obtained very quickly - in only few iterations.

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