Topological Design of Structures with Stress Constraints

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Abstract

Topology optimisation is well recognised as one of the most effective and powerful tools in the field of multidisciplinary and structural optimisation. It can be applied to the conceptual design of a wide variety of practical engineering applications. It is also renowned as a tool for dealing with the synthesis of compliance mechanisms. Traditionally, the design problem is to find a structural topology under predefined loads while optimising nonlinear design objective with the mass or volume being constrained. The disadvantage of this approach is that it always needs the further design step to ensure structural safety requirements. This paper presents an alternative topology design approach in which the design problem is the classical mass minimisation with stress constraints. The use of numerical strategy ground element filtering to suppress topology checkerboards is detailed and illustrated. The optimum results show that the GEF method can suppress topology checkerboard. The resulting topologies are illustrated and compared to the optimal topology obtained from solving the compliance minimisation problem.

Keywords: Ground Element Filtering, Topology Optimisation, Mass Minimisation, Stress

1. Introduction

Topology optimisation is a numerical tool implemented in the conceptual design stage. Structural topological design is an optimisation problem that is set to find the best possible layout of a structure such that optimising the design objective value whilst meeting predefined constraints. A lot of research work has been made towards this design technology while a number of design approaches have been well-established e.g. Solid Isotropic Material with Penalisation (SIMP) [1] and homogenisation method [2, 3]. Topological design is traditionally carried out by employing finite element analysis and numerical optimisation techniques. For a microstructure-based approach, with a predefined structural design domain being assigned, a structure is discretised into a number of finite elements called ground elements. Topology design variables are element density distribution which implies that locations at where element density is nearly zero form voices in the structure whereas the others represent material existence. Note that the term element density here defines a particular parameter that can affect structural merit and result in reasonable topologies of a structure. It could be element thickness for a 2D case.

The classical topology optimisation problem is posed to minimise structural compliance (equivalent to maximising global structural stiffness) whereas structural mass is constrained. The other objective functions often used are natural frequencies and buckling factors. A few attempts have been made to apply the classical mass minimisation with stress constraints but it seemed to be unsuccessful due to some numerical difficulties [4] e.g. the problem of checkerboard formation. It has been illustrated that, for the finite element types such as a 4-node membrane, a topology with checkerboards is artificially stiffer. Such a problem can be alleviated by applying the higher order finite element formulation. However, there remain some drawbacks i.e. it is computationally expensive when using a great number of ground elements. Therefore, the use of the 4-node finite element formulation with an efficient numerical scheme for checkerboard prevention is the more popular and efficient approach. A number of numerical schemes have been proposed to suppress the checkerboards e.g. sensitivity filtering technique [1] and checkerboard constraint [5]. Another simple but effective numerical strategy for checkerboard-free design is the ground element filtering technique (GEF), which is presented in references [6, 7]. It should be noted that the authors have applied this approach to deal with some particular applications several times and named it differently from GEF.

This paper presents the use of the numerical strategy GEF to find a structural topology where the problem is set to be mass minimisation with stress constrains. The numerical technique is detailed, illustrated and implemented to solve the 2D structural topology design problem. The optimum topologies are illustrated and compared to that obtained from solving the classic compliance minimisation problem. It is shown that the GEF method can suppress topology checkerboard. The technique enables an unconventional topology design problem to be solved.

2. Topology Optimisation

A generic constrained optimisation problem is posed to find the solution of design variables that optimise the value of design objective while fulfilling predefined constraints. For topological optimisation, since it is operated on the early stage of a design process, some of the structural constrains can be ignored and the design problem can be simplified as:

$$\min_{\boldsymbol{\rho}} : f(\boldsymbol{\rho}) \tag{1}$$

Subject to

$$m(\boldsymbol{\rho}) = r.m(\mathbf{1})$$

$$\mathbf{0} < \boldsymbol{\rho}_l \le \boldsymbol{\rho} \le \mathbf{1}$$

where $\boldsymbol{\rho}$ is the vector of topological design variables having lower and upper bounds as $\boldsymbol{\rho}_l$ and $\mathbf{1}$ respectively

$f(\boldsymbol{\rho})$ is an objective function

$m(\boldsymbol{\rho})$ is structural mass

and r is the ratio of structural mass to the maximum mass.

