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Finite Element Method for Analysis of Moisture Content in Porous Media

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Abstract

The Galerkin finite element method for unsteady two-dimensional water flow in saturated-unsaturated soils of the standard head-based form according to the Richards' equations is presented. The discretization of the flow domain is obtained by using triangular finite elements with linear shape functions. Unsaturated flow behavior in porous media is governed by nonlinear differential equation, and an iterative procedure must be employed for solution. Time integration is performed by using a finite difference backward Euler scheme. Fine grid spacings may also be required to effectively capture sharp moisture fronts, as well as small time step is needed to avoid the solution instability.

1. Introduction

Many Thailand's historic structures and buildings have suffered from surface deterioration. Water infiltrates into these structures during rainy season, and then flows back during dry season toward surfaces where it evaporates. This water may carry chemical compositions that cause severe damage to the structures. In this paper, the moisture conditions are studied to predict the water movement in unsaturated porous media.

In virtually all studies of the saturated and unsaturated zone, the fluid motion is assumed to follow the classical Richards' equation [1] and flow rates are calculated upon application of Darcy's law. Water moves through porous media in response to two forces, the capillary potential gradients and the gravity. Problems related to two-dimensional, isothermal transient water transfer in unsaturated homogeneous isotropic porous media are highly nonlinear nature of the governing partial differential equation and the solutions of this equation are valid under certain restrictive initial and boundary conditions. To solve for solution, standard iteration techniques such as Picard and Newton methods may be used. Moreover, mass lumping is employed to improve solution convergence and stability behavior [2]. Time approximation is normally based on a fully implicit (backward Euler) scheme.

An excellent tool for checking numerical models of unsaturated flow in porous media is from analytical solutions. However, because of the problems above, only a limited number of analytical solutions is available. Moisture distribution problem is presented as the first example to compare the analytical solution with the numerical solution. The vertical flow in soil column [3], is used as the second example to study vertical movement of water in porous media during rainfall. A more complex geometry is simulated in the third example, which is a historic pagoda at Wat Pansat, Chiangmai [4], in order to evaluate and demonstrate the efficiency of the finite element method.

2. Flow in Porous Media and Finite Element Formulation

2.1 Governing Equation

The unsaturated flow behavior in two-dimensional x-y coordinates is governed by the differential conservation of mass,

$$\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial x} + \frac{\partial q}{\partial y} = 0 \tag{1}$$

with the flux q described by Darcy's law [5],

$$q = -K(\psi)\frac{\partial \psi}{\partial x} - K(\psi)\left[\frac{\partial \psi}{\partial y} + 1\right] \quad (2)$$

where ψ is the pressure head according to the Richards' equation [6],

$$\theta^* \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial x}\left(K(\psi)\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K(\psi)\frac{\partial \psi}{\partial y}\right) + \frac{\partial K(\psi)}{\partial y} \quad (3)$$

In the above equations, $\theta^* = \partial\theta/\partial\psi$ is the specific moisture capacity function, K is the hydraulic conductivity, θ is the moisture content, y denotes the vertical dimension assuming positive upward, and the porous medium is assumed to be isotropic.

2.2 Finite Element Formulation

Finite element equations for determining nodal pressure heads can be derived from the governing differential equation using the Galerkin method [7,8]. These equations can be written in matrix form as,

$$[C(\psi)]\{\dot{\psi}\} + [K(\psi)]\{\psi\} = \{Q(\psi)\} \quad (4)$$

where $\{\dot{\psi}\}$ is the time rate of change of the nodal pressure heads, $\{\psi\}$ is the vector that contains unknowns of the element nodal pressure head components, $[C]$ is the capacitance matrix, $[K]$ is the hydraulic conductance matrix, and $\{Q\}$ is the load vector. These matrices are given by,

$$[C(\psi)] = \int_A \theta^* \{N\}[N]\,dA \quad (5a)$$

$$[K(\psi)] = \int_A [B]^T [k][B]\,dA \quad (5b)$$

$$\{Q(\psi)\} = -\int_A \{G\}\,k(\psi)\,dA \quad (5c)$$

In the above Eq. (5a-c), $[N]$ is the pressure head interpolation matrix, $[B]$ is the pressure head gradient interpolation matrix, $[k]$ is the soil hydraulic conductance matrix, $\{G\}$ is the gravity load vector, and A is the element area.

