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Stress singularity analysis around the vertex point in three-dimensional bonded joints under thermal loading

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Abstract

The stress distribution around the vertex point of dissimilar materials in three-dimensional joints under thermal loading are investigated using BEM based on Rongved's fundamental solutions. Stress distributions for the material combination in the singularity region on the Dundurs composite plane are investigated. The influences of thermal expansion coefficients and loading conditions on the stress distribution are examined. It can be found that the stress distributions around the vertex point were different from those at the apex in two-dimensional bonded joints.

Keywords: Thermoelasticity; Thermal stress; Stress singularity; Three-dimensional joints; Dissimilar materials; BEM

1. Introduction

Stress singularities at the interface in the bonded joints of dissimilar materials are induced by mechanical loading or thermal loading. Thermal stresses are caused by differences in elastic properties and thermal expansion coefficients in dissimilar materials joints. The stress singularities generally exist at the vertex in three-dimensional joints of dissimilar materials. Li et al. (1992) reported the results of stress analysis for dissimilar materials using three-dimensional BEM based on Kelvin's fundamental solutions. In the analysis, the interface must be divided using finer meshes along the stress singularity lines, and huge memory and time consuming procedures are required for accurate analysis. Then, Koguchi (1997) investigated the stress singularity in three-dimensional bonded joints using three-dimensional BEM based on Rongved's fundamental solutions. Rongved's fundamental solutions (1955) satisfy boundary conditions at the interface. Therefore, the number of nodes and elements necessary for accurate analysis decreases, because the BEM based on Rongved's fundamental solutions does not require the interface area of dissimilar materials joints to be divided into elements. Koguchi et al. (2003) also used the fundamental solution for two-phase transversely isotropic materials to investigate the stress singularity fields in three-dimensional bonded joints using three-dimensional BEM. Furthermore, Prukvilailert and Koguchi (2005) reported on stress singularity analysis around a point on the stress singularity line in three-dimensional bonded joints using three-dimensional BEM based on Rongved's fundamental solutions. However, this previous research focused only on the stress singularity distributions in three-dimensional bonded joints under mechanical loading. The distributions of the stress fields around the vertex in three-dimensional joints of dissimilar materials under thermal loading have not been made clear so far.

In recent years, there has been much research on thermal stresses at the interface in two-dimensional bonded joints. Munz and Yang (1992, 1994 and 1995) investigated the stress singularities and stress intensity factors near the free edge of a junction between dissimilar materials subjected to mechanical or thermal loading using the eigenfunction expansion method. It is well-known that three-dimensional BEM is useful to efficiently analyze the stress fields in three-dimensional joints, since only surfaces are divided into meshes for analysis. Cruse et al. (1977) and Rizzo and Shippy (1977) determined the boundary integral equation for three-dimensional thermoelasticity. The thermoelastic integral equation was also derived using the body force analogy (Karami and Kuhn, 1992; Cheng et al., 2001).

In this paper, we investigate the stress singularity fields around the vertex point in three-dimensional joints of dissimilar materials under thermal loading using BEM based on Rongved's fundamental solutions. The material combinations are mapped on the $\alpha_{2D}-\beta_{2D}$ Dundurs' composite plane (Dundurs, 1969) for the order of stress singularity in a form of power-law singularity, λ_a, in plane strain condition as shown in Fig. 1. These parameters are defined as

$$\alpha_{2D}=\frac{Km_{(2)}-m_{(1)}}{Km_{(2)}+m_{(1)}}$$
$$\beta_{2D}=\frac{K\left(m_{(2)}-2\right)-\left(m_{(1)}-2\right)}{Km_{(2)}+m_{(1)}} \qquad (1)$$

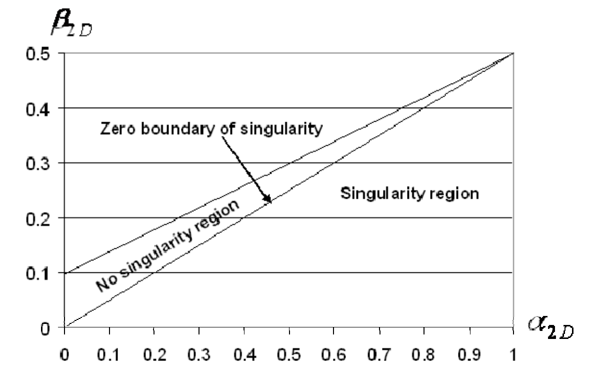

Figure 1. Dundurs' composite plane

where

$$K = \frac{G_{(1)}}{G_{(2)}} \tag{2}$$

$$m_{(h)} = \begin{cases} 4\left(1-\nu_{(h)}\right) & \text{for plane strain} \\ \dfrac{4}{1+\nu_{(h)}} & \text{for plane stress} \end{cases} \qquad (h=1,2) \tag{3}$$

in which $G_{(h)}$ is the shear modulus and $\nu_{(h)}$ is the Poisson's ratio. The subscript h of these material properties represents the material region; subscript 1 refers to the region of material 1 and subscript 2 refers to the region of material 2.

