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A Simple Lumped-parameter Model of Parallel and Counter Flow Heat Exchangers

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Abstract

A simple lumped parameter model of the parallel and counter-flow heat exchangers, based on their static and structural characteristics, is developed by using the method of weighted residuals (MWR) with 2nd order approximation for estimating their dynamic characteristics. The 2nd order simple model describes the dynamics characteristics not only to the inlet flow rate changes but also to the inlet temperatures changes of liquids in both shell and tube sides. It has been found from simulation results obtained from the proposed model that the dynamic characteristics of the heat exchangers are dependent upon four parameters, i.e. the number of transfer units k_1 and k_2 which are depended upon the steady state temperature of liquid 1 and liquid 2, and the residence times of the liquids τ_1 and τ_2. In addition, the experimental results are also presented to verify the validity of the model.

Keywords: Parallel and Counter Flow Heat Exchangers, Dynamics Characteristics, Method of Weighted Residuals, Simple Lumped Parameter Model

1. Introduction

Heat exchangers have been used extensively in nearly every industrial process such as chemical processes, air-conditioning systems and so forth. With the evolution of many processes, precise and fast operations for startup, shutdown, emergency situation and load changes were required. The traditional design based on the steady state data become inadequate, and attention has been focused on the understanding and on investigating dynamic behavior of the heat exchangers in order to perform the most appropriate control, design and operation. Particularly, attention has presently been paid to the energy saving problems and the heat exchanger control is one of the important topics of the energy conservation.

Dynamic characteristics of the heat exchangers have extensively been investigated and reported by several researchers from both theoretical and experimental points of views using distributed-parameter models, since P. Profos proposed the first dynamic modeling of the simple percolation-type heat exchanger. From practical point of view, the distributed-parameter models are useful and convenient for investigating the dynamic characteristics of the heat exchangers in the frequency domain simulations, but not appropriate at all for time domain analysis. As a consequence, simple lumped-parameter models with low order approximation are considerably required in practice, especially for the simulation and optimal control design of the thermal systems. The first lumped-parameter models of the parallel and counter flow heat exchangers were proposed by H. Kanoh. However, these lumped-parameter models are not based on their static and structural characteristics. As a consequence, they are practically inappropriate for simulating the dynamic behavior of any thermal system consisting of heat exchangers. The first simple lumped-parameter model of the steam liquid heat exchangers, developed by using by using the method of weighted residual (MWR) and based on their static and structural characteristics, was first presented by S. Kawai and the author, in case that the input variable is the inside tube flow rate. It has been found that the simulation results obtained from the lumped-parameter model shows good agreement with the simulation results obtained from the distributed-parameter model.

In this paper, a simple lumped-parameter model of the parallel and counter flow heat exchangers has been developed by using the MWR method and based on their static and structural characteristics, in case that the input variables is the inside tune flow rate which are more difficult to analyze because of their non linear model. The proposed model has subsequently been employed for theoretical investigations on the dynamic characteristics of the heat exchangers. In addition, the validity of the proposed model has also been verified by conducting some experiments of step response subject to the flow rate changes on the inside tube liquid.

2. Fundamental equations

Fundamental equations of the parallel and counter flow heat exchangers, depicted schematically in Fig.1, are derived under the following assumptions.

