การประชุมวิชาการเครือข่ายวิศวกรรมเครื่องกลแห่งประเทศไทยครั้งที่ 18
18-20 ตุลาคม 2547 จังหวัด-ขอนแก่น

Analysis of Heat Transport and Water Infiltration in Granular-Porous Media (One-and-Two Dimensional Models)

การวิเคราะห์การส่งถ่ายความร้อนและการแทรกซึมของน้ำในเกรนูลาร์-โพรัส มีเดีย (แบบจำลอง 1 และ 2 มิติ)

ผดุงศักดิ์ รัตนเดโช (P. Rattanadecho)
Department of Mechanical Engineering,
Faculty of Engineering, Thammasat University (Rangsit Campus),
Klong Luang, Pathumthani, Thailand, 12120
Tel (02) 564-3001-9 Fax (02) 564-3010 E-mail: ratphadu@engr.tu.ac.th Webpage: http://www.engr.tu.ac.th/~ratphadu

Abstract

The characteristics of heat transport and water infiltration in granular porous media due to supplied hot water are investigated experimentally and numerically. The distributions of water content and temperature are predicted for one- and two-dimensional models assuming the local thermal equilibrium among water and particles at any specific space. The predicted temperature distributions are compared with the experimental results. Most importantly, the effects of particle sizes, initial water content and supplied water flux on heat transport and flow kinetics are examined. It is found that using a larger particle size results in a faster infiltration rate and forms a wider infiltration layer, especially in the direction of gravity. However, an extension of the heated layer is not as much that of the infiltration layer because the temperature of water infiltration gradually drops due to upstream heat transport.

1. Introduction

Understanding of heat transport in granular packed bed or porous media with water infiltration due to the force of capillary suction is essential in a variety of soil science and chemical engineering applications such as temperature control of soil, recovery of geothermal energy, thermal energy storage, and various reactors in chemical industry. Up to the present time, the related problem of water infiltration in porous media has been investigated both experimentally and numerically by many researchers, for example; Campbell (1985), Bear (1972) and Thomas and King (1991). However, few research reports have been published for heat transport in porous media with water infiltration, except the problem of permafrost (Harlan 1973, Kennedy and Lielmezes 1973, Guymon and Luthin 1974) and drying technology and phase change problem (Nasrallah and Perre 1988, Kaviany and Mittal 1977, Rogers and Kaviany 1992, Ratanadecho et al. 2001a, Ratanadecho et al. 2001b, Ratanadecho et al. 2002, Rattanadecho 2004a, Rattanadecho 2004b and Rattanadecho 2004c).

The purpose of this paper is to clarify the characteristics of heat transport with water infiltration in granular porous media experimentally and numerically. Most importantly, the effects of particle sizes, supplied water flux and initial water content on the flow kinetics are examined. The result presented here provides a basis for fundamental understanding of heat transport and water infiltration in granular materials.

2. Experiment

Figure 1 shows the experimental apparatus for one-dimensional heat transport in granular porous media with water infiltration. The test column, 60 mm inner diameter and 400 mm in height, is made of acrylic resin and equipped with stainless steel screen at the bottom of the bed to prevent the movement of particles. The test column is filled with a mixture of water and spherical soda-lime glass beads with the average diameter (d) of 0.15 mm or 0.4 mm. The water pumped from a tank is heated at a certain temperature in a heating section and is uniformly supplied at the top of granular porous media through a distributor. The test column is covered with insulation to reduce heat loss. The distribution of temperature in granular porous media is measured with Cu-C thermocouples with a diameter of 0.1 mm. Thermocouples are set at 20 mm interval along the axis of granular porous media. The penetration of infiltration front in the granular porous media is determined by interpolating the prescribed temperature from the adjacent thermocouple reading.

Figure 2 shows the experimental apparatus for two-dimensional heat transport in granular porous media with water infiltration. A rectangular test cell with inner dimensions of 300 mm in length, 210 mm in height and 50 mm in width, is made of acrylic resin. The entire test cell is covered with 60 mm thick Styrofoam on all sides to minimize the effects of heat losses and condensation of moisture at the walls. The test cavity is filled with a mixture of water and uniform size spherical glass beads with a diameter of 0.15 mm or 0.4 mm. The heated water is uniformly supplied through a distributor located on the surface of the granular porous media with inside dimensions of 10 mm in length and 50 mm in width. The distributions of temperature of the granular porous media at 24 locations are recorded by a data logger connected to a computer. In order to check the heat losses through test cell for justifying the adiabatic condition, the experiments are performed with and without insulation around the test cell.

