ETM046

การประชุมวิชาการเครือข่ายวิศวกรรมเครื่องกลแห่งประเทศไทยครั้งที่ 19
19-21 ตุลาคม 2548 จังหวัดภูเก็ต

Nodeless Finite Element Method for 2D Heat Transfer Problems

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Abstract

Nodeless finite element method is presented to predict the temperature distribution for heat transfer problems. The paper first describes 2D heat transfer theory. The finite element formulations based on nodeless element, the computational procedure and its boundary conditions are then represented. The validated examples with analytical solution of the proposed technique are rectangular plate with periodic temperature problem and rectangular plate with internal heat generation as well as periodic temperature problem. The solutions show that the nodeless finite element method can be employed to predict the temperature distribution efficiently.

1. Introduction

Finite element method is applied to solve heat transfer problems for a decade. Steady–state heat transfer can be predicted by applying the method of weighted residual (MWR). The various element types and their element interpolation functions are widely developed such as 3–node triangle with linear element interpolation function, 6–node triangle with quadratic element interpolation function, 10–node triangle with cubic element interpolation function, and etc.

This paper presents nodeless finite element method to solve 2D steady state heat transfer problem. Nodeless triangular elements and their element interpolation functions are described. Then, the computational procedure and its boundary conditions are shown. Next, the computational solutions by nodeless finite element method are validated with the exact solution and the finite element solutions using linear triangular element, respectively.

2. Theory

2.1 Governing differential equation

Steady–state heat transfer problem is governed by energy equation as following.

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)=Q \qquad (1)$$

where k is the thermal conductivity, T is the temperature and Q is the internal heat generation.

2.2 Element interpolation functions and finite element matrices [1]

Nodeless triangular element consists of 3 nodes and 3 nodeless per element as shown in figure 1.

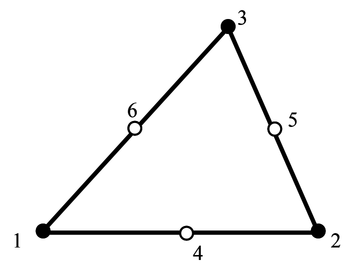

Figure 1. Nodeless element and its connectivity.

Its element interpolation functions are in the form,

$$N_i(x,y) = \frac{1}{2A}\left(a_i + b_i x + c_i y\right) \qquad i = 1, 3 \qquad (2)$$

$$N_4 = 4N_1 N_2 \;\; ; \;\; N_5 = 4N_2 N_3 \;\; ; \;\; N_6 = 4N_1 N_3 \qquad (3)$$

where

$$a_i = x_j y_k - x_k y_j \;,\; b_i = y_j - y_k \;,$$
$$c_i = x_k - x_j \qquad i, j=1,3$$

a_i , b_i , and c_i coefficients are obtained by cyclically permuting the subscripts, and A is the triangular area.

After applying MWR in equation 1, the finite element equation based on Bubnov–Galerkin method is obtained to solve 2D heat transfer problem.

$$\left([K_c]+[K_h]\right)\{T\} = \{Q_c\}+\{Q_Q\}+\{Q_q\}+\{Q_h\} \tag{4}$$

where

$$[K_c] = \int_A [B]^T [k][B] dA$$

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial x} & \frac{\partial N_5}{\partial x} & \frac{\partial N_6}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} & \frac{\partial N_4}{\partial y} & \frac{\partial N_5}{\partial y} & \frac{\partial N_6}{\partial y} \end{bmatrix}$$