1. Temperature and velocity of both fluids are uniform at any flow cross section.

2. Heat conduction in the axial direction is negligible in both fluids and all component of the heat exchanger.

3. Heat loss from the shell surface is negligible.

4. Both fluids are incompressible and their properties are constant.

5. Overall heat transfer coefficient is uniform along tube length and depends upon the velocities of both fluids.

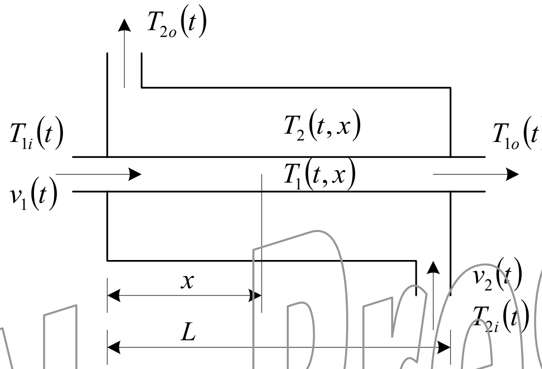

Fig. 1 A model of the counter flow heat exchanger

Under the assumption mentioned and considering the heat balance on the tube-side and shell side fluid, the fundamental equations of the heat exchanger are derived.

For the tube side:

$$\frac{\partial T_1}{\partial t} + v_1 \frac{\partial T_1}{\partial x} = \frac{UA}{w_1}\left(T_2 - T_1\right) \tag{1}$$

For shell side:

$$\frac{\partial T_2}{\partial t} \pm v_2 \frac{\partial T_2}{\partial x} = \frac{UA}{w_2}\left(T_1 - T_2\right) \tag{2}$$

A fundamental equation of the header, shown in Fig. 2, is derived under the following assumptions.

1. No heat is lost from the header

2. Heat capacity of the header in negligible

3. The fluid inside the header is perfectly mixed

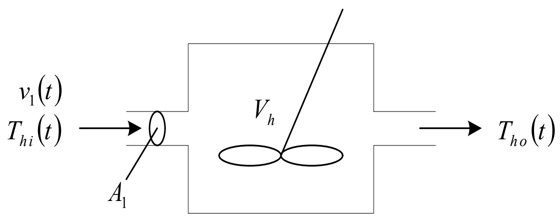

Fig. 2 A model of the header

$$\frac{\partial T_{ho}}{\partial t} = \frac{A_1 v_1}{V_h}\left(T_{hi} - T_{ho}\right) \tag{3}$$

3. A lumped parameter model

In converting the fundamental equations of the heat exchanger into dimensionless equations, the dimensionless space coordinate is defined based on the tube length and can be expressed as the following equation.

$$x^* = \frac{2x}{L} - 1 \tag{4}$$

The dimensionless time is defined based on the resident time of the inside tube fluid and can be expressed as the following equation.

$$t^* = \frac{t}{\tau_1} \tag{5}$$

The dimensionless temperature is defined based on the inlet temperatures of the inside tube fluid and the shell side fluid, and can be expressed as the following equation.

$$T^* = \frac{T - \overline{T}_{2i}}{\overline{T}_{1i} - \overline{T}_{2i}} \tag{6}$$

An overall heat transfer coefficient of the heat exchanger is defined as depending upon the flow rates of both tube side and shell side fluids, and can be expressed as:

$$\Delta U = \overline{U}\left\{\left(1 + f_1\right)^{q_1}\left(1 + f_2\right)^{q_2} - 1\right\} \tag{7}$$

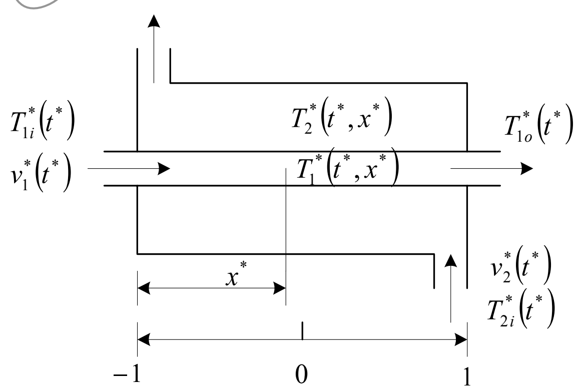

Fig. 3 Dimensionless model of the heat exchanger

where

$$f_1 = \frac{\Delta v_1}{v_1} = \frac{\Delta \dot{m}_1}{\dot{m}_1} \tag{8}$$

$$f_2 = \frac{\Delta v_2}{v_2} = \frac{\Delta \dot{m}_2}{\dot{m}_2} \tag{9}$$

In addition, the following dimensionless parameters are defined:

$$k_1 = \frac{UA}{w_1}, \quad k_2 = \frac{UA}{w_2}, \tau^* = \frac{\tau_2}{\tau_1} \qquad (10)$$

By considering the variation of the variables from the steady state and using the previously stated dimensionless variables, the dimensionless dynamical equations of the heat exchanger are derived.