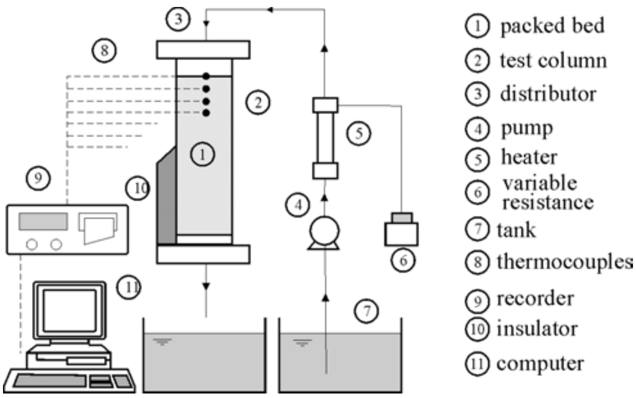
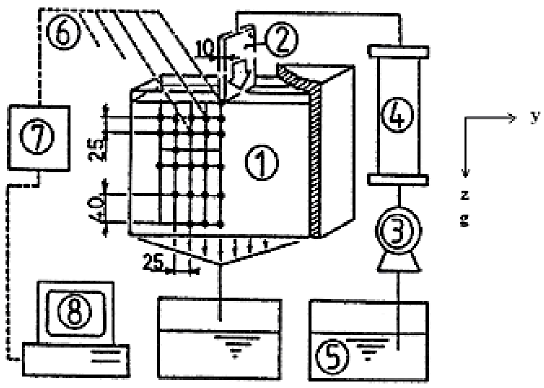


Fig.1 Experimental apparatus for one- dimensional heat transport with water infiltration



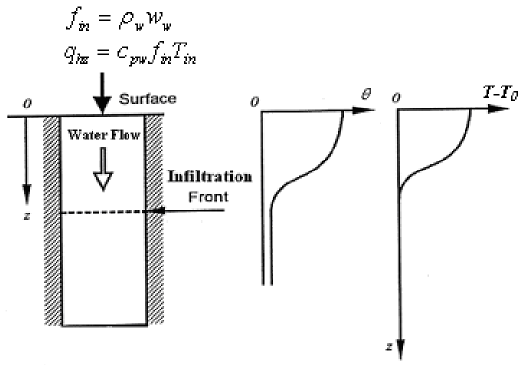
1. Granular porous media 2. Distributor
3. Pump 4. Heater
5. Tank 6. Thermocouples
7. Recorder 12. Computer

Fig. 2 Experimental apparatus for two-dimensional heat transport with water infiltration

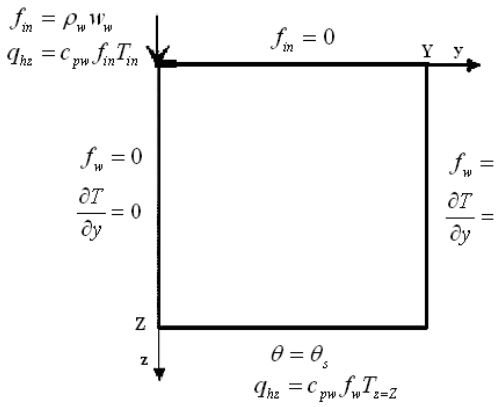
3. Analysis of Heat and Water Transport in Granular porous media

Figures 3(a) and (b) show the physical models for one and two-dimensional heat transport with water infiltration in granular porous media, respectively. Initially, the system is at a uniform temperature and water content. At time t>0, the hot water is uniformly supplied at the top of granular porous media through a distributor and the infiltration front is then formed. The following simplified assumptions are made in the analysis.

1. The porous medium is isotropic, homogeneous and has uniform porosity. Therefore, the volume average model for isotropic and homogeneous material can be used in the theoretical modeling and analysis.
2. The water infiltration in granular porous media is governed by the Darcy's law.
3. The movement of water vapor can be neglected.
4. The volumetric change due to temperature gradient is negligible.
5. The natural convection is absent.
6. There is thermal equilibrium between the water and the matrix at any specific space in granular porous media.



