An Approximate Solution for Single Helmholtz-resonator Silencer

Ming LOKITSANGTONG Pongsak KUMMOL
Department of Mechanical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang
3 Moo 2 Chalongkrung Road, Ladkrabang, Bangkok 10520, Thailand
Tel: 0-23264197 Ext. 107 Fax: 0-23264198 E-mail: klming@kmitl.ac.th

Abstract

The problems on acoustics are generally solved via the wave equation in particular those related to the performance of a Helmholtz-resonator silencer. In the present paper, an alternative approach has been suggested. The sound propagating into the cavity of the Helmholtz resonator behaves in a manner analogous to an oscillation of a single-degree-of-freedom (SDOF) system. The air column in the connector of the resonator can be envisaged as a single mass of the system. The results obtained from this methodology are compared to those based on the wave equation and those from the experiment. The comparison is rather promising. This approximate method can be used for the preliminary study of the Helmholtz silencer

1. Introduction

Plane wave propagated along the longitudinal axis of a duct can be described by the acoustic linear wave theory. The assumption of plane wave traveling in a viscous stationary medium as in the case of a duct installed with a single Helmholtz resonator-type silencer is valid to the extent that the wavelength of the sound is very much longer than the cross-sectional dimension of the duct. Davis et al. has performed some rigorous calculations and experiments on silencers with the single resonators of different types based on the linear wave theory[1]. In general, the sound propagation in any fluid exists in the way that satisfies the wave equation and the imposed boundary conditions. However, in modeling the acoustic behavior of a fluid region which is small if its principal dimensions are very much less than an acoustic wavelength, it is acceptable to define an equivalent mechanical system to replace the acoustic system[2]. The analysis of this single-degree-of-freedom(SDOF) oscillatory system has been employed in this paper to study the frequency response and to estimate the sound transmission characteristic of a single Helmholtz resonator silencer. The results of the study are then compared with the ones from experiments and the linear wave theory.

2. Frequency response

A silencer composed of a single Helmholtz resonator, which is mounted on one side of a duct terminating with the anechoic end, is depicted in Fig.1. The fluid column of effective length l_e and cross-sectional area S in the connector moves as a whole

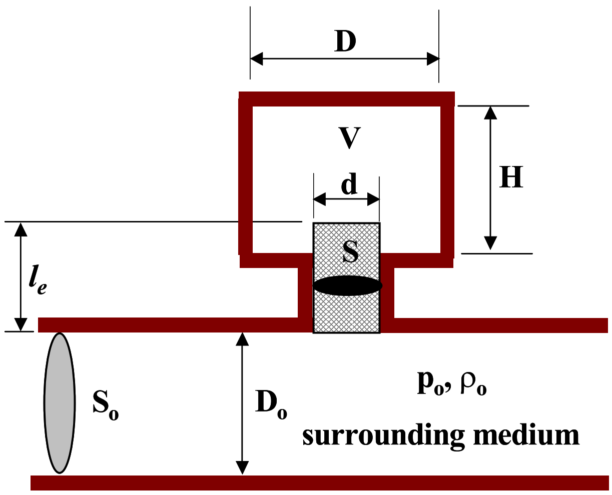

Fig.1 Model of lumped mass in a silencer

and provides the lumped mass element. A volume of fluid of magnitude V which is alternately compressed and expanded by the movement of the fluid in the connector, provides the stiffness of the element. At the opening to the main duct, there is a radiation of sound into the surrounding medium leading to the dissipation of acoustical energy and providing the damping element. Figure 2 describes this oscillatory lumped mass as a viscously damped spring-mass system excited by a harmonic force F. The differential equation of motion can then be written as

$$M\ddot{\xi} + C\dot{\xi} + K\xi = F \qquad (1)$$

where M is a mass, C the damping coefficient and K the spring stiffness. The expressions for M, K and C can be derived as follows

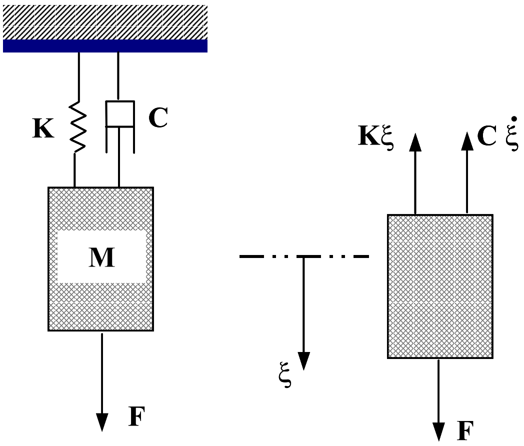

Fig.2 Single-degree-of-freedom (SDOF) system

:- for the inertial term, the mass of fluid column in the connector is given by

$$M = \rho_o S l_e \qquad (2)$$

where effective length l_e [2]equals to the length of the connector plus $S^{1/2}/3$.