For tube side:

$$\begin{aligned}&\frac{\partial \Delta T_1^*\left(t^*, x^*\right)}{\partial t^*} + 2\left(1+f_1\right)\frac{\partial \Delta T_1^*\left(t^*, x^*\right)}{\partial x^*} \\ &= -k_1\left\{f_1 - F\left(f_1, f_2\right)\right\}\left\{\Delta T_2^*\left(t^*, x^*\right) - \Delta T_1^*\left(t^*, x^*\right)\right\} \\ &\quad + k_1\left\{1 + F\left(f_1, f_2\right)\right\}\left\{T_2^*\left(x^*\right) - T_1^*\left(x^*\right)\right\}\end{aligned} \qquad (11)$$

For shell side:

$$\begin{aligned}&\frac{\partial \Delta T_2^*\left(t^*, x^*\right)}{\partial t^*} \pm \frac{2}{\tau^*}\left(1+f_2\right)\frac{\partial \Delta T_2^*\left(t^*, x^*\right)}{\partial x^*} \\ &= \frac{k_2}{\tau^*}\left\{F\left(f_1, f_2\right) - f_2\right\}\left\{\Delta T_1^*\left(t^*, x^*\right) - \Delta T_2^*\left(t^*, x^*\right)\right\} \\ &\quad + \frac{k_2}{\tau^*}\left\{1 + F\left(f_1, f_2\right)\right\}\left\{T_1^*\left(x^*\right) - T_2^*\left(x^*\right)\right\}\end{aligned} \qquad (12)$$

Equations (11) and (12) are a dimensionless distributed-parameter model of the heat exchangers, based on their static and structural characteristics.

where

$$F\left(f_1, f_2\right) = \left(1+f_1\right)^{q_1}\left(1+f_2\right)^{q_2} - 1 \qquad (13)$$

And initial conditions are:

$$\Delta T_1^*\left(t^*, -1\right) = \Delta T_{1i}^*\left(t^*\right) \qquad (14)$$

$$\Delta T_2^*\left(t^*, +1\right) = \Delta T_{2i}^*\left(t^*\right) \qquad (15)$$

By considering energy balance at the steady state, the number of transfer units based on the heat capacity rate of the inside tube fluid $\left(k_1\right)$ and the number of transfer unit based on the heat capacity rate of the shell side fluid $\left(k_2\right)$ can be expressed as:

$$k_1 = \frac{T_{1i} - T_{1o}}{\Delta T_L} \qquad (16)$$

$$k_2 = \frac{T_{2o} - T_{2i}}{\Delta T_L} \qquad (17)$$

The lumped-parameter model of the heat exchanger is derived by applying the method of weighted residuals (MWR) to the equations (11) and (12), and using the trial function $\left(1+x^*\right)u^T\left(x^*\right)$ as weighted function. Finally the lumped parameter model is derived and can be expressed as:

$$\begin{aligned}\dot{a}\left(t^*\right) &= \left\{A_1 + A_2 F\left(f_1, f_2\right) + A_3 f_1 + A_4 f_2\right\}a\left(t^*\right) \\ &\quad + B_1 b\left(t^*\right) + B_2\left\{1 + F\left(f_1, f_2\right)\right\}b\left(t^*\right) \\ &\quad + y_1 F\left(f_1, f_2\right) + y_2 F\left(f_1\right) + y_3 F\left(f_2\right)\end{aligned} \qquad (18)$$