$$\frac{\partial N_1}{\partial x} = \frac{y_2 - y_3}{2A} \quad , \quad \frac{\partial N_1}{\partial y} = \frac{x_3 - x_2}{2A}$$
$$\frac{\partial N_2}{\partial x} = \frac{y_2 - y_1}{2A} \quad , \quad \frac{\partial N_2}{\partial y} = \frac{x_1 - x_3}{2A}$$
$$\frac{\partial N_3}{\partial x} = \frac{y_1 - y_2}{2A} \quad , \quad \frac{\partial N_3}{\partial y} = \frac{x_2 - x_1}{2A}$$
$$\frac{\partial N_4}{\partial x} = \frac{(y_2 - y_3)N_2 + 2(y_3 - y_1)N_1}{A}$$
$$\frac{\partial N_4}{\partial y} = \frac{(x_3 - x_2)N_2 + 2(x_1 - x_3)N_1}{A}$$
$$\frac{\partial N_5}{\partial x} = \frac{(y_3 - y_1)N_3 + 2(y_1 - y_2)N_2}{A}$$
$$\frac{\partial N_5}{\partial y} = \frac{(x_1 - x_3)N_3 + 2(x_2 - x_1)N_2}{A}$$
$$\frac{\partial N_6}{\partial x} = \frac{(y_1 - y_2)N_1 + 2(y_2 - y_3)N_3}{A}$$
$$\frac{\partial N_6}{\partial y} = \frac{(x_2 - x_1)N_1 + 2(x_3 - x_2)N_3}{A}$$

$$[K_h] = \int_S h\{N\}\lfloor N \rfloor dA$$
$$\{Q_c\} = \int_S \left(k\frac{\partial T}{\partial x} n_x + k\frac{\partial T}{\partial y} n_y\right)\{N\} dS$$
$$\{Q_Q\} = \int_A Q\{N\} dA$$
$$\{Q_q\} = \int_S q_s\{N\} dS$$
$$\{Q_h\} = \int_S hT_\infty\{N\} dS$$

3. Examples

A rectangular plate with sinusoidal thermal load is presented as the first example to validate nodeless finite element method for 2D steady–state heat transfer. The results gained from the nodeless finite element calculation are compared to the exact solution and the approximation solutions using 3–node triangular element. Then, applying internal heat generation in plate to evaluate the performance of nodeless finite element to solve 2D steady–state heat transfer problem.

3.1 Rectangular plate with sinusoidal temperature

Rectangular plate with dimension 0.5x1 unit is applied with a sinusoidal temperature function at the top edge and the constant zero temperature is applied along the left edge as shown in figure 2.

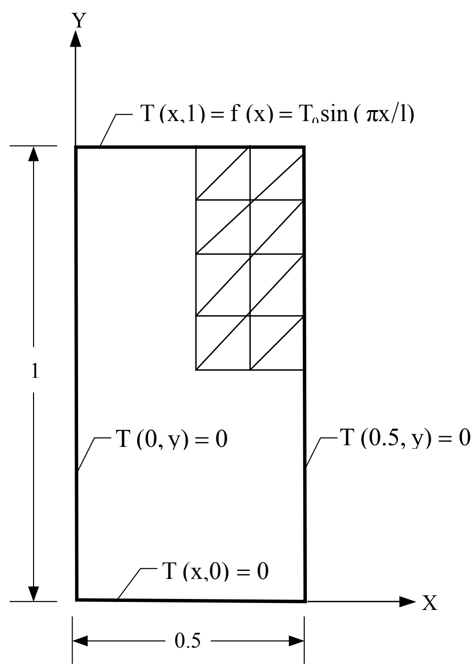

Figure 2. Problem statement of rectangular plate.

This problem has an exact solution in the form,

$$T(x,y) = \frac{\sin(2\pi x)\times\sinh(2\pi y)}{\sinh(2\pi)} \tag{5}$$

where $T_o = 1.0$

After applying nodeless finite element method, the solution of temperature distribution is shown in figure 3.

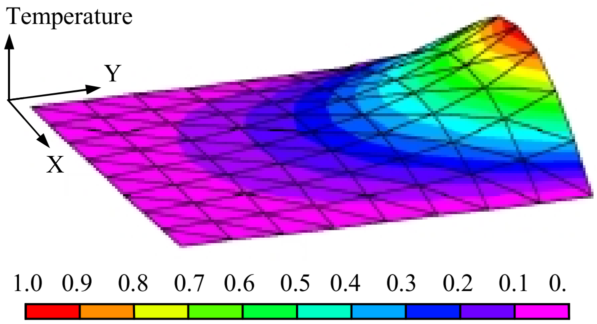

Figure 3. Temperature distribution on rectangular plate.

To validate the accuracy of nodeless finite element method, the rectangular plate is discretized in several finite element model i.e. 32 elements, 48 elements, 64 elements, 72 elements, 96 elements, 112 elements, 256 elements, and 400 elements as shown in figure 4.

