The 17th Conference of Mechanical Engineering NETwork of Thailand
15-17 October 2003 Prachin Buri

Estimation of Sound Transmission Loss for a Design of Single Helmholtz Resonator-Type Silencers.

Ming Lokitsangtong Pongsak Kummol
Department of Mechanical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang
3 Moo 2 Chalongkrung Road, Ladkrabang, Bangkok 10520, Thailand
Tel: 0-23264197 Ext. 107 Fax: 0-23264198 E-mail: klming@kmitl.ac.th

Abstract

Linear wave theory is amenable to "spreadsheet" type manipulations that can subsequently be used in a design for a single Helmholtz resonator-type silencer. In this paper, algorithms have been developed to take advantage of the linear wave theory and the transfer matrix methodology. The method is illustrated by applying it to a square and a circular duct installed with different-size resonator and a stationary medium is inside the duct. Results of transmission loss for a frequency range from linear wave theory computations and experiments are compared.

1. Introduction

Plane wave propagated along the longitudinal axis of a duct can be described by the acoustic linear wave theory. The assumption of plane wave traveling in a viscous stationary medium as in the case of a duct installed with a single Helmholtz resonator-type silencer is valid to the extent that the wavelength of the sound is very much longer than the cross-sectional dimension of the duct. Davis et al. has performed some rigorous calculations and experiments on silencers with the single resonators of different types based on the linear wave theory[1]. The two state variables, i.e. the sound pressure p and the volume velocity q on the two sides of the resonator can be related by the transfer or transmission matrix[2]. This is amenable to "spreadsheet" type computations that can facilitate in the design process. This paper will illustrate how a spreadsheet application can be used to implement analysis and design of a single Helmholtz resonator-type silencers. The results of the computational method are then compared with the ones from experiments.

2. Equations

A silencer composed of a single Helmholtz resonator, which is mounted on one side of a rectangular or a circular duct terminating with the anechoic end, is depicted in Fig.1. Munjal derived a transfer matrix in connection with sound pressure and mass rate at the duct sections directly at the front and back of a resonator[2]. If volume velocity is used instead of mass rate in the above matrix, matrix elements A,B,C,D may be rewritten as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2M + \frac{Z_r}{Z_0}} \begin{bmatrix} M + \frac{Z_r}{Z_0} & M^2 Z_0 \\ \frac{1}{Z_0} & M + \frac{Z_r}{Z_0} \end{bmatrix} \qquad (1)$$

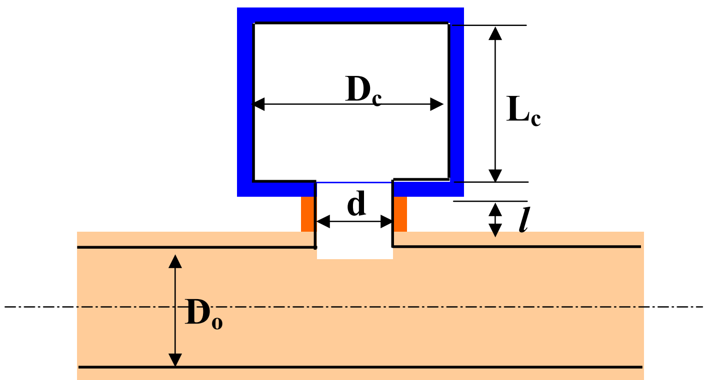

Fig.1 Model of silencer

In eq. (1), M is Mach number of mean flow passing over the resonator and Z_o is the characteristic impedance of the duct given by

$$Z_0 = \frac{\rho_0 c}{S_0} \qquad (2)$$

where ρ_o is mean density of medium, c is sound speed, and additionally Z_r is the acoustic impedance of a resonator given by

$$Z_r = R + j(X + R) \qquad (3)$$

where R is connector resistance and X is resonator reactance with the resistance term omitted. Therefore the dimensionless impedance Z_r/Z_o may be expressed by

$$\frac{R}{Z_0} = \frac{16}{\left(\frac{d}{\ell_e}\right)\left(\frac{d}{D_0}\right)^2}\sqrt{\frac{\mu f}{\pi \rho_0 c^2}} \qquad (4)$$