3. Examples

3.1 Steady State Moisture Distribution

For steady state problem without gravity effect and with constant hydraulic conductivity, the Richards' equation reduces to,

$$K\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) = 0 \quad (6)$$

For the domain and the boundary conditions as shown in Fig. 1, the exact solution is,

$$\psi(x, y) = \sin\frac{\pi x}{2}\, \sinh\frac{\pi y}{2} / \sinh\frac{\pi}{2} \quad (7)$$

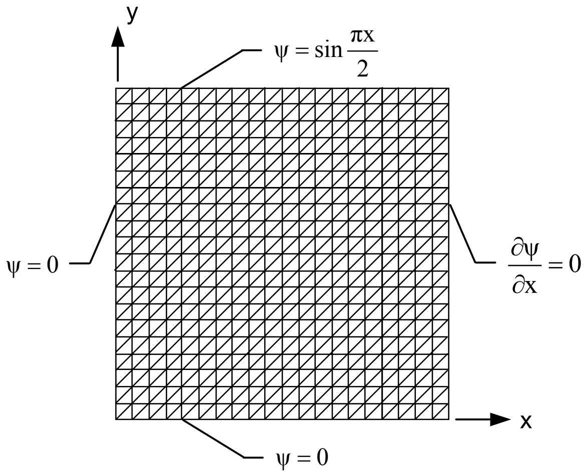


Fig. 1 - Finite element model and boundary conditions.

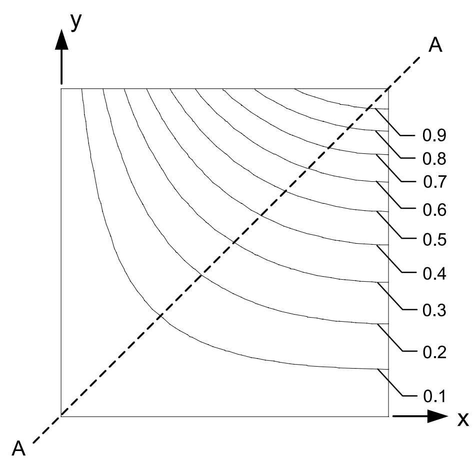


Fig. 2 - Pressure head contours.

The finite element model in Fig. 1 consists of 441 nodes and 800 elements. Figure 2 shows the computed pressure head contours. Figure 3 shows the comparison of computed and exact solutions in section AA.

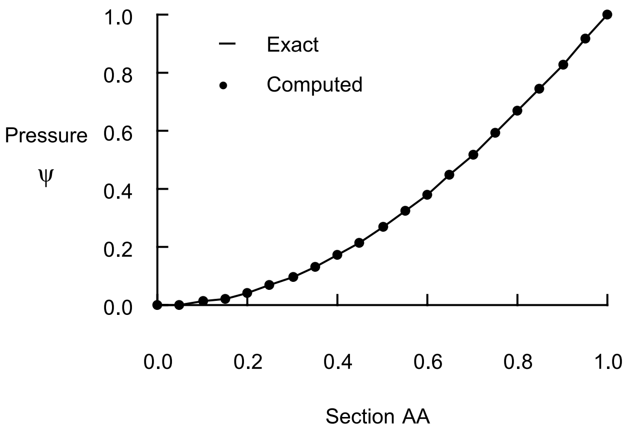


Fig. 3 - Comparative pressure distributions along section AA.

3.2 Vertical Flow in Soil Column

The finite element model of the soil column as shown in Fig. 4 consists of 451 nodes and 800 elements,