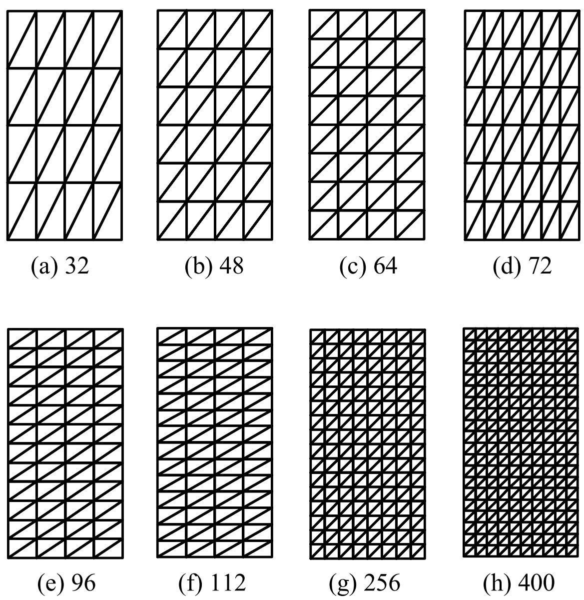

Figure 4. Finite element number of rectangular plate.

Figure 5 shows temperature solution along x direction at y = 0.5 of nodeless finite element solution comparing with exact solution and linear triangular element solution. Figure 6 displays temperature solution along y direction at x = 0.25. The results express a good accuracy of nodeless finite element method in both of 32 elements and 400 elements.

The computational error has been collected in every models and displayed in figure 7 and 8. The results show that nodeless element has fewer errors than linear element in every element number.

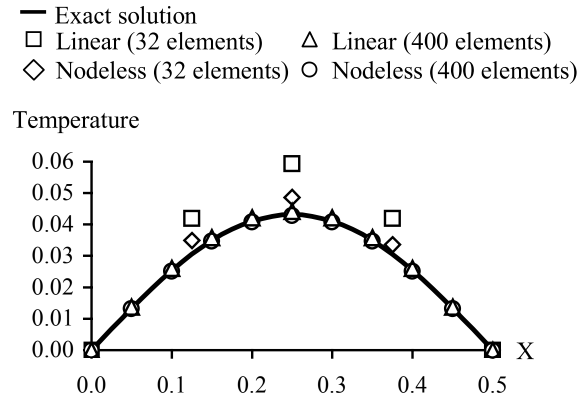

Figure 5 Temperature distribution along x axis at y = 0.5.