(a)



(b)

Fig.3 Physical models; (a) One-dimensional model (b) Two-dimensional model

3.1 Water Transport

It is assumed that Darcy's law may hold for the flow of liquid in unsaturated granular porous media. Therefore, water flux f_w can be represented by using matric potential (ψ):

$$f_w = k\left(\frac{\partial \psi}{\partial z}\right) \quad (1)$$

where *k* is hydraulic conductivity. The parameters, ψ and *k* are considered to be a functions of volumetric water content (θ). The relationships between three parameters are assumed to be approximated by the following equations proposed by Campbell (1985):

$$\psi = \psi_e\left(\frac{\theta}{\theta_s}\right)^{-b} \quad (2)$$

$$k = k_s\left(\frac{\theta}{\theta_s}\right)^{2b+3} \quad (3)$$

$$k = k_s\left(\frac{\psi}{\psi_e}\right)^{2+3/b} \quad (4)$$

where *b* is the exponent constant obtained from the characteristic curve of matric potential-water content in granular porous media. k_s and θ_s are the saturated hydraulic conductivity and saturated water content, respectively. ψ_e is called the air entry potential and means the matric potential when water movement starts on the maximum gap in granular porous media. For the water infiltration in y-z plane with z-direction under a gravitational effect, the conservation of the water is given as:

$$\rho_w\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial y}\left(k\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial z}\left[k\left(\frac{\partial \psi}{\partial z} - g\right)\right] \quad (5)$$

where ρ_w is water density, g is gravitational acceleration and subscripts y and z denote the coordinates. Since this equation is nonlinear, it is convenient to use the Kirchhoff integral transformation. In this study, the author recommends to use an integral form of matric flux potential as shown below:

$$\phi = \int_{\psi_e}^{\psi} k(\psi)d\psi \quad (6)$$

Substitute Eq. (6) into Eq. (5), the conservation equation of the water is:

$$\rho_w\frac{\partial \theta}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} - g\frac{\partial k}{\partial z} \quad (7)$$

Equations (2)-(4) are also transformed respectively as:

$$\phi = \phi_e\left(\frac{\theta}{\theta_s}\right)^{b+3} \quad (8)$$

$$k = k_s\left(\frac{\theta}{\theta_s}\right)^{2b+3} \quad (9)$$

$$k = k_s\left(\frac{\phi}{\phi_e}\right)^{(2b+3)/(b-3)} \quad (10)$$

Heat Transport

The energy conservation equation in the granular porous media is given by:

$$\frac{\partial}{\partial t}\left[\left(\rho c_p\right)_T T\right] = \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) - \frac{\partial q_{hy}}{\partial y} + \frac{\partial}{\partial z}\lambda\left(\frac{\partial T}{\partial z}\right) - \frac{\partial q_{hz}}{\partial z} \quad (11)$$

where $\left(\rho c_p\right)_T$ is the effective heat capacitance of water-mixture, T is temperature, λ is the effective thermal conductivity depending on water content; and q_h is the energy transfer occurring due to water infiltration. Under thermal equilibrium conditions and using the volumetric average technique, the effective heat capacity is given by:

$$\left(\rho c_p\right)_T = \rho_w c_{pw}\theta + \rho_p c_{pp}\left(1-\theta_s\right) \quad (12)$$

where subscripts w and p denote water and particle, respectively. Based on the experimental results of Rattanadecho (2001b) using glass beads saturated with water, the effective thermal conductivity is represented as a function of the saturation degree of water content (*s*= θ/θ_s),

$$\lambda = \frac{0.8}{1+3.78e^{-5.95s}} \quad (13)$$

The energy transfer to y- and z-directions due to water infiltration is, respectively, given by:

$$q_{hy} = c_{pw}\left(k\frac{\partial \psi}{\partial y}\right)T \quad (14)$$

$$q_{hz} = c_{pw}k\left(\frac{\partial \psi}{\partial z} - g\right)T \quad (15)$$

3.2 Boundary Conditions and Initial Conditions

The boundary conditions proposed for the one and two-dimensional models are shown in Fig.3(a) and Fig. 3(b), respectively. The initial conditions are given by uniform initial temperature and water content.

