# UAV Path Planning for Maximum Coverage Surveillance of Area with Different Priorities 

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#### Abstract

This paper focuses on the surveillance problem on the area with different priorities using Unmanned Aerial Vehicle. This work develops a path planning algorithm for UAV surveillance in order to maximize the coverage. The UAV is modeled as a hybrid system in order to simplify the aircraft dynamics such that it fits into the aspect of path planning. The different parts of the area could have different priorities. The priority indicates that the UAV should pay more attention in those places than other region. The concept of entropy is employed to handle such issue.. Entropy assigned on each part is correspondent to higher priority on the cell. The definition of coverage is introduced as a summation of entropy. The objective of the planner is to maximize the coverage with the shortest time while the constraints on UAV, such as velocity and turning radius, are satisfied. The area is covered more than $97 \%$ in most cases.


Keywords: Path planning, Surveillance, UAVs.

## 1 Introduction

One of the most important applications of Unmanned Aerial Vehicles (UAVs) is in area surveillance or border patrol [3]. There are many advantages over traditional method such as lower cost, more agility, and real-time information. The aerial photographs provide more accuracy due to current camera technology. There have been many works on target tracking/identification or search using UAVs ( [2], [9], [8]).

The use UAV is more common and frequent because it is more affordable, more reliable, and more importantly it can reduce a number of casualties in dangerous environment. Therefore, the UAV path planning has became more important issue that
must be taken into account. The paths have effect on the performance such as fuel consumption, amount of area coverage, and traveling time. In order to maximize the benefits of UAVs especially in a developing country like Thailand, the path has to be carefully and systematically planned. The UAV path planning has gains many interests in aerospace community in the past decade ( $[4],[7],[11]$ ).

One of the widely used search/surveillance patterns is lawn-mower [5], [10]). It is arguably the best approach available. It is chosen because of its continuous curve and better performance in smaller area. However, it provides very little flexibility in term of the shape of area and different priority on some certain area. Moreover, the capability of UAVs, such as minimum turning radius and velocity, creates constraints that affect the performance significantly. These constraints distinguish UAV path planning from others (i.e., robots).

This work intends to investigate effects of the lawn-mower parameters on surveillance performance. A more formal description of the area including different priorities of the area is considered. The modified configuration based on the knowledge of parametric study of typical lawn-mower pattern is developed. The area is mapped using the entropy concept. The concept allows the modeling of different priority by increasing entropy in the more important area. The revisits are the key to entropy reduction of such area. The coverage is formally defined as summation of all entropy. The performance of the developed approach is evaluated by amount of coverage. The objective is to plan paths to reduce as much entropy as possible in the minimum time. The dynamics of UAV must be taken into account to insure the feasibility of path.

The paper is organized as follows: first the aircraft dynamics is formulated as a hybrid system. Then the area and camera footprint descriptions are given. The number of visiting on each path is determined

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based on initial entropy of the area. Finally, the path is generated and simulation results are shown.

## 2 Hybrid Modeling

Hybrid Modeling is one of the most powerful modeling tools available today. A complex highly nonlinear system such as aircraft dynamics can be simplified so that it is easier to work on. The aircraft dynamics is decomposed based on its basic movements such as turning, going straight. In this paper, a two dimensional plane $(x, y)$ is considered. There are three discrete states of the aircraft: 1) going straight, 2) turning left, 3) turning right. It is noted that a more complex state such as turning left slowly can be easily included. The model is simplified here to better evaluate the path performance.