And the equations of the outlet temperature of the inside tube and shell side fluids are:

$$\begin{bmatrix}\Delta T_{1o}^* \\ \Delta T_{2o}^*\end{bmatrix} = \begin{bmatrix}2u^T(1) & 0 \\ 0 & 2u^T(-1)\end{bmatrix} a\left(t^*\right) + \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix} b\left(t*\right) \qquad (19)$$

The equations (18) and (19) are a general lumped-parameter model of the heat exchangers based on their static and structural cahracteritics.

where

$u\left(x^*\right) =$ normalization Legendere's polynomial

$a\left(t^*\right) =$ state vector

$b^T\left(t^*\right) = \left[\Delta T_{1i}^*\left(t^*\right), \Delta T_{21}^*\left(t^*\right)\right]$

$$A_1 \equiv \begin{bmatrix}-\left(k_1 + 2S^{-1}T_1\right) & k_1 S_1^{-} S_2 \\ \frac{k_s}{\tau^*} S_2^{-1} S_1 & -\frac{1}{\tau^*}\left(k_2 I + 2S_2^{-4} T_2\right)\end{bmatrix}$$

$$A_2 \equiv \begin{bmatrix}k_1 I & k_1 S_1^{-1} S_2 \\ \frac{k_2}{\tau^*} S_2^{-1} S_1 & \frac{k_2}{\tau^*} I\end{bmatrix}$$

$$A_3 \equiv \begin{bmatrix}-2S_1^{-1}T_1 & 0 \\ 0 & 0\end{bmatrix}$$

$$A_4 \equiv \begin{bmatrix}0 & 0 \\ 0 & -\frac{2}{\tau^*} S_2^{-1} T_2\end{bmatrix}$$

$$B_1 \equiv \begin{bmatrix}-\sqrt{2}\, S_1^{-1} e_1 & 0 \\ 0 & -\sqrt{2}\, S_2^{-1} e_2\end{bmatrix}$$

$$B_2 \equiv \begin{bmatrix}-\sqrt{2}\, k_1\, S_1^{-1} e_1 & \sqrt{2}\, k_1 S_1^{-1} e_1 \\ \frac{\sqrt{2} k_1}{\tau^*} S_2^{-1} e_1 & \frac{\sqrt{2}\, k_1}{\tau^*} S_2^{-1} e_1\end{bmatrix}$$

$$y_1 \equiv \begin{bmatrix}-k_1 S_1^{-1} \\ \frac{k_2}{\tau^*} S_2^{-1} C\end{bmatrix} \qquad y_2 \equiv \begin{bmatrix}k_1 S_1^{-1} C \\ 0\end{bmatrix}$$

$$y_3 \equiv \begin{bmatrix} 0 \\ -\frac{k_2}{\tau^*} S_2^{-1} C \end{bmatrix}$$

$$S_1 \equiv \int_{-1}^{1} \left(1+x^*\right) u\left(x^*\right) u^T\left(x^*\right) dx^*$$

$$T_1 \equiv \int_{-1}^{1} u\left(x^*\right) \frac{d}{dx^*} \left\{\left(1+x^*\right) u^T\left(x^*\right)\right\} dx^*$$

$$S_2 \equiv \int_{-1}^{1} \left(1+x^*\right) u\left(x^*\right) u^T\left(x^*\right) dx^*$$

$$T_2 \equiv \int_{-1}^{1} u\left(x^*\right) \frac{d}{dx^*} \left\{\left(1-x^*\right) u^T\left(x^*\right)\right\} dx^*$$

$$e_1 \equiv [1,0,..........0]$$

$$C \equiv \int_{-1}^{1} u\left(x^*\right) \left\{T_1^*\left(x^*\right) - T_2^*\left(x^*\right)\right\} . dx^*$$

The transfer function of the heat exchanger, for general case, is derived from the MWR models with 2nd order approximation, and can be expressed as follows.

$$\Delta T_{1o}^* \equiv \frac{G_{F1}(s)}{G(s)} f_1(s) + \frac{G_{F2}(s)}{G(s)} f_2(s) + \frac{G_{T1}(s)}{G(s)} \Delta T_{1i}^* + \frac{G_{T2}(s)}{G(s)} .\Delta T_{2i}^* \qquad (20)$$

where

$$G(s) = (s\tau_1)^4 + b_3(s\tau_1)^2 + b_1(s\tau_1) + b_0 \qquad (21)$$

$G_{F1}(s)$, $G_{F2}(s)$, $G_{T1}(s)$ and $G_{T2}(s)$ are listed in Table 1.