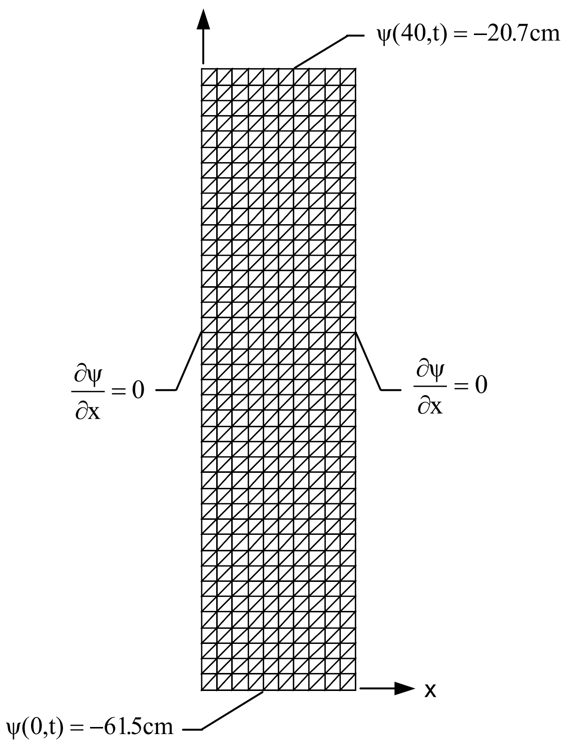


Fig. 4 - Finite element model and boundary conditions for vertical flow problem.

The material properties are given by [9],

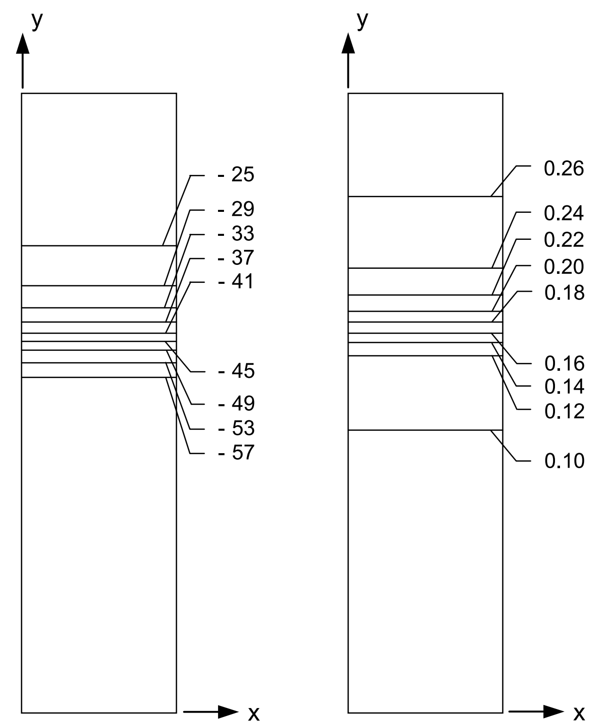
$$\theta(\psi) = \frac{\alpha(\theta_s - \theta_r)}{\alpha + |\psi|^\beta} + \theta_r$$

$$K(\psi) = K_s \frac{\delta}{\delta + |\psi|^\gamma}$$

where θ_s and θ_r are the saturated and residual water contents of the soil, K_s is the saturated hydraulic conductivity, and α, β, δ, and γ are model parameters determined from laboratory experiments.

The value of above parameters are θ_s = 0.287, θ_r = 0.075, α = 1.611x10^6, β = 3.96, K_s = 0.00944 cm/s, δ = 1.175x10^6, and γ = 4.74. The initial condition is $\psi(z,0)$ = -61.5 cm. The process lasted 360 s.

Figure 5 shows the predicted pressure head and moisture content in the soil column after six minutes.



a) Pressure head contours. b) Moisture head contours.

Fig. 5 - Predicted pressure head and moisture content for vertical flow.

3.3 Moisture Content in a Pagoda

A historic pagoda at Wat PanSat in Chiangmai is used to predict the propagation of the moisture content. Figure 6 shows the finite element model with 4,305 nodes and 7,924 triangles with the boundary conditions shown in the figure. Initially, the pressure head is given as -30.5 cm. Figure 7 shows the predicted pressure head contours after five minutes. With the predicted pressure head, the moisture content can then be computed. Figure 8 shows the computed moisture content at 10, 20, 30, and 60 minutes. The figures highlight the propagation behavior of the moisture content from the outer surface toward the pagoda interior.