:- for the stiffness term , the force exerted on the column of fluid by a pressure change dp is given by means of compressibility and isentropy relations by

$$Sdp = -S(\gamma p_o \frac{dV}{V}) = -S\frac{\gamma P_o}{V}(S\xi) = -S^2\frac{c^2\rho_o}{V}\xi$$

where ρ_o , p_o are the density and pressure of fluid in duct, γ the specific heat constant, and c the sound speed. Therefore the spring stiffness is given by

$$K = \frac{\rho_o c^2 S^2}{V} \qquad (3)$$

:- for the dissipation term, starting similarly as above, the force is given via energy and kinematics relations by

$$Sdp = S(-\rho_o\dot{\xi}d\dot{\xi}) = -S(\rho_o\frac{\omega\lambda}{2\pi})\dot{\xi} = -S\frac{\omega kS}{2\pi}\rho_o\dot{\xi}$$

where ω is the angular frequency of the excitation, λ the wavelength and k the wave number equals to $\frac{\omega}{c(1-Ma^2)}$, in which Ma is the Mach number[3]. Hence the damping coefficient can be written as

$$C = \frac{\rho_o\omega k S^2}{2\pi} \qquad (4)$$

After substituting M, C, and K equation (1) becomes

$$\rho_o S l_e\ddot{\xi} + \frac{\rho_o\omega k}{2\pi}S^2\dot{\xi} + \frac{\rho_o c^2 S^2}{V}\xi = F \qquad (5)$$

Equation (5), by letting x=Sξ, and replacing F by pS, can be reduced to

$$\frac{\rho_o l_e}{S}\ddot{x} + \frac{\rho_o\omega k}{2\pi}\dot{x} + \frac{\rho_o c^2}{V}x = p \qquad (6)$$

If the harmonic excitation pressure p is written as $\overline{p}e^{j\omega t}$, then the volume velocity $\dot{x}$ will also be harmonic and is given by $qe^{j(\omega-\psi)}$, where $\overline{p}$ and q denote the corresponding amplitudes, t the time, ψ the phase angle, and $j=(-1)^{1/2}$. After their substitution into eq(6), the characteristic acoustic impedance of the resonator can be obtained as

$$Z = \frac{\rho_o\omega k}{2\pi} + j\left(\frac{\omega\rho_o l_e}{S} - \frac{\rho_o c^2}{V\omega}\right) \qquad (7)$$

and the specific acoustic impedance is given by

$$z' = z'_c + j(z'_m + z'_k) \quad .$$

$$z' = \frac{\omega k S_o}{2\pi c} + j\left(\frac{\omega l_e S_o}{Sc} - \frac{cS_o}{V\omega}\right) \qquad (8)$$

The resonance frequency is obtained from the free undamped vibration as

$$f_r = \frac{\omega_n}{2\pi} = \frac{c}{2\pi}\sqrt{\frac{S}{Vl_e}} \qquad (9)$$

The amplification α is written as

$$\alpha = 10\log_{10} \frac{1}{(\omega/\omega_n)^2\left[4\zeta + \left\{(\omega/\omega_n) - \frac{1}{(\omega/\omega_n)}\right\}^2\right]}$$ (10)

where ζ is the damping factor. The relationship for eq(10) has been shown in Fig.3.

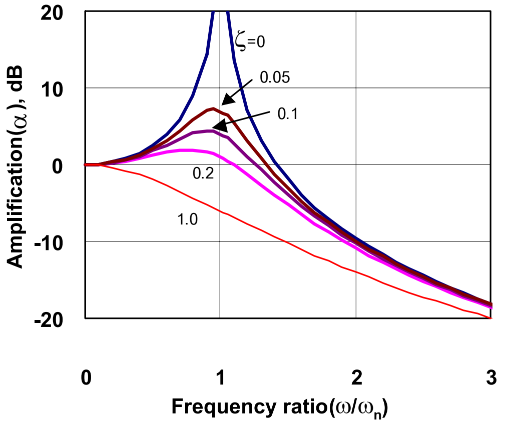

Fig.3 Frequency response for the resonator

3. Transmission loss characteristic

By considering the energy and the continuity relations for a volume of fluid in the duct adjacent to the opening of the resonator, the fluctuating pressure for incident wave p_i and for reflecting wave p_r can be written as

$$p_{i_1} + p_{r_1} = p_{i_2} + p_{r_2} = p_{i_3} + p_{r_3} \tag{11}$$

and the volume velocity q_i and q_r for incident and reflecting wave are given by

$$q_{r_1} + q_{i_2} + q_{i_3} = q_{i_1} + q_{r_2} + q_{r_3} \tag{12}$$

Here the subscripts 1, 2, and 3 are referred to the fluid surface in duct at the fore, the aft, and at the opening of the resonator respectively. For non-reflecting end p_{r_2}, p_{r_3}, q_{r_2}, and q_{r_3} all vanish. Equations (11) and (12) become

$$p_{i_1} + p_{r_1} = p_{i_2} = p_{i_3} \tag{13}$$

$$q_{r_1} + q_{i_2} + q_{i_3} = q_{i_1} \tag{14}$$

Writing volume velocity in eq(14) in terms of pressure, the following equation is obtained.