2. BEM for thermoelasticity

The stress and displacement fields at a point in the joints with high stress are examined using BEM, which requires less memory than the FEM, especially in the case of three-dimensional joints. Here, Rongved's fundamental solutions satisfying the boundary conditions at the interface in dissimilar materials are applied in our analysis. For thermoelasticity with a uniform temperature variation in dissimilar materials, the boundary integral equation is derived as follows:

$$\begin{aligned} C_{ij}\boldsymbol{u_j}(P) &= \int_S \left(\boldsymbol{t_j}(Q)U_{ij}(P,Q) - T_{ij}\left(P,Q\right)\boldsymbol{u_j}\left(Q\right)\right) dS\left(Q\right) \\ &+ \int_S \left(\left(\boldsymbol{n_j}M\varphi\right)U_{ij}(P,Q)\right) dS\left(Q\right) \end{aligned} \tag{4}$$

where S is the surface of the dissimilar materials model excluding the interface area, P and Q are points on the boundary, C_{ij} is the C-matrix derived from the configuration around a boundary point P, and U_{ij} and T_{ij} are Rongved's fundamental solutions for displacements and surface tractions, respectively. Parameter φ is a uniform temperature variation from the stress-free state. The term M varies according to the location of an element. We can define M as

$$M = \begin{cases} \dfrac{2G_{(1)}\alpha_{T1}\left(1+\nu_{(1)}\right)}{\left(1-2\nu_{(1)}\right)} & \text{in Material region 1} \\ \dfrac{2G_{(2)}\alpha_{T2}\left(1+\nu_{(2)}\right)}{\left(1-2\nu_{(2)}\right)} & \text{in Material region 2} \end{cases} \tag{5}$$

where α_{T1} and α_{T2} are the thermal expansion coefficients for material 1 and for material 2, respectively.

A very fine mesh division is used to obtain an accurate stress distribution. Then, the stress state at internal points can be derived. First, the strain-displacement relation is written as

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{6}$$

The stress-strain relation for thermoelasticity is given by

$$\sigma_{ij}^{(h)} = 2G_{(h)}\varepsilon_{ij} + N\delta_{ij}\varepsilon_{kk} - M\delta_{ij}\varphi \tag{7}$$

where

$$N = \begin{cases} \dfrac{2G_{(1)}\nu_{(1)}}{\left(1-2\nu_{(1)}\right)} & \text{in Material region 1} \\ \dfrac{2G_{(2)}\nu_{(2)}}{\left(1-2\nu_{(2)}\right)} & \text{in Material region 2} \end{cases} \tag{8}$$

Then, substitution of Eq. (6) into Eq.(7) gives

$$\sigma_{ij}^{(h)} = G_{(h)}\left(u_{i,j} + u_{j,i}\right) + N\delta_{ij}u_{k,k} - M\delta_{ij}\varphi \tag{9}$$

Finally, the stress σ_{ij} at the internal point, ξ, can be obtained by differentiating Eq. (4) and substituting into Eq. (9) as follows:

$$\begin{aligned} \sigma_{ij}^{(h)}(\xi) &= \int_S \left(t_l(Q)D_{ijl}^{(h)}(\xi,Q) - V_{ijl}^{(h)}(\xi,Q)u_l(Q)\right) dS(Q) \\ &+ \int_S \left(n_l M\varphi\right) D_{ijl}^{(h)}(\xi,Q) dS(Q) - M\delta_{ij}\varphi \end{aligned} \tag{10}$$

where the third-order tensor components $D_{ijl}^{(h)}\left(\xi,Q\right)$ and $V_{ijl}^{(h)}\left(\xi,Q\right)$ are obtained by substituting Rongved's fundamental solutions $U_{ij}\left(\xi,Q\right)$ and $T_{ij}\left(\xi,Q\right)$, respectively, in the stress-displacement equations as follows:

$$D_{ijl}^{(h)}\left(\xi,Q\right) = G_{(h)}\left(U_{il,j}\left(\xi,Q\right) + U_{jl,i}\left(\xi,Q\right)\right) + N\delta_{ij}U_{kl,k}\left(\xi,Q\right) \tag{11}$$

$$V_{ijl}^{(h)}\left(\xi,Q\right) = G_{(h)}\left(T_{il,j}\left(\xi,Q\right) + T_{jl,i}\left(\xi,Q\right)\right) + N\delta_{ij}T_{kl,k}\left(\xi,Q\right) \tag{12}$$

where δ_{ij} is the Kronecker delta.