3.3 Calculation Procedure

The governing equations of water and heat transport are solved numerically by using the finite control volume method. The basic strategy of finite control volume discretization method is to divide the domain into a number of control volumes and then integrate the conservation equations over this control volume over an interval of time ($t, t+\Delta t$) so that a set of algebraic equations are obtained. In these equations, there are two space increments ($\Delta y, \Delta z$) and one time increment (Δt). The magnitude of $\Delta y, \Delta z$ and Δt cannot be chosen arbitrarily. In order to have a stable solution of Eqs. (7) and (11).

Furthermore, at each time increment, the nodal values of θ and T are solved iteratively and convergence is checked on both these variables. The Newton-Raphson method is employed at each iteration for quicker convergence (Ratanadecho et al. 2002).

4. Results and Discussions

4.1 Matric Potential and Hydraulic Conductivity

Figure 4 shows the variation of matric potential with volumetric water content in granular porous media for the different particle sizes obtained from the present experiments. It is clearly seen that the matric potential decreases with increasing water content. In the case of the same water content, the small particle size corresponds to a higher matric potential. The solid lines in this figure show the relationships obtained by a least squares method fitting to the form of Eq. (2). Next, the hydraulic conductivities for the granular porous media saturated with water are obtained from the experiments of steady water flow under the constant matric potential gradient in the granular porous media.

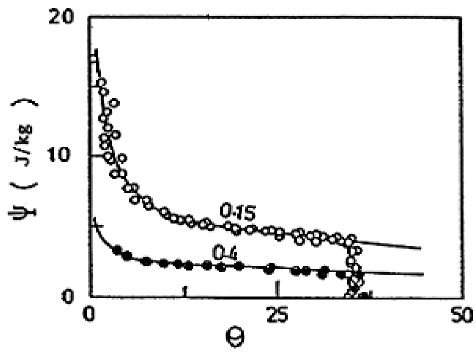


Fig.4 Change of matric potential with volumetric water content

Figure 5 shows the change of water flux with the matric potential gradient in the granular porous media. The relationship between water flux and the matric potential gradient is almost linear and the slope directly corresponds to the hydraulic conductivity of each particle size. It is found that larger particle size corresponds to a higher saturated hydraulic conductivity. Additionally, the characteristics of water infiltration in granular porous media obtained here were already shown in Rattanadecho 2004b.

One-Dimensional Heat and Water Transport

Figures 6-8 show the one dimensional profiles of water infiltration and heat transport in z-direction.

Fig.5 Change of water flux with matric potential gradient in saturated state

Figure 6 shows the limit infiltration time (t) and the total quantity of water ($\bar{f}_m$) infiltrating into granular porous media as a parameters of the supplied water flux. Corresponding to the explanation above, a greater supplied water flux leads to shorter limit infiltration time and less total water quantity Nevertheless, only the former case ($f_{in} \le g.k_s$) is discussed in the this paper.

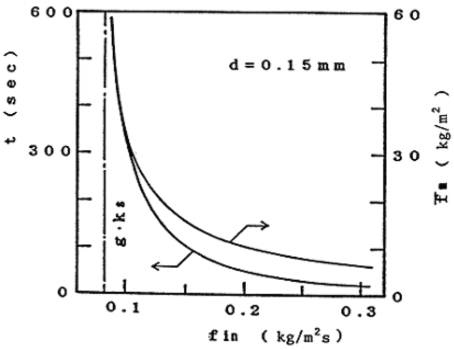


Fig.6 Limit infiltration time and total water quantity ($\bar{f}_m$)

Figure 7 shows the distribution of water content and temperature at different times as a parameter of supplied water flux. The supplied water flux, f_{in}=0.247 kg/(m^2.s), is almost equal to $g.k_s$ g in the granular porous media of 0.4 mm diameter. It is observed that a greater supplied water flux leads to a higher water content within the granular porous media and forms a wider infiltration layer, i.e., infiltration front. The temperature in the granular porous media rises due to the water infiltration, but the heated layer does not extend as much as the infiltration layer. This means that heat transport hardly occurs in the layer close to the infiltration front because the temperature of water infiltrating there has already dropped due to upstream heat transport. The predicted results for the temperature distributions are in agreement with the experimental results.