There are two types of states as follows,

1. Discrete State, $\mathbf{x}_{D}$ :

$$
\begin{equation*}
\mathbf{x}_{D} \in[0,1,2] \tag{1}
\end{equation*}
$$

It indicates going straight, turning left, and turning right respectively.
2. Continuous states, $\mathbf{x}$ : Here, position of the aircraft can be described using four variables $(x, y, V, \psi)$, where $(x, y)$ indicates location, $V$ is total velocity, and $\psi$ indicates heading.

$$
\begin{equation*}
\mathbf{x}=\{x, y, V, \psi\} \tag{2}
\end{equation*}
$$

The aircraft dynamics can then be represented as,

$$
\begin{array}{r}
\dot{x}(t)=V \cos (\psi(t)) \\
\dot{y}(t)=V \sin (\psi(t)) \\
\dot{V}(t)=a(t) \\
\dot{\psi}(t)=\omega(t) \tag{6}
\end{array}
$$

where $a(t)$ and $\omega(t)$ are the linear acceleration and angular velocity in yawing direction respectively. They are considered as input of the system. The input is different for each discrete state. The angular velocity,$\omega(t)$, is zero where $x_{D}=0$

$$
\omega(t)= \begin{cases}=0 & \text { if } x_{D}=0 \\ <0 & \text { if } x_{D}=1 \\ >0 & \text { if } x_{D}=2\end{cases}
$$

A simple hybrid model is illustrated in Figure 1. TO complete the model, the condition in discrete state transition must be provided. These conditions will be given by the path planner as discussed later. The hybrid model can be easily simulated using Matlab Stateflow.


Figure 1: A Simple Hybrid Model of Aircraft Dynamics

## 3 Area Description

A rectangular shape area is considered here. There are different priorities on different section of the area. This is very useful in an event to resource/disaster surveillance. The user can integrate the knowledge into the path planner to improve performance. For example, in disaster evaluation the village area is more important than forest. One constraint is that every part in the area must be explored. No discontinuity is allowed because it results in non-convex problem that is much more difficult to solve. A more general description of the area should be included in further work.

### 3.1 Entropy representation

In order to mathematically represent the different priorities on different parts, the concept of entropy is employed. Entropy in this paper indicates the uncertainties of the area. Therefore, higher entropy can be assigned to the area with more priorities as shown in Figure 2. In the figure, the area is gridded into cells. Entropy is assigned to each cell based on its priority. The entropy shows here is also normalized (the minimum entropy is one).

Another constraint imposed in this paper is that the same entropy must be presented for an entire strip or column. This is essential for lawn-mower pattern that is considered here. A relaxation of this constraint must be done through another search pattern.

Consider an area with size of $N$ rows and $M$ columnsThe coverage, $C O V$, can then be defined by using the summation of overall entropy.

$$
\begin{equation*}
C O V=1-\frac{\sum_{i}^{N} \sum_{j}^{M} N E_{i j}}{E_{0}} \tag{7}
\end{equation*}
$$

where $E_{i, j}$ is the entropy in cell $(i, j)$. The $i$ index

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Figure 2: Area Representation using Entropy
indicates the coordinate in $y$ axis while the $j$ index indicates the coordinate in $x$ axis. The variable $E_{0}$ is the summation of initial entropy. Therefore, in the beginning where $\sum_{i}^{N} \sum_{j}^{N} E_{i j}=E_{0}$ the coverage is zero.

### 3.2 Camera Footprint

The most common tool in surveillance is camera. The footprint of a camera is rectangular. It is assumed that the area under footprint is explored. The more thorough survey can be accomplished by revisiting such that the new and previous footprints overlap. Because the normalized entropy is utilized here, one visit from the camera results in one unit entropy reduction. Figure 3 shows one unit entropy reduction on cell $(2,5)$ and $(3,5)$.


Figure 3: Effect of Camera Footprint on the Area Entropy

However, because of aircraft dynamics, the footprint of the camera also depends on the heading unless there is a separate camera control. This creates an additional issue as the aircraft turns. An example of the footprint as the aircraft heading is $45^{\circ}$ is shown in Figure 4. The footprint with size of $(2 \times 1)$ is employed through out the paper for the sake of
simplicity.