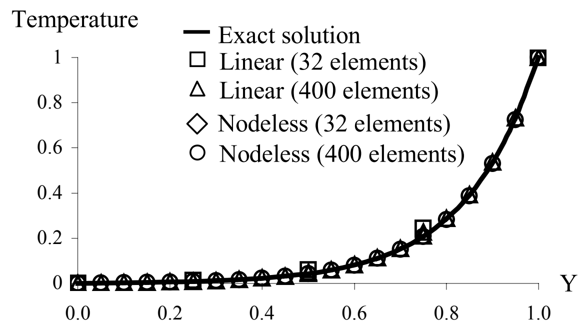

Figure 6 Temperature distribution along y axis at x = 0.25

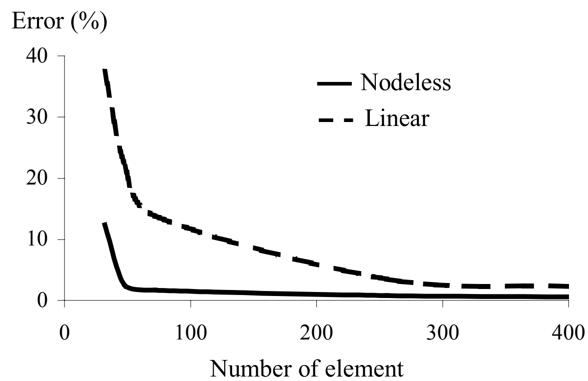

Figure 7 Error of temperature along x axis at y = 0.5

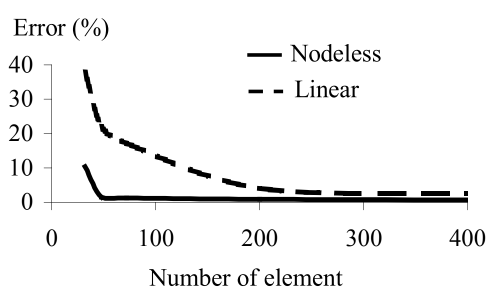

Figure 8 Error of temperature along y axis at x = 0.25

3.2 Rectangular plate with sinusoidal temperature and internal heat generation

This example is more complicated than the previous example. The dimension of geometry is similar to the first example. Thermal load consists of sinusoidal temperature at the top edge and the internal heat generation.

The exact solution of this problem is in the infinite series form as shown below,

$$T(x,y) = \frac{(1-(x-0.5a)^2)}{2} - \frac{16}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n \cos((2n+1)\pi(x-0.5a)/2)\cos((2n+1)\pi(y-0.5b)/2)}{(2n+1)^3\cos((2n+1)\pi/2)} + \frac{\sin(2\pi x)\times\sinh(2\pi y)}{\sinh(2\pi)} \qquad (6)$$

where a = 1.0, b = 1.0

Finite element model consists of 400 elements and 231 nodes. Figure 9 shows temperature solution computed by nodeless finite element method.

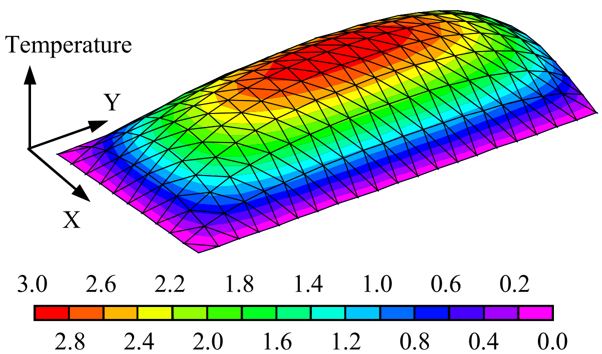

Figure 9 Temperature distribution of second problem.

The computational solution is plot with respect to distance in x direction at y = 0.5 and y direction at x = 0.25 as shown in figure 10 and 11, respectively.

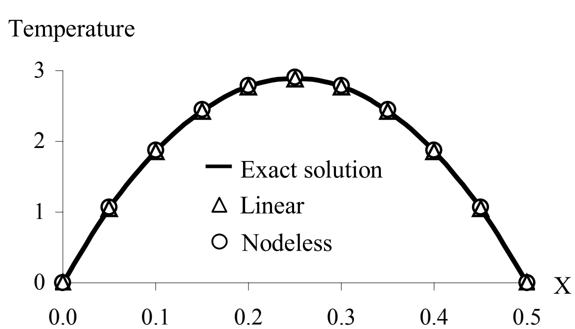

Figure 10 Temperature distribution along x axis at y = 0.5 of rectangular plate with sinusoidal temperature at top edge and internal heat generation.

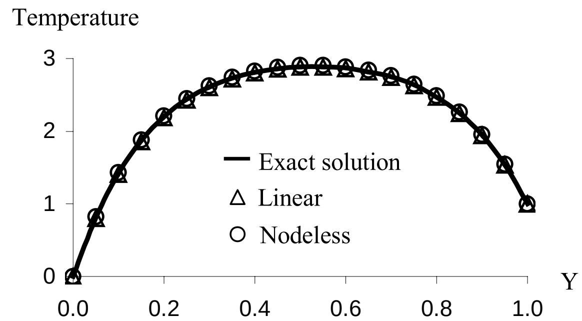

Figure 11 Temperature distribution along y axis at x = 0.25 of rectangular plate with sinusoidal temperature at top edge and internal heat generation.

4. Conclusions

Nodeless finite element method is presented to predict 2D heat transfer problem. The proposed method is validated with two examples and comparing with exact solution and finite element method using linear triangular element. The results clearly show that the nodeless element method give a higher accuracy than the linear element in solving 2D steady-state heat transfer problem.

5. Acknowledgement

The author is pleased to acknowledge the National Research Council of Thailand (NRCT) for supporting this research work.

6. References

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