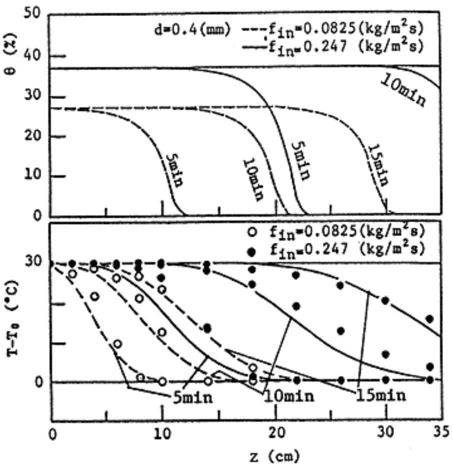


Fig.7 Distributions of water content and temperature packed bed

Figure 8 shows the effect of particle size on the distributions of water content and temperature under the same supplied water flux. A larger particle size leads to a faster infiltration rate and forms a wider infiltration layer. The heated layer also expands somewhat with a larger particle size. However, it is evident from the figure that the discrepancy of the heated layers is smaller compared to that of infiltration layers.

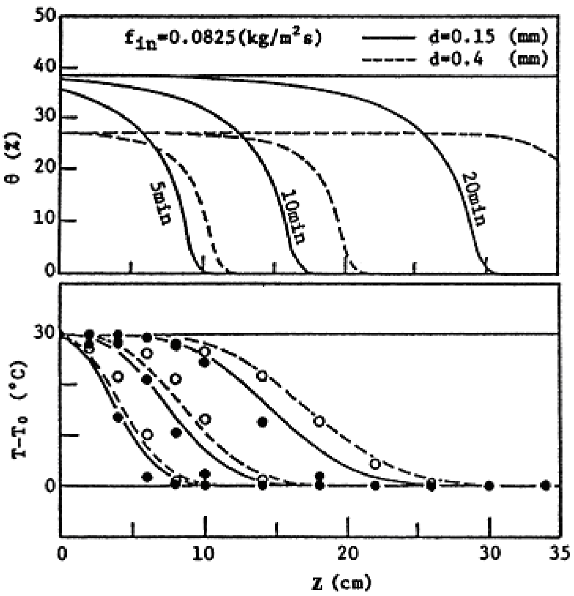


Fig. 8 Effect of particle size on the distributions of water content and temperature

To clarify the phenomena of heat transport in granular porous media, the distributions of the rate of heat storage per unit volume at different times are presented in Fig. 9. In the early stage of water infiltration, it can be seen that the maximum rate of heat storage is large and close to the leading edge of the granular porous media. As water infiltration progresses, the value of the maximum rate decreases and its position gradually moves to downstream side, resulting in the formulation of a wider layer.

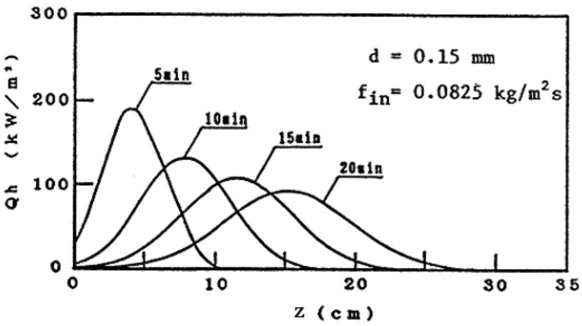


Fig.9 Distribution of the rate of heat storage per unit volume at different times

Two-Dimensional Heat and Water Transport

The two-dimensional heat and water infiltration in y- and z-directions will now be discussed with the aids of Figs. 10-14. Figure 10 shows the effect of particle size on water infiltration under the constant supplied water flux. In the early stage of infiltration, the infiltration layer expands uniformly in both y- and z-directions because matric potential has more influence than a gravity potential. As water infiltration progresses, gravitational effect becomes superior to the matric potential and the infiltration layer expands wider in z-direction which is the direction of gravity. This trend is more obvious for a larger particle size having a lower matric potential.