Figure 4: Effect of Camera Footprint on the Area Entropy while UAV is turning

Another interesting sensor in surveillance is Lidar [6]. Lidar has one major advantage over camera: it is less sensitive to the weather than camera is. The footprint of Lidar is also rectangle shape. Therefore, the same approach can be employed for both camera and Lidar.

## 4 Lawn-Mower Search Pattern

Lawn-mower pattern is one of the most widely used search pattern because of its simplicity and guarantee of continuity and smoothness of the path.


Figure 5: Lawn-mower Pattern

It can be seen in Figure 5 that the pattern consists of two major movements: 1)Straight Line and 2)Semicircle. The semicircle section can be simulated using the turning left/right state in hybrid model as discuss previously. The dependency of the footprint on the aircraft heading as mentioned has a significant effect of coverage of the lawn-mower pattern. In fact, many search patterns suffer from this dependency.

Figure 6 show path and coverage of a typical lawnmower pattern for a $10 \times 9$ rectangular area. It is

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Figure 6: Lawn-Mower Path and Its Final Entropy for Initial Entropy of "1" Every Cells
assumed that entropy is one in every cell. It can be seen that the corner or border of the area are often missed. The coverage defined by Equation 7 in this case is 0.9485 . The coverage would also be reduced if the turning radius is larger.

One of the simplest ways to avoid this is to delay the Semicircle movement of the path such that the aircraft can cover more area before turning. In the other words, the straight path is extended. Therefore, the traveling time and fuel consumption is also increased. This problem can be solved or alleviated by determining the optimal speed along the path. In the early stage of investigation where the area is gridded into cells as shown, the UAV is assumed to move from one cell to another. Therefore, the path is connecting of short straight line. In order to achieve the more realistic path, the cell size can be smaller. However, a bigger cell size is utilized in this paper to explore the different aspects of the lawn-mower pattern.

## 5 Path Planning

Assume that the footprint is $2 \times 1$ as mentioned, if entropy of all cells is one, it is apparent that the most effective path is the ones with pitch (path spacing) equal to two as shown in Figure 5. The path here refers to the possible path that is the vertical grid
line. The assumption on footprint size can be generalized by constructing a cell with the same height as the footprint and half a width of footprint.
Because each "lane" could have different priorities, the pitch must be adjusted corresponding to the entropy. Consider a case in Figure 2 where the lane 2 has entropy of two, the entropy in this lane could be reduced by either moving along path $x=1$ or $x=3$ line. In this simple case, it is quite obvious that the UAV should travel along path $x=1$ and $x=2$ lines in order to reduce entropy of lane 2 to zero. In this section, a simple set of rules to determine the number of revisits is developed. Furthermore, the decision on how to revisit the path must also be considered carefully.

### 5.1 Number of Revisits

It can be seen that the number of revisits on each line depends on entropy of the surrounding lanes. Figure 7 illustrates the parameters that are employed in this subsection.


Figure 7: Description of parameters to determine a number of revisits

The number of revisits of path $\mathbf{A}, N_{v s}(A)$, depends on entropy on the left, right lane and the second right lane $\left(E_{L}, E_{R 1}\right.$ and $\left.E_{R 2}\right)$. Consider the path from left to right, it can be assumed that left entropy, $E_{L}$ depends solely on $N_{v s}(A)$. This assumption can be done with updated left entropy by reducing it by $N_{v s}(Z)$ that is determined in the previous step. Therefore, the UAV must travel along path A at least $E_{L}$ because entropy of the left lane can only be reduced through path A. The upper bound of $N_{v s}(A)$ is defined by the maximum between $E_{L}$ and $E_{R 1}$.

$$
\begin{equation*}
E_{L} \leq N_{v s}(A) \leq \max \left(E_{L}, E_{R 1}\right) \tag{8}
\end{equation*}
$$

If $E_{L} \geq E_{R 1}$, it is obvious that $N_{v s}(A)=E_{L}$. Otherwise, the possibility of $N_{v s}(B)$ must also be