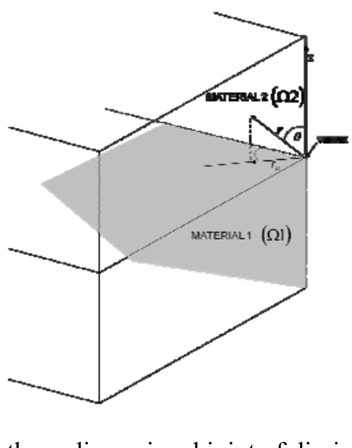

Figure 2. A three-dimensional joint of dissimilar materials with the origin at the vertex point

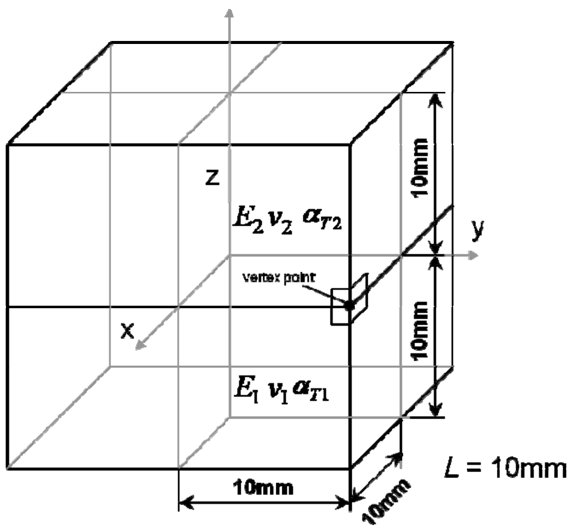

Figure 3. Model for analysis of a three-dimensional joint

A typical model employed in our calculation is shown in Fig. 3. The total number of nodes and elements are 3067 and 1370, respectively. A very fine mesh division is located around the vertex point. For the boundary conditions, the displacements in the *x*-direction and the *y*-direction are free at all surfaces of the model. The displacement in the *z*-direction at the upper surface and side surfaces of the model is free, whereas that at the lower surface is fixed to zero.

3. Results and Discussion

In this section, thermal loading due to a uniform temperature variation ($\varphi = \Delta T$: constant) is applied to the three-dimensional joint model. The material combinations of the joint are chosen so as to locate in the singularity region on the Dundurs' composite plane. Material properties are first chosen as $E_{(1)}$ = 206GPa, $v_{(1)}$=0.3, $E_{(2)}$=52.6742GPa, $v_{(2)}$=0.26316. The corresponding Dundurs' parameters ($\alpha_{2D} = 0.6$, $\beta_{2D} = 0.2$) are in the singularity region. The thermal expansion coefficient of material 1, α_{T1}, is $1.0\times10^{-6}\ K^{-1}$ and of material 2, α_{T2}, is $5.0\times10^{-6}\ K^{-1}$. A uniform temperature variation, ΔT, is $-100K$, which means that the temperature in the joint decreases from the stress-free state (ΔT is negative, indicating a cooling-down condition). The upper part of the model (material 2) allows more contraction than the lower part of the model (material 1, which has a lower value of the thermal expansion coefficient).

The stress distribution of $\sigma_{\theta\theta}$ at the interface ($\theta = 0^\circ$) around the vertex point along the dimensionless distance r/L in the present BEM analysis for a uniform temperature variation $(\Delta T = -100K)$ is shown in Fig. 4. For comparison, we also provide the stress distributions of $\sigma_{\theta\theta}$ in two-dimensional bonded joints computed using the formulation developed by Munz and Yang (1992) in plane strain condition. It can be seen that the stress distribution of $\sigma_{\theta\theta}$ for three-dimensional bonded joints is similar to that for two-dimensional bonded joints, but the magnitude is larger. The stress distribution of σ_{zz} in the upper interface (z/L = 0.00005) and in the lower interface (z/L = -0.00005) along r_o/L are shown in Fig. 5. In the upper interface, Material 2, there is Young's modulus less than those of Material 1, and it is noticed that the stress distribution of $\sigma_{\theta\theta}$ is maximum inside the region not at the interface. Next, the stress distributions of $\sigma_{\theta\theta}$ for various uniform temperature variations and various values of α_{T2} when α_{T1} is fixed to $1.0\times10^{-6}K^{-1}$ are investigated and shown in Fig. 6. The magnitude of the stress $\sigma_{\theta\theta}$ is proportional to the value of a uniform temperature variation according to the Linear Theory of Elasticity, and the magnitude also increases as the value of α_{T2} increases.