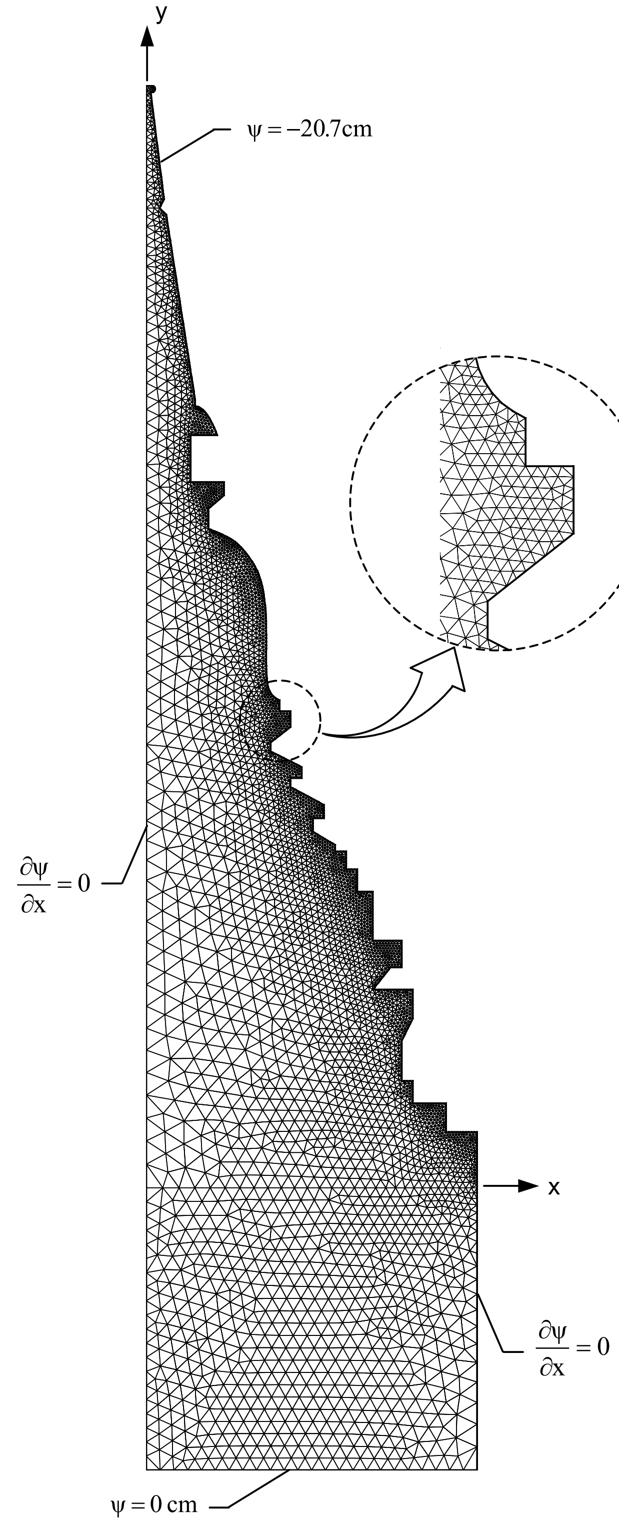


Fig. 6 - Finite element model and boundary conditions for a pagoda at Wat Pansat, Chiangmai.