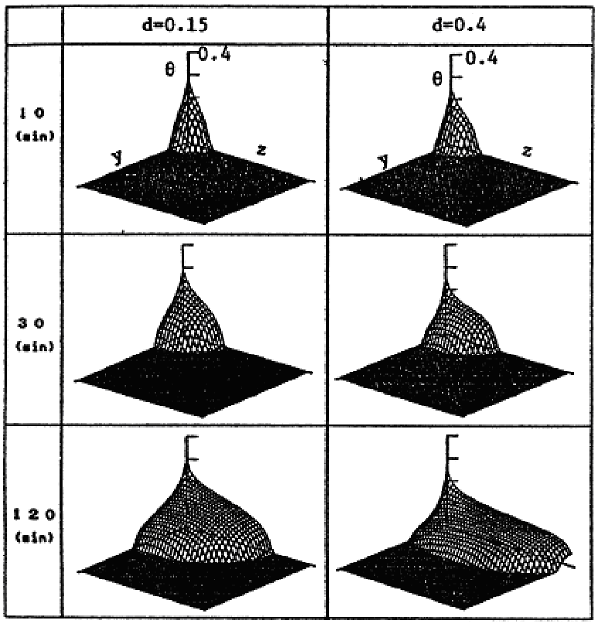


Fig.10 Effect of particle size on two-dimensional water

infiltration

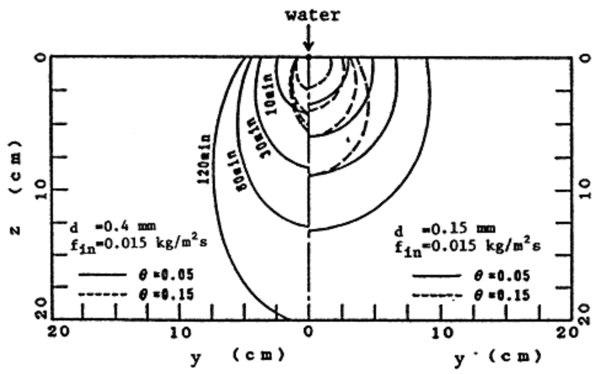


Fig.11 Distribution of constant water content at different times

Figure 11 shows the distributions of constant water content line as the parameters of initial water content and particle size at different time. The solid lines represent initial water content (θ) of 0.05 and the dotted lines represent initial water content of 0.15. It is seen that the lines of initial water content of 0.05 are nearly identical to the infiltration front as shown in Fig. 10. The following discussion refers to the effect of particle size on water flow mechanism under the same condition. It is evident from the figure that, in the case of larger particle size (d=0.4 mm), the infiltration front expands wider in z-direction for case of lower initial water content (θ=0.05) as compared with a smaller particle size (d=0.15 mm) because the gravity becomes superior to the matric potential. However, the infiltration front in the case of small particle size expands wider in y-direction in comparison with a larger particle size due to stronger effect of the matric potential.

Furthermore, the infiltration front for the case of higher initial water content (θ=0.15) becomes steeper for both particle sizes that the infiltration front for the case of lower initial water content. This may be due to the fact that the formation of higher initial water content around each particle in unsaturated granular porous media allows gaps between pores to be narrower so that the infiltration front is difficult to expand widely in all directions. In contrast to that case of lower initial water content, infiltration front expands wider in case of smaller particle size. This may be due to the fact that the effect of the matric potential becomes the important role for the conditions of higher initial water content particularly the smaller particle size, as presented in Fig. 4.

Figures 12 and 13 show the effect of particle size on the expansion of the heated layer due to infiltration of supplied hot water, which corresponds to the conditions those shown in Figs. 10 and 11. The heated layer expands somewhat in z-direction at a larger particle size, but the effect of particle size on the expansion of heated layer is smaller as compared to the infiltration layer. In Fig.13, it is found that the isothermal lines are always wider in z-direction and narrower in y-direction for a larger particle size and differ from the distribution of water content as shown in Fig. 11.