Table 1 Transfer function of the heat exchanger

$G(s)$	$(s\tau_1)^4 + b_3(s\tau_1)^3 + b_2(s\tau_1)^2 + b_1(s\tau_1) + b_0$
b_3	$2\{k_1 + 3 + (k_2+3)/\tau^*\}$
b_2	$k_1^2 + 2\{k_1k_2 + 6(k_1+k_2) + 18\}/\tau^* + 12$ $6k_1 + (k_2^2 + 6k_2 + 12)/\tau^*$
b_1	$6[\{k_1^2 + (k_2+6)k_1 + 4(k_2+3)\}/\tau^* + \{k_2^2 + (k_1+6)k_2 + 4(k_1+3)\}/\tau^{*2}$
b_0	$12\{k_1^2 + k_2^2 - 2k_1k_2 + 6(k_1+k_2) + 12\}/\tau^{*2}$
$G_{F1}(s)$ $G_{F2}(s)$	$(6k_2/\tau^*)\{a_{F3}(s\tau_1)^3 + a_{F2}(s\tau_1)^2 + a_{F1}(s\tau_1) + a_{F0}\}$

a_{F3}	$c_1(\alpha_2-\alpha_3)$	
a_{F2}	$c_2k_1(\alpha_1-\alpha_4) + c_1[\{2(k_1+3) + k_2/\tau^*\}(\alpha_2-\alpha_3) + 2(3\alpha_2-\alpha_3)/\tau^*]$	
a_{F1}	$c_2k_1\{(k_1+k_2/\tau^*)(\alpha_1-\alpha_4) + 2(1-1/\tau^*)(3\alpha_1-\alpha_4)\} + c_1[\{k_1^2+6k_1+12 + k_2(k_1+6)/\tau^*\}(\alpha_2-\alpha_3) + 4(k_1+3)(3\alpha_2-\alpha_3)/\tau^*]$	
a_{F0}	$(2c_2k_1/\tau^*)\{(k_2-k_1-6)(3\alpha_1-\alpha_4) + 6(\alpha_1-\alpha_4) + (2c_1/\tau^*)\{k_1^2-k_1k_2$ $6k_1+12)(3\alpha_2-\alpha_3) + 6k_2(\alpha_2-\alpha_3$	
c_1	$q_1(G_{F1}(s)), q_2-1(G_{F1}(s))$	
c_2	$1-q_1(G_{F1}(s)), -q_2(G_{F2}(s))$	
$k_1 \neq k_2$	α_1	$\{\exp(k_1)-\beta_1\}/\beta_2$
	α_2	$\{\exp(k_2)-\beta_1\}/\beta_2$
	α_3	$\{\exp(k_1)+2\exp(k_2) - 3\beta_1\}/\beta_2$
	α_4	$\{\exp(k_2)+2\exp(k_1) - 3\beta_1\}/\beta_2$
	β_1	$\{\exp(k_2)-\exp(k_1)\}/(k_2-k_1)$
	β_2	$k_2\exp(k_2) - k_1\exp(k_1)$
$k_1 = k_2$	α_1, α_2	$-1/\{2(k_1-1\}$
	α_3, α_4	$1/\{2(k_1-1\}$
$G_{T1}(s)$	$(12k_2/\tau^*)\{(s\tau_1)^2 + (k_1+k_2/\tau^*)(s\tau_1) + 12/\tau^{*\}}$	
$G_{T2}(s)$	$(s\tau_1)^4 + a_{T23}(s\tau_1)^3 + a_{T22}(s\tau_1)^2 + a_{T21}(s\tau_1) + a_{T20}$	
a_{T23}	$2\{k_1+3+(k_2-3)/\tau^*\}$	
a_{T22}	$k_1^2+6k_1+12+2\{k_1k_2+6(k_2-k_1) - 18\}/\tau^* + (k_2^2-6k_2+12)/\tau^*$	
a_{T21}	$6[-\{k_1^2-(k_2-6)k_1-4(k_2-3)\}/\tau^* + \{k_2^2-(k_1+6)k_2+4(k_1+3)\}/\tau^{*2}]$	
a_{T20}	$12\{k_1^2+k_2^2-2k_1k_2+6(k_1-k_2) + 12\}/\tau^*$	