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considered through $E_{R 1, R 2}$. If $E_{R 2}$

$$
\begin{equation*}
E_{R 1}-N_{v s}(A) \leq N_{v s}(B) \leq \max \left(E_{R 1}, E_{R 2}\right) \tag{9}
\end{equation*}
$$

Combined with the upper bound of $N_{v s}(A)$ in Equation 8, the bound of $N_{v s}(B)$ becomes,

$$
\begin{equation*}
E_{R 1}-N_{v s}(A) \leq N_{v s}(B) \leq \max \left(E_{R 1}-E_{L}, E_{R 2}\right) \tag{10}
\end{equation*}
$$

Therefore, the other two cases must be considered. If $E_{R 2} \geq E_{R 1}-E_{L}$, then $N_{v s}(A)=\max \left(E_{L}, 2 E_{R 1}-\right.$ $\left.E R_{2}\right)$. Otherwise, $N_{v s}(A)=E_{L}$.

The number of revisiting can then be described as,
$N_{v s}(A)= \begin{cases}\max \left(E_{L}, 2 E_{R 1}-E R_{2}\right) & \text { if } E_{R 2} \geq E_{R 1}-E_{L} \geq 0 \\ E_{L} & \text { otherwise }\end{cases}$

The algorithm is concluded in Table 1

Table 1: The Number of Revisiting Algorithm
1: Construct path along the shorter side of the area. Let the path spacing be half of footprint width.
2: Let the most left path (not including border) be path $\mathbf{A}$. The other parameters are defined as shown in Figure 7.
3: Let the revisiting number of path $\mathbf{A}, N_{v s}(A)$ be as described in Equation 11.
4: Update the right lane entropy, $E_{R}=E_{R}$ $N_{v s}(A)$.
5: Let path $\mathbf{B}$ be path $\mathbf{A}$. The other parameters are defined as shown in Figure 7
6: Repeat step 3 until path $\mathbf{A}$ is right border of the area.

For an example given in Figure 2, the algorithm results in one visit on path $x=[1,2,4,6,8]$. For a sake of simplicity, path $x=i$ is denoted as $p_{i}$ here. The result is consistent to what is expected. The variable, $N_{v s}$ then becomes,

$$
N_{v s}=\left\{\begin{array}{llllllll}
1 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}\right\}
$$

The vector $N_{v s}$ indicates the visiting number for path 1 to 8 . Another variable named " $N_{T V}$ " is introduced as the total number of visiting.

$$
\begin{equation*}
N_{T V}=\sum_{i=1}^{N_{\text {path }}} N_{v s}\left(p_{i}\right) \tag{12}
\end{equation*}
$$

where $N_{\text {path }}$ is the total number of paths. Therefore, $N_{T V}$ equals to 5 here. This example is employed through out the paper.

### 5.2 Path Generation

With the assigned revisiting number, the path must be constructed such that it agrees with actual aircraft dynamics. With turning radius constraint, UAV cannot travel up and down the same path effectively. There must be a minimum space between the paths such that the vehicle can turn and switch path.

It is reasonable to assume that the turning radius is greater of equal to the width of the footprint that is equivalent to two cells. This statement implies that UAV can travel from one path to another path that is at least two cells apart. Consider the previous example on Figure 2, visiting number on path $x=[1,2,4,6,8]$ is one. The UAV cannot travel from path $x=1$ to $x=2$ because of turning constraint. In this subsection, an algorithm is developed to determine the optimal path order within UAV performance. The UAV path is then generated based on the specific order.

The path order must be planned with two factors in mind 1)turning radius and 2)minimum traveling time. They can be taken into account through a $N_{\text {path }} \times 1$ vector, $C_{\text {turn }}$. The element $i^{t h}$ in $C_{\text {turn }}$ indicates the level of difficulty in switching to between two paths with $i-1$ cell spacing. The value of $C_{\text {turn }}$ utilized in this example is as follow,

$$
C_{\text {turn }}=\left\{\begin{array}{llllllll}
H & H & 1 & 2 & 3 & 4 & 5 & 6
\end{array}\right\}
$$

where $H \gg 1$ is a very large number. The given set of values implies that it is impossible to switch within the same path or to the adjacent path, hence $C_{\text {turn }}(1)=C_{\text {turn }}(2) \rightarrow \infty$. It also indicates that the larger the spacing, the higher the level of difficulty is.