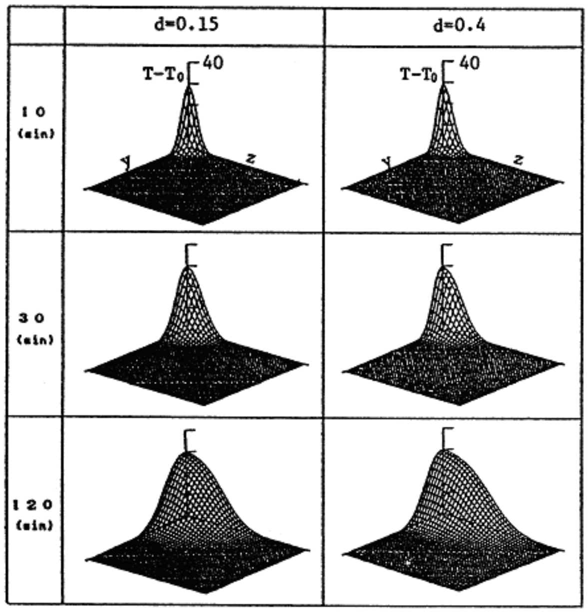


Fig. 12 Effect of particle size on two-dimensional heat transport with water infiltration

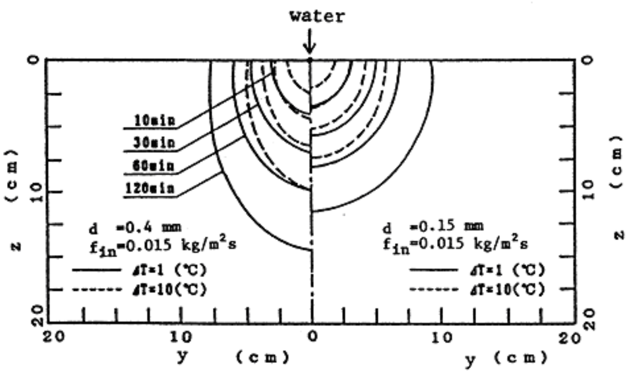


Fig.13 Effect of particle size on isothermal line in granular packed bed

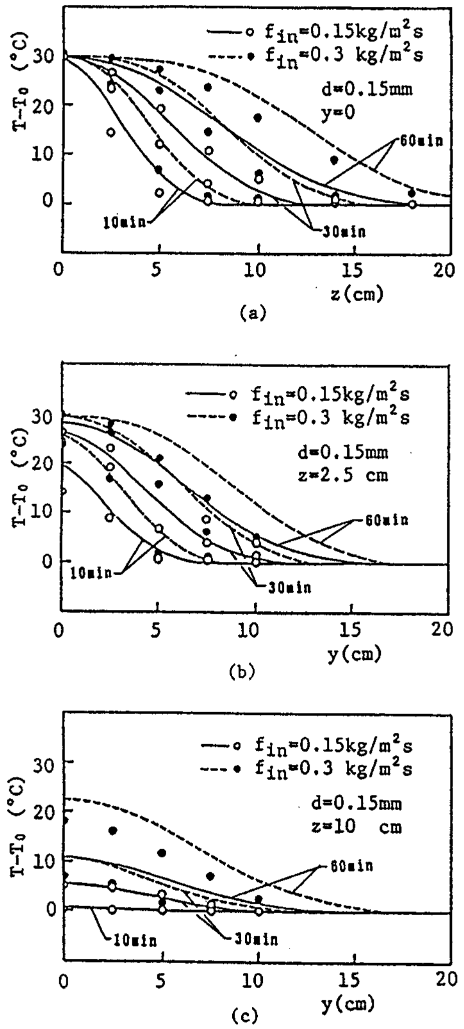


Fig. 14 Comparison between the predicted and the experimental temperature distributions

Figure 14 shows the comparison between the predicted and experimental temperature distributions in the specified directions: (a) z-direction on y=0, (b) y-direction on z=2.5 cm and (c) y-direction on z=10 cm. Here, the difference of water flux is considered to be a main parameter. It is observed that as water infiltration progresses, the heated layers expand in y- and z-directions. A greater supplied water flux results in a wider heated layer and displays a faster temperature rise. The observation of temperature profiles depicted in Fig. 14 verifies that the match between the predicted results and experimental data is qualitatively consistent, with the predicted results exhibiting the same overall trend of the experimental profiles. However, the predicted results display slightly over-predicted experimental data. The source of discrepancy may be attributed to the effect of heat loss through a test cell during experiment. Numerically, the discrepancy may be attributed to uncertainties in the thermal property.