The fundamental equation of the header is converting into dimensionless equation using the dimensionless time defined as follows:

$$t^* \equiv \frac{t}{T_h} \tag{22}$$

where

$$T_h = \frac{V_h}{A_1 v_1}$$

By considering the variations of the variables from the steady state condition and used the dimensionless variables, the dimensionless equation of the header can be expressed as follow.

$$\frac{\partial \Delta T^*_{ho}\left(t^*\right)}{\partial t^*} = \left(1 + f_1\right)\left\{\Delta T^*_{hi}\left(t^*\right) - \Delta T^*_{ho}\left(t^*\right)\right\} \tag{23}$$

Under the assumption that the liquid inside the header is perfectly mixed, the delay caused by the header can be represented by the following transfer function.

$$\frac{\Delta T^*_{ho}(s)}{\Delta T^*_{hi}(s)} = \frac{1}{1 + sT_h} \tag{24}$$

4. Derivation of a low lumped-parameter model

The suitable approximation order of the lumped parameter model of the heat exchangers, derived in the previous section, is firstly determined by comparing the simulation results obtained from the lumped-parameter model with 2nd and 4th order approximations with the exact results obtained from the distributed-parameter model for the case of the inside tube flow rate change. It has been found that the simulation results of the proposed model with 2nd and 4th order approximations are in good agreement with each other. As a result, only the simulation result of the lumped-parameter model with 2nd order approximation and the exact results are presented in Fig. 4.

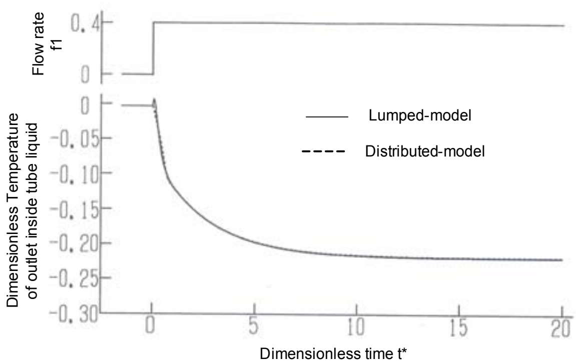

Fig.4 Comparison between the responses of the lumped-parameter model with of the distributed-parameter model

It can be seen from Fig. 4 that the step response of the outlet temperature of the tube side liquid, obtained from lumped-parameter model, is in good agreement with the response obtained from the distributed-parameter model. As a result, a conclusion can be drawn that the lumped model is appropriate for investigating the dynamic characteristics of the heat exchanger as compared with the distributed model.

5. Experimental results

In order to verify the validity of the low order lumped-parameter of the heat exchangers, experiments of both step and frequency response subjected to the flow rate changes of the inside tube liquid have been conducted. The experimental apparatus employed in this investigation is schematically depicted in Fig. 5.