The path order can be considered through permutations of all possible orders. For a total visiting number as defined in Equation 12, there are $\left(N_{T V}-1\right)$ ! possible permutations. The subtraction of one is for the starting position constraint. Although the number grows rapidly, there are many permutations that can be pruned out using very low computation effort.

Consider the $i^{\text {th }}$ permutation, $\mathbf{p}_{i}=$ $\left\{p_{i 1} p_{i 2} p_{i 3} \cdots\right\}$, the cost of the permutation is,

$$
\begin{equation*}
C\left(\mathbf{p}_{i}\right)=\sum_{j=2}^{N_{T V}-1} C_{\text {turn }}\left(\mathcal{S}\left(p_{i j}, p_{i(j-1)}\right)\right) \tag{13}
\end{equation*}
$$

where $p_{i j}$ is the $j^{t h}$ order path of permutation $\mathbf{p}_{i}$ and $\mathcal{S}\left(p_{m}, p_{n}\right)$ is the spacing between path $m^{t h}$ and $n^{t h}$.

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The optimal permutation is then defined as,

$$
\begin{equation*}
\mathbf{p}_{o p t}=\arg \min _{\forall \mathbf{p}_{i}} C\left(\mathbf{p}_{\mathbf{i}}\right) \tag{14}
\end{equation*}
$$

For the previous example, the optimal permutation with path 1 as starting location is $\mathbf{p}=$ $\left\{p_{1} p_{4} p_{2} p_{6} p_{8}\right\}$. The path is then constructed based on the optimal permutation as shown in Figure 8. The UAV travels from path 1 to 3 , then back to path 2 to satisfy the turning radius constraint. From path 2 , the UAV travels to path 6 and 8 respectively. The coverage as defined in Equation 7 is 0.9725 .


Figure 8: Path and Final Entropy for Initial Entropy as shown in Figure 2

The smoothness of the path can be guaranteed using smaller grid size or by a path smoother [1]. The formulation presented here has enough flexibility for different turning radius, level of turning difficulty, and starting constraint. A user can tune parameters, such as $C_{\text {turn }}$, path spacing, so that the result satisfies the requirement.

Figure 9 shows an example where initial entropy of the left area is twice larger than the right area. The approach results in two visits on path 1 and 3. Therefore, the variable $N_{v s}=\left\{\begin{array}{lllllll}2 & 0 & 2 & 0 & 1 & 0 & 1\end{array}\right\}$ and $N_{T V}=6$. The optimal permutation is $\mathbf{p}_{\text {opt }}=$ $\left\{\begin{array}{llllll}1 & 3 & 1 & 3 & 5 & 7\end{array}\right\}$. The figure shows two repetitions on path 1 and 3 . The coverage is 0.9713 .


Figure 9: Initial Entropy and Path with Coverage of 0.9713

## 6 Conclusion

The UAV surveillance problem on the area with different priorities is presented. The aircraft is described as a hybrid system, while the area is modeled by utilizing entropy concept. The area is represented by cells with assigned entropy. The high entropy indicates higher priority on the cell. The objective of the planner is to reduce as much entropy as possible with the shortest time while the constraints on UAV, such as velocity and turning radius, are satisfied. The definition of coverage is introduced as summation of entropy. It is employed as performance indication. The planner is decomposed two parts: determination of revisiting number and path generation. The first component determines the number of revisits that is required on each path. Then, the second part determines the optimal order of paths that satisfy the UAV constraint. Together the path is constructed. Simulations show more than $97 \%$ coverage.

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