Optimization of Silencers of Multiple Helmholtz Resonators for Higher Sound Transmission Loss

Ming Lokitsangtong
Department of Mechanical Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang
3 Moo 2 Chalongkrung Road, Ladkrabang, Bangkok 10520, Thailand
Tel: 0-23264197 Ext. 107 Fax: 0-23264198 E-mail: klming@kmitl.ac.th

Abstract

This paper presents a study on the optimization of sound transmission loss across silencers of multiple Helmholtz resonators. It has been found in earlier studies that there is a significant increase in the sound transmission loss for silencers with multiple Helmholtz resonators compared to those with single ones. The optimization studies presented in this work further validate this concept. The acoustic and geometric properties of the component are treated as the design variables with the objective to maximize the sound transmission loss across the silencer. Appropriate constraints are imposed to maintain acoustic and geometric integrity.

1. Introduction

The Helmholtz resonator-type silencers have found their applications in many situations on industrial noise control. Engine exhaust noise pollutes the street environment and ventilation fan noise enters dwellings along with the fresh air. Work on the analysis of Helmholtz resonator to implement the design of silencers for exhaust and ventilation systems has been going on for a few decades. The sound transmission loss across silencers of single Helmholtz resonator has been studied in detail by Davis et al[1]. The set-back of a single Helmholtz resonator silencer is due to its applicability limited only on a rather narrow band of frequency. An investigation on multiple Helmholtz resonator silencers[2] has revealed a widen bandwidth and an increase on the transmission loss. The optimizations of structures with the objectives of noise reduction have been related mostly to the sound generated from vibration of the structures themselves. Optimal analyses on the sound transmission across structures have generally been studied in relation to the structures preventing sound from transmitting through. The work on the optimization of the Helmholtz resonator-type silencers, to the best of the author's knowledge, has not been mentioned.

In this paper, the acoustic and geometric properties of the silencer components are treated as the design variables with the objective of maximizing the sound transmission loss across the silencer. The quasi-Newton method with the BFGS update formula has been employed in the study.

2. Theory

2.1 The simulation problem

A model silencer with multiple Helmholtz resonators, which are arranged on both of the opposite sides of a square duct is shown in Fig. 1. The sound transmission loss TL across the silencer in terms of incident sound pressures at the junctions just before and just after the silencer which are denoted by p_{Ii} and p_{IIi} respectively, can be written as

$$TL = 10\log\left|\frac{p_{Ii}}{p_{IIi}}\right|^2 \tag{1}$$

The acoustic and geometric properties selected as design variables in this study are defined as resonance phase angle $k_r L$ and geometric parameter $(1/S_o)\sqrt{VS/l_e}$ which have the following meanings.

In the acoustic design variable

$$k_r = \frac{2\pi f_r}{c(1-M^2)} \tag{2}$$

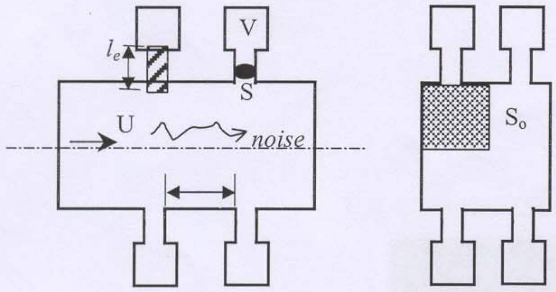


Fig. 1 Model of Multiple Helmholtz resonator-type silencer

where k_r denotes the wave number at resonance frequency, f_r denotes the resonance frequency of each resonator, c denotes the sound velocity, M denotes the Mach number, and L denotes the connecting length of the adjacent resonators. In the geometric design variable, S_o denotes the cross-sectional area of equivalent duct for an individual resonator, V denotes the resonance chamber volume, S denotes the cross-sectional area of the connector and l_e denotes the effective length of connector. The simulation has been carried out by means of the transfer matrix method[2]. By using the least square method, the mathematical model at resonance frequency can be expressed as

$$TL\,(x_1,x_2) = 8.3388 + 1.7044\,x_1 - 11.7468\,x_1^2 + 2.8490\,x_2 + 34.9545\,x_1 x_2 - 31.0609\,x_1^2 x_2 - 1.0994\,x_2^2 - 10.8795\,x_1 x_2^2 + 10.3046\,x_1^2 x_2^2 \quad (3)$$

where x_1 is $(1/S_o)\sqrt{VS/l_e}$, and x_2 is $k_r L$.