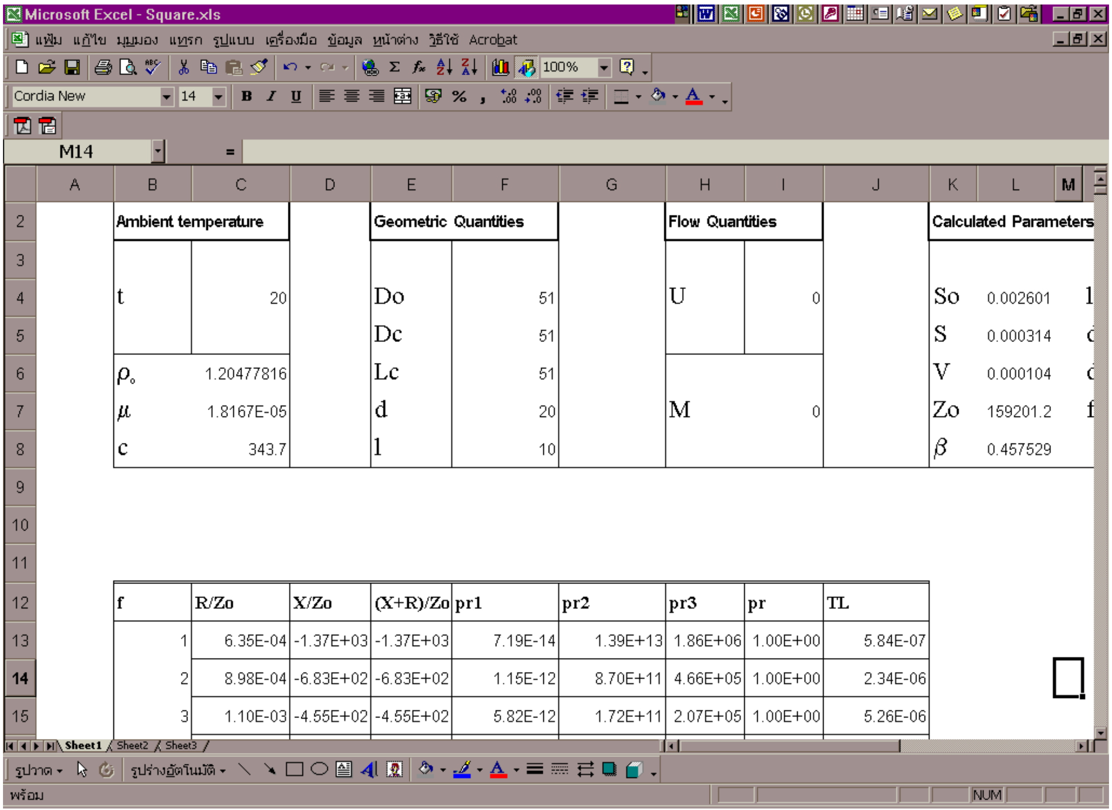

Fig.2 Spreadsheet setup

and

$$\frac{X}{Z_0} = S_0\left(\frac{f}{f_r} - \frac{f_r}{f}\right)\sqrt{\frac{\ell_e}{VS}} \qquad (5)$$

where μ denotes dynamic viscosity of medium, f frequency, and f_r resonance frequency given by

$$f_r = \frac{c}{2\pi}\sqrt{\frac{S}{V\ell_e}} \qquad (6)$$

$$k = \frac{2\pi f}{c(1 - M^2)} \qquad (7)$$

In the above equations, V denotes volume of resonance chamber, S indicates cross-sectional area of connector, and Do, So denote width or diameter and cross-sectional area of a duct respectively.

Thus sound pressure p_I and volume velocity q_I at the silencer entrance may be related to both quantities p_{II} and q_{II} at its exit, which can be given as

$$\begin{pmatrix} p_I \\ q_I \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} p_{II} \\ q_{II} \end{pmatrix} \qquad (8)$$

The above p_I, q_I, p_{II}, and q_{II} are written as

$$p_I = p_{Ii} + p_{Ir} \qquad (9)$$

$$q_I = \frac{1}{Z_0}(p_{Ii} - p_{Ir}) \qquad (10)$$

$$p_{II} = p_{IIi} \qquad (11)$$

$$q_{II} = \frac{p_{IIi}}{Z_0} \qquad (12)$$

where p_{Ii} and p_{Ir} are, respectively, the incident and reflected pressures at the silencer entrance and p_{IIi} transmitted pressure in the tail duct.

The transmission loss TL is defined as

$$TL = 10\log\left|\frac{p_{Ii}}{p_{IIi}}\right|^2 \qquad (13)$$

3. Implementation on a spreadsheet application

The analysis is implemented on the Microsoft Excel[3] for a square and a circular duct with a single Helmholtz resonator for the tested configuration as described in Appendix A. The motion of fluid inside is also included. The end correction factor β employed in the calculation of effective length I.e. has been adopted from experimental results in ref.[4], and experimental formulae have been obtained as

$$\beta = e^{-0.51416}\left(\frac{d}{l}\right)^{-0.3467} \qquad (14)$$

for a square duct and

$$\beta = e^{-0.3495}\left(\frac{d}{l}\right)^{-0.3971} \qquad (15)$$

for a circular duct.

The effective length l_e, the actual length l and the diameter d of connector are related in the form

$$l_e = l + \beta d \qquad (16)$$

By manipulating eq's (8)-(12), the following outcome is arrived

$$\frac{p_{Ii}}{p_{IIi}} = \frac{1}{2}\left(A + \frac{B}{Z_o} + Z_o C + D\right) \qquad (17)$$

After expanding out the real and imaginary parts of the matrix elements and a final absolute term is derived as

$$\left|\frac{p_{Ii}}{p_{IIi}}\right|^2 = \frac{1}{4[(2M+a)^2+b^2]^2}[\{(2M+a)(M^2 + 2M + 1 + 2a) + 2b^2\}^2 + b^2\{2M - M^2 - 1\}^2] \qquad (18)$$

where a=R/Z_o and b=(X+R)/Z_o

Equation(18) has been implemented on the spreadsheet application as depicted in Fig.2.