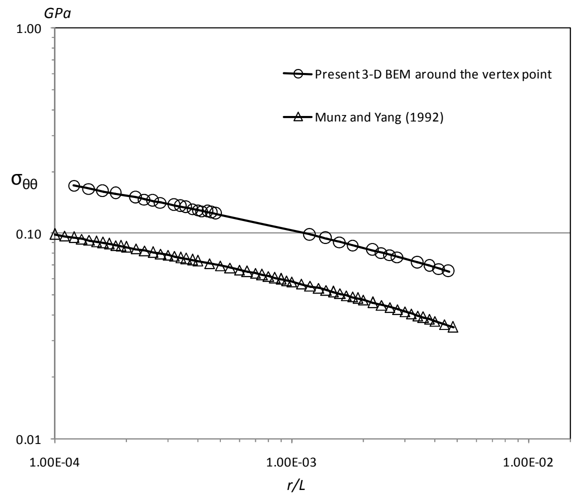

Figure 4. Stress distributions of $\sigma_{\theta\theta}$ at the interface around the vertex point for a uniform temperature variation ($\Delta T = -100K$).

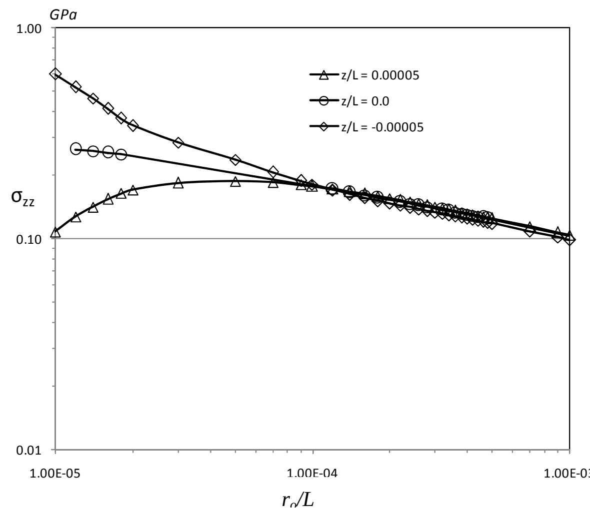

Figure 5. Stress distributions of σ_{zz} for z/L (= 0.00005, 0.0, -0.00005) along r_o/L

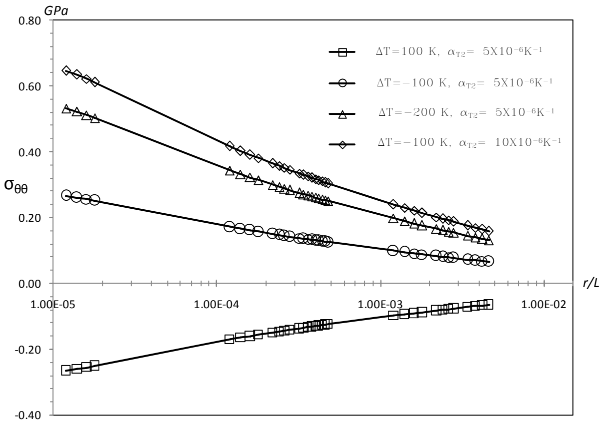

Fig. 6. Stress distributions of $\sigma_{\theta\theta}$ at the interface for various uniform temperature variations and thermal expansion coefficient in a semi-log scale.

6. Conclusion

In the present paper, we created a three-dimensional BEM program for thermoelasticity based on Rongved's fundamental solution satisfying the boundary condition at the interface. As a result, accurate analysis using the present BEM program required less memory and was less time consuming than BEM analysis based on Kelvin's fundamental solutions or FEM analysis. The distributions of stress singularity fields around the vertex point for dissimilar materials in three-dimensional bonded joints under thermal loading were presented and compared with the results in the previous research studies. For a uniform temperature variation applied to three-dimensional bonded joints, the magnitude of stress distributions were proportional to the temperature variation, ΔT, and depended on the difference in the thermal expansion coefficients.

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