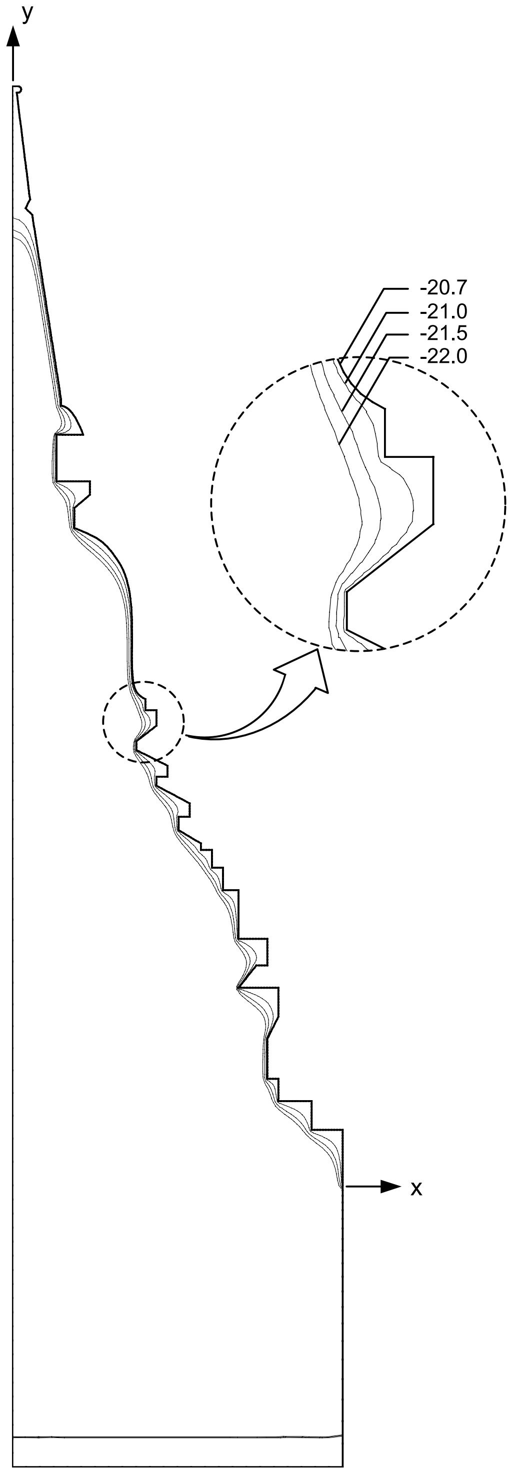


Fig. 7 - Predicted pressure head contours of the pagoda after five minutes.

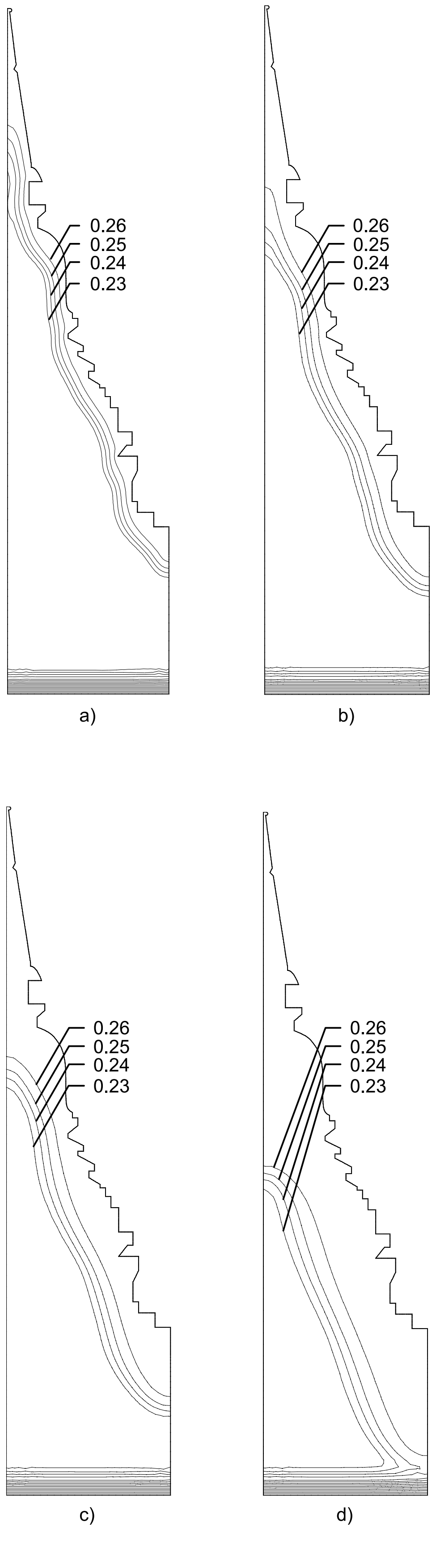


Fig. 8 - Predicted moisture content contours of the pagoda at a) 10 min. b) 20 min. c) 30 min. d) 60 min.

4. Conclusions

A finite element method for analysis of moisture content in porous media was presented. The finite element equations were derived from the governing differential equation of the unsaturated flow behavior in two dimensions according to the Richards' equation. The formulation was validated for simple problems that have exact solutions prior to applying to more complex problems. Finally, the formulation was evaluated by analyzing the moisture content propagation behavior of a historic pagoda in Wat Pansat, Chiangmai.

5. Acknowledgements

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