2.2 The optimization problem

To optimally design a multiple HelmHoltz resonator-type silencer so that its TL is maximal, a multivariable constrained non-linear optimization problem has to be dealt with. The optimization can be stated as

$$\text{minimize} \quad f(\mathbf{x}) = -TL(\mathbf{x}),\ \mathbf{x} = (x_1,x_2)^T,\ \mathbf{x} \in R^2 \quad (4)$$

$$\text{subject to} \quad 0 \le x_1 \le 1.0 \quad (5)$$

$$\text{and} \quad 0 \le x_2 \le \pi \quad (6)$$

3. Numerical procedure

The optimization technique used in this paper is the quasi-Newton method with the BFGS update formula which has effective algorithm to determine a descent direction vector. An overview of the method is given next. Further details of this method can be found in reference[3].

Consider the problem to search for $\mathbf{x}^*$ such that $f(\mathbf{x}^*) \le f(\mathbf{x})$ for $\mathbf{x} \in R^n$. First, the starting design or point $\mathbf{x_o}$ is chosen. The updating $\mathbf{x}$ can be determined by

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{d}_k \quad (7)$$

where $\mathbf{x}_k$ is the current point at the k^{th} iteration, k=0 corresponds to the starting point. The descent direction vector d_k is given by

$$\mathbf{d}_k = -\mathbf{H}_k \nabla f(\mathbf{x}_k) \quad (8)$$

where $\mathbf{H}_k$ is the approximation of the inverse of the Hessian matrix ie

$$\mathbf{H}_k \cong [\nabla^2 f(\mathbf{x}_k)]^{-1} \quad (9)$$

and the BFGS update formula which was suggested by Broyden,

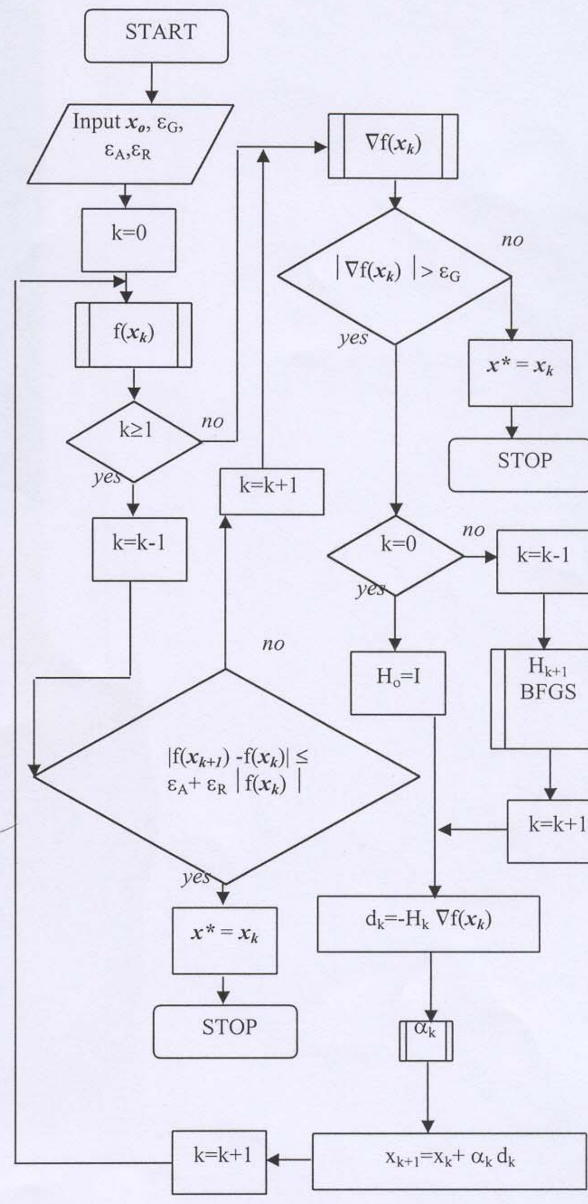


Fig. 2 Flow chart of the optimization

Fletcher, Goldfarb, and Shanno is expressed as

$$H_{k+1} = H_k - \left(\frac{\delta_k \gamma_k^T H_k + H_k \gamma_k \delta_k^T}{\delta_k^T \gamma} \right) + \left(I + \frac{\gamma_k^T H_k \gamma_k}{\delta_k^T \gamma_k} \right) \frac{\delta_k \delta_k^T}{\delta_k^T \gamma_k} \quad (10)$$

where $\delta_k = \mathbf{x}_{k+1} - \mathbf{x}_k$ and $\gamma_k = \nabla f(\mathbf{x}_{k+1}) - \nabla f(\mathbf{x}_k)$. The step size or traveling distance α_k can be determined by line search method to satisfy

$$\text{minimize } f(\alpha) \equiv f(\mathbf{x}_k + \alpha \mathbf{d}_k) \quad (11)$$