$$\frac{p_{r_1}}{Z_o} + \frac{p_{i_2}}{Z_o} + \frac{p_{i_3}}{Z} = \frac{p_{i_1}}{Z_o} \tag{15}$$

where Z_o denotes the characteristic acoustic impedance of air in duct and equals to $\rho_o c / S_o$. Eliminating p_{r_1} and p_{i_3} in eq's (13) and (15), one obtains

$$\frac{p_{i_1}}{p_{i_2}} = 1 + \frac{Z_o}{2Z} \tag{16}$$

The transmission loss can then be written as

$$TL = 10\log_{10}\left|\frac{p_{i_1}}{p_{i_2}}\right|^2 = 10\log_{10}\left[1 + \frac{1+4R}{4(R^2+X^2)}\right] \tag{17}$$

where $R = z'_c$ and $X = z'_m + z'_k$

4. Results and conclusions

The frequency response of single Helmholtz resonator is

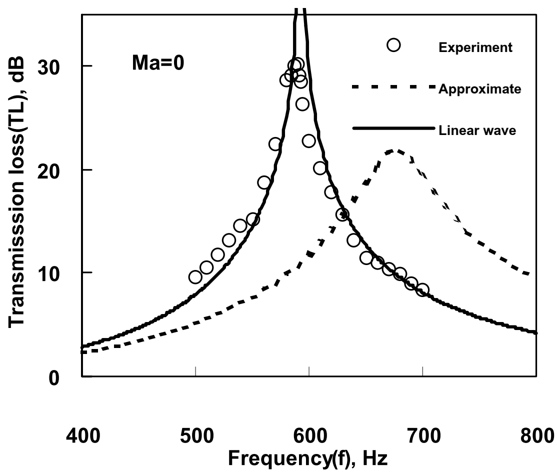

Fig.4 Comparison of transmission characteristic without duct flow

shown in Fig.3. The amplification α of the resonator represents the ratio of the maximum excess power in the resonator and the maximum external operating power. These curves indicate that the dissipation factor has a large influence on the amplification in the region of resonance.

Figures 4 and 5 show the transmission loss characteristic of a duct as obtained from the linear wave theory[4], an experimental test[5], and the present approach. The comparison of the results demonstrates a convincingly high degree of agreement. In the case of no air flow in duct, the linear wave theory can predict the performance of the resonator very well, but the approximate method gives a somewhat higher resonance frequency of about 15% and a lower resonance transmission loss of roughly 30%. When there is an air flow in duct, the linear wave theory and the approximate approach indicate a same amount of drop and rise in resonance frequency respectively. The transmission loss characteristic at resonance also follows the

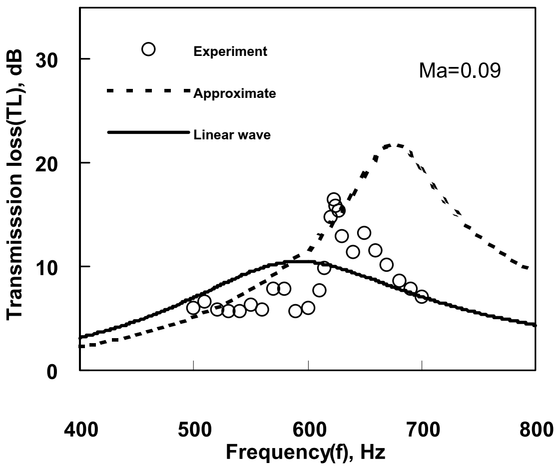

Fig.5 Comparison of transmission characteristic with duct flow

same trend.

The approximate method based on the SDOF oscillatory system is proposed as a convenient platform for solving preliminary design problems. An improvement for this model of a single Helmholtz resonator-type silencer can be achieved by somehow modifying the dissipation term.

Appendix A

The sampled silencer has the following dimensions:

Dimension of duct (mm)	48
Diameter of resonance chamber (mm)	105
Length of resonance chamber (mm)	30
Diameter of connector (mm)	40
Length of connector (mm)	20
Ambient temperature (^{O}C)	23
Density of air (kg/m^3)	1.193
Speed of sound (m/s)	346

References

[1]. D. Davis, Jr., G. Stevens, Jr., D. Moore and G. Stokes,"Theoretical and Measured Attenuation of Mufflers at Room Temperature without Flow, with Comments on Engine-exhaust Muffler Design," NACA Technical Note 2893, 1954.

[2]. F. Fahy, "Foundation of Engineering Acoustics," Academic Press, 2001.

[3]. P. Mungur and G.M.L. Gradwell, "Acoustic Wave Propagation in a Sheared Fluid Contained in a Duct," J. Sound Vib., **9** (1969) No.1 pp.28-48.

[4]. M. Lokitsangtong, and P. Kummol, "Estimation of Sound Transmission Loss for a Design of Single Helmholtz Resonator-Type Silencers," The 17th Conference of Mechanical Engineering NETwork of Thailand, 15-17 October 2003 Prachin Buri.

[5] T. Khieo-im, T. Ingkaprasitkul, and W. Khajardparn, "Development of Noise Control Device for Exhaust Pipe," Thesis for Bachelor Degree of Engineering, King Mongkut's Institute of Technology Ladkrabang, 2003. (in Thai)