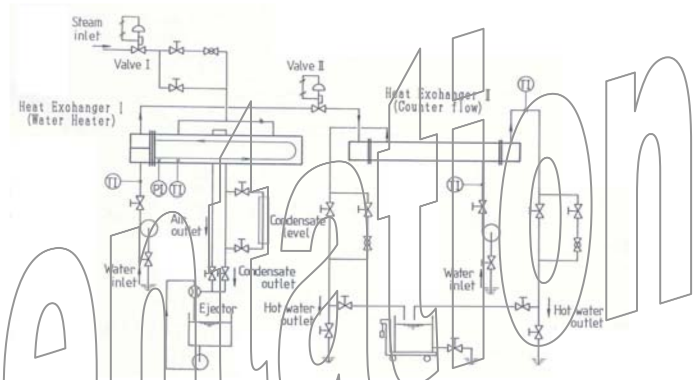

Fig.5 Schematic diagram of the experimental apparatus

In order to verify the validity of the proposed lumped-parameter model, experiments of step response subjected to the inside tube flow rate change have been conducted. One of the experimental results of the step response as compared with the results obtained from the lumped-parameter model is illustrated graphically in Fig. 6.

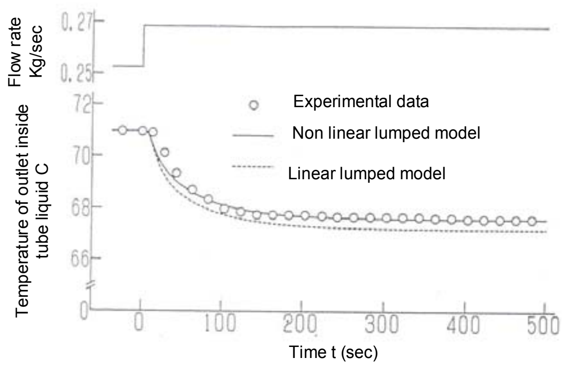

Fig.6 Step responses of the lumped-parameter model and the experimental data

6. Conclusions

In this paper, the simple lumped-parameter model of the parallel and counter flow heat exchangers is firstly developed. The proposed model is subsequently employed to investigate the dynamic characteristics of the heat exchangers subject to the liquid flow rate changes. Experiment of step response has also been conducted in order to verify the validity of the proposed model. And the following conclusions can be drawn from the preceding theoretical analyses and experimental data.

1. The simple lumped-parameter model, developed by the MWR method with 2nd order approximation, gives good approximation for investigating the dynamic characteristics of the heat exchangers due to the liquid flow rate changes.

2. It has been found from the proposed model based on the static and structural characteristics that the dynamic characteristics of the heat exchangers due to the liquid flow rate changes are dependent upon four dimensionless parameters. They are the number of transfer units k_1 and k_2 which are depended upon the steady state temperature of liquid 1 and liquid 2, and the residence times of the liquids τ_1 and τ_2 which are depended upon the heat exchanger length.

3. The experimental data show that the lumped-model gives good approximation results for the case of the liquid flow rate changes. This confirms that the proposed lumped model is appropriate for investigating the dynamic characteristics of the heat exchangers due to the liquid flow rate changes.

7. Nomenclature

A :	heat transfer area	m^2
A_1 :	cross section area of header inlet	m^2
c_p :	specific heat of liquid	$kJ/kg.K$
k_i :	number of transfer unit	
L :	length of tube	m
t :	time	$\sec$
t^* :	dimensionless time	
T :	temperature	^0C
T^* :	dimensionless temperature	
U :	overall heat transfer coefficient	kW/m^2K
v_i :	velocity of liquid	$m/\sec$
V_h :	volume of header	m^3
w_i :	heat capacity of liquid	$kJ/\sec-K$
x :	space coordinate	m
x^* :	dimensionless space coordinate	
τ :	resident time of liquid	$\sec$
$\Delta(\ )$:	deviation from steady state	

Subscripts

1 : tube side
2 : shell side
h : header
i : inlet
o : outlet
V : volume

Superscripts

$(\ )^T$: matrix transpose of ( )
$-$: steady state value

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