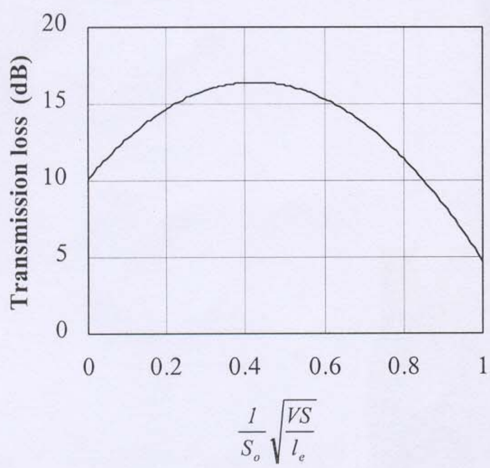
The stopping criteria are

$$|\nabla f(x_k)| \le \varepsilon_G \quad (12)$$

for optimality condition and

$$|f(x_{k+1}) - f(x_k)| \le \varepsilon_A + \varepsilon_R f(x_k) \quad (13)$$

where ε_G is a tolerance on the gradient, ε_A is absolute tolerance on the change in function value and ε_R is relative tolerance. The numerical procedure is depicted as a flow chart in Fig. 2.



$\frac{1}{S_o}\sqrt{\frac{VS}{l_e}}$

Fig. 3 Contour curve of TL and $\frac{1}{S_o}\sqrt{\frac{VS}{l_e}}$

4. Numerical results

The proposed optimization approach is developed based on the accuracy of the mathematical model described in the previous section and applied to the multiple Helmholtz resonator-type silencer. The contour curves are illustrated in Fig's 3 and 4. Figure 5 depicts a surface plot of the objective function. The optimum geometric parameter of 0.424 and resonance phase angle of 1.566 radian have been obtained, and the maximum transmission loss is 16.4101 dB.

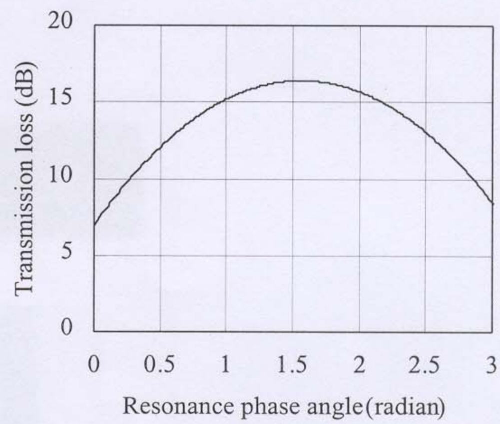


Fig. 4 Contour curve of TL and $k_r L$

5. Conclusions

The optimization study of sound transmission loss across a multiple Helmholtz resonator-type silencer has been presented in this paper. The quasi-Newton method with BFGS update formula has been used as the optimization algorithm. The main objective of the optimization is to maximize the sound transmission loss across the silencer subject to the resonance phase angle constraint and the geometric parameter constraint. The optimization study indicates that optimal sound transmission loss can be achieved with the help of resonator arrangements.

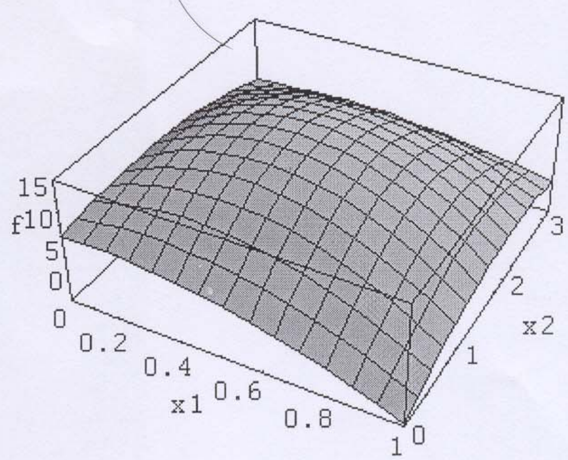


Fig. 5 Surface plot of $TL\left(\frac{1}{So}\sqrt{\frac{VS}{le}};krL\right)$

References

[1] D. Davis, Jr., G. Stevens, Jr., D. Moore and G. Stokes, "Theoretical and Measured Attenuation of Mufflers at Room Temperature without Flow, with Comments on Engine-exhaust Muffler Design," NACA Technical Note 2893, 1954.

[2] M. Lokitsangtong, M. Sakamoto, S. Murakami, and P. Kummol, "Transmission Loss Characteristics of a Silencer with Multiple Helmholtz Resonators," The Eight Western Pacific Acoustic Conference, Melbourne, Australia, 7-9 April 2003.

[3] A.D. Belegundu, and T.L. Chandrupatla,"Optimization Concepts and Applications in Engineering," Prentice-Hall, 1999.