Conclusions

The experiments and numerical analysis presented in this paper describe many of the important interactions during heat transport with water infiltration within granular porous media. The following are the conclusions of this work:

1. A generalized mathematical model of heat transport with water infiltration within granular porous media is proposed. It is used successfully to describe the flow phenomena under various conditions.
2. As the distribution profiles between heated layer and infiltration layer are compared, it is observed that the heated layer does not extend as much as the infiltration layer. This means that heat transport hardly occurs in the layer close to the infiltration front because the temperature of water infiltrating gradually drops due to heat transport upstream. Furthermore, the effect of particle size on the discrepancy of heated layers is smaller compared to that of the water content layers.
3. It is found that the gravity, the matric potential and initial water content have clearly exhibited influence on the infiltration front and heated layer. The predicted results are in agreement with the experimental results.
4. It is possible to use the present model for analysis of numerous other applications (e.g. temperature and moisture movement in the ground, the stability of buried electrical cable and underground soil heating). Our future aim is to validate the investigation of heat transport and unsaturated infiltration in granular porous media especially in multi-layered models.

Acknowledgement

The authors are pleased to acknowledge ***Thailand Research Fund (TRF)*** and ***Faculty of Engineering, Thammasat University*** for supporting this research work.

References

[1] Bear, J. (1972). *Dynamics of fluids in porous media*, Elsevier, New York.

[2] Ben Nasrallah, S., and Perre, P. (1988). “Detailed study of a model of heat and mass transfer during convective drying of porous media.” *Int. J. Heat and Mass Transfer*, 31, 957-967.

[3] Campbell, G.S., (1985). *Soil Physics with Basic*, Elsevier, New York.

[4] Guymon, G.L., and Luthin, J.N. (1974) “A coupled heat and moisture transport model for arctic soils.” *Water Resources Research*, 10, 995-1001.

[5] Harlan, R.L. (1973). “Analysis of coupled heat-fluid transfer in partially frozen soil, *Water Resources Research*, 9, 1314-1323.

[6] Kennedy, G.F., and Lielmezes, J. (1973). “Heat and mass transfer of freezing water-soil system.” *Water Resources Research*, 9, 395-400.

[6] Kaviany, M., and Mittal, M. (1987) “Funicular state in drying of porous slab.” *Int. J. Heat and Mass Transfer*, 30, 1407-1418.

[7] Rogers, J.A. and Kaviany, M. (1992). “Funicular and evaporative-front regimes in convective drying of granular beds.” *Int. J. Heat and Mass Transfer*, 35, 469-479.

[8] **Ratanadecho, P**., Aoki, K., and Akahori, M. (2001a). “Experimental and numerical study of microwave drying in unsaturated porous material.” *Int. Commun. Heat Mass Trans.*, 28, 605-616.

[7] **Ratanadecho, P**., Aoki, K., and Akahori, M. (2001b). “A Numerical and experimental study of microwave drying using a rectangular wave guide.” *Drying Technology International Journal*, 19, 2209-2234.

[8] **Ratanadecho, P**., Aoki, K., and Akahori, M. (2002). “Influence of irradiation time, particle sizes and initial moisture content during microwave drying of multi-layered capillary porous materials.” *ASME J. Heat Transfer*, 124, 151-161.

[10] **Rattanadecho, P**., (2004a), Experimental and numerical study of solidification process in unsaturated granular packed bed.” *AIAA J. Thermophysics and Heat Transfer*, 18, 87-93,

[11] **Rattanadecho, P**., (2004b), The numerical and experiment investigation of heat transport and water infiltration in granular packed bed due to supplying hot water (One- and –two dimensional models).” *ASCE Engineering Mechanics J.* (in press)

[12] **Rattanadecho, P,** (2004c), The experimental and theoretical analysis of heat and mass transfer mechanism during convective drying of multi-layered porous packed bed, *AIAA J. Thermophysics and Heat Transfer* (in press).

[13] Thomas, H.R., and king, S.D. (1991). “Coupled temperature/capillary potential variations in unsaturated soil.” *J. Engrg. Mech. ASCE.*, 117(11), 2475-2491.