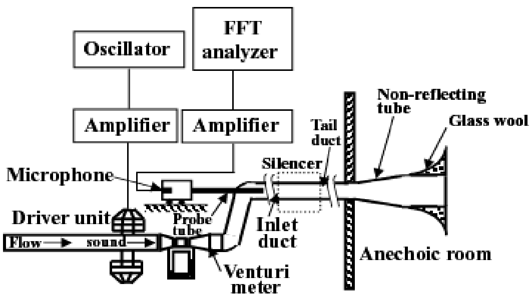

Fig.3 Schematic diagram for the experimental set-up

4. Experimental apparatus and method

The experimental apparatus used in this investigation is shown schematically in Fig.3. The sound was produced by an oscillator feeding through an amplifier and conducted to the system by means of two loud speakers. The sound which passed over the single Helmholtz resonator continued down through the tailpipe to the termination, which consisted of glass wool surrounded by an involute tube. The propagating signal was detected with the probe tube microphone traversing axially along the test section.

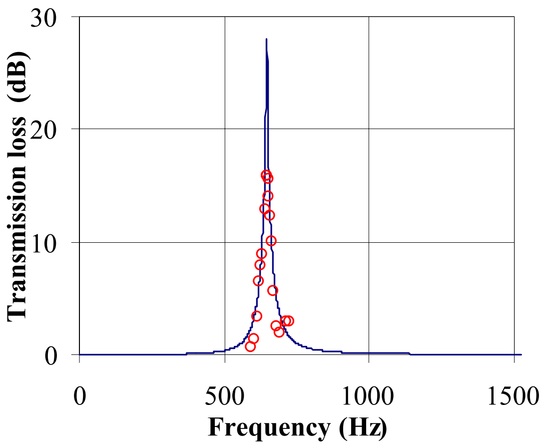

Fig.4 Comparison of experimental and spreadsheet results for square duct

5. Results and conclusions

Figures 4 and 5 show the transmission loss characteristic of a square and a circular duct respectively as obtained from the speed sheet calculation and experimental test[5]. The comparison of the results demonstrates a convincingly good agreement.

The spreadsheet is proposed as a very convenient and effective platform for solving design problems. A better design of a single Helmholtz resonator-type silencer can be achieved.

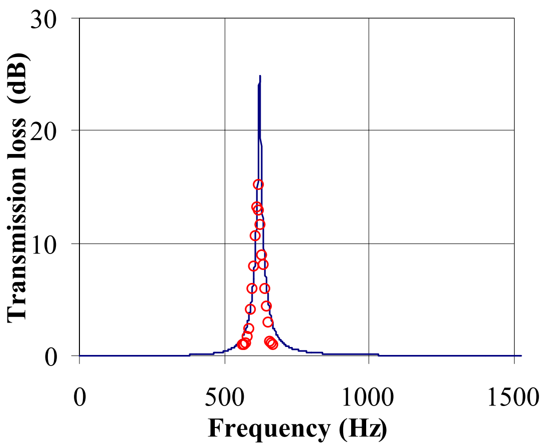

Fig.5 Comparison of experimental and spreadsheet results for circular duct

Appendix A

The tested silencer has the following dimensions:

	Square	Circular
Dimension of duct (mm)	70	83
Diameter of resonance chamber (mm)	50.8	50.8
Length of resonance chamber (mm)	60	60
Diameter of connector (mm)	29.7	29.7
Length of connector (mm)	25	25
Ambient temperature (^{O}C)	25	25
Flow velocity (m/s)	0	0

References

[1]. D. Davis, Jr., G. Stevens, Jr., D. Moore and G. Stokes,"Theoretical and Measured Attenuation of Mufflers at Room Temperature without Flow, with Comments on Engine-exhaust Muffler Design," NACA Technical Note 2893, 1954.

[2]. M.L. Munjal, "Velocity Ratio-cum-Transfer Matrix Method for the Evaluation of a Muffler with Mean Flow," J. Sound Vib., **39**(1975) No.1 pp.105-119.

[3]. Microsoft Corporation, "User's Guide Microsoft Excel 2000," Microsoft Corporation, 2000.

[4]. M. Hototzuka, "Characteristics for the Helmholtz Type Silencer," Thesis for Master Degree of Engineering, Tokai University 2000. (in Japanese)

[4]. P. Mungur and G.M.L. Gradwell, "Acoustic Wave Propagation in a Sheared Fluid Contained in a Duct," J. Sound Vib., **9**(1969) No.1 pp.28-48.

[5]. T. Padsumran, P. Puksiriwongchai, and S. Phuwipadawat, "Acoustic Study of Helmholtz Type Silencer," Thesis for Bachelor Degree of Engineering, King Mongkut's Institute of Technology Ladkrabang, 2002. (in Thai)