The design problem can be thought of as the plan of using limited material to have an optimum value of design merit. The traditional objective functions are structural compliance, eigenfrequency and buckling factor [4]. Figure 1 displays a particular topology design process of a 2D plate. Note that, for 2D design, element density is represented by element thickness. Apart from checkerboard, some inevitable numerical difficulties involving the design are: local optimum resulting in less effective design, and many optimum results of one design domain with various element mesh resolutions. Another drawback of this design approach is that shape and sizing optimisation are always needed to perform ensuring that the structure fulfils all safety requirements.

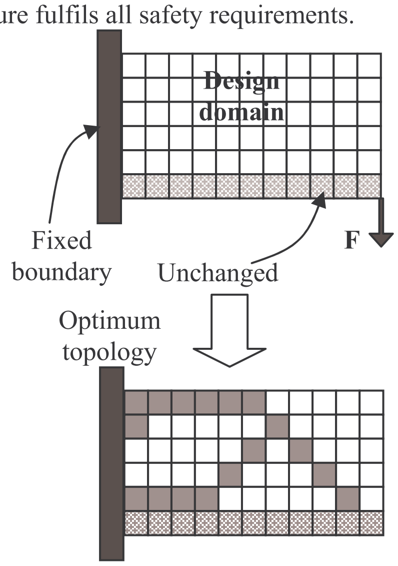

Figure 1 Topological design process

3. Ground Element Filtering

GEF is a simple numerical technique exploiting interpolation for approximating the values of finite element densities from the known density distribution on another domain. The grid of design variables can be achieved by discretising the structural domain with the resolution different from the ground element resolution. Figure 2 shows a rectangular design domain being meshed into n ground finite elements whereas the design variables have m elements. Let $\mathbf{r}_j^0$ be the position vectors of the m centre points of the design variable grids (plus sign) and $\mathbf{r}_k^v$ be the position vectors of the centre points of the n ground elements ('o' sign). By using radial-basis function interpolation, the densities at the centre points of the ground finite elements,$\boldsymbol{\rho}$, can be computed from the given densities at the design variables' centre points, $\boldsymbol{\rho}^{GEF}$, by the relation

$$\boldsymbol{\rho} = \mathbf{C}\mathbf{A}^{-1}\boldsymbol{\rho}^{GEF} = \mathbf{T}\boldsymbol{\rho}^{GEF} \tag{2}$$

where

$$\mathbf{C} = [c_{ij}]_{n\times m} = [f(d(\mathbf{r}_k^v, \mathbf{r}_j^0))] = [f(d_{kj})]$$

$$\mathbf{A} = [a_{ij}]_{m\times m} = [f(d(\mathbf{r}_i^0, \mathbf{r}_j^0))] = [f(d_{ij})]$$

$$f(d_{ij}) = d(\mathbf{r}_i, \mathbf{r}_j)$$

and

$$d(\mathbf{r}_i, \mathbf{r}_j) = \sqrt{(\mathbf{r}_i - \mathbf{r}_j)^T(\mathbf{r}_i - \mathbf{r}_j)}\,.$$

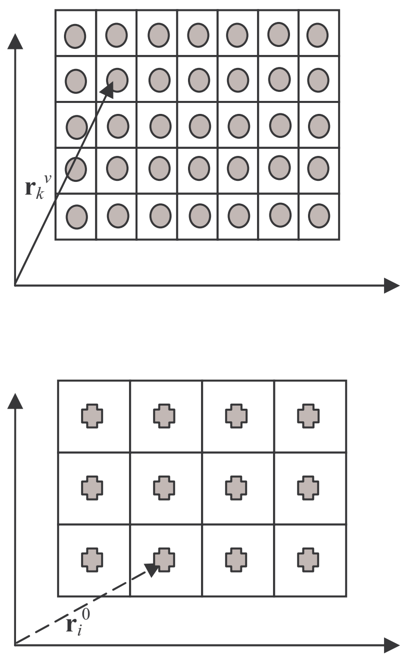

Figure 2 ground element and design variable grids

The resolution of the design variable grid must be lower than that of the ground finite element grid. The process can be seen as the ground element being filtered by the densities values on the design variable elements, thus, it is termed ground element filtering. For more details, see references [6] and [7]. Figure 3 demonstrates the mapping of densities from the GEF domain to the densities at the finite element centre points. It is shown that the actual structural configuration (on the finite element domain) is controlled by the density values from the GEF design domain. Due to the less resolution on the GEF domain, the checkerboard and/or one-node connected hinge patterns which appear on the GEF domain are automatically prevented on the ground finite element domain. This means that if the optimisation process is carried out on the GEF domain, checkerboards are allowed to occur but they will not appear on the actual topology, which is on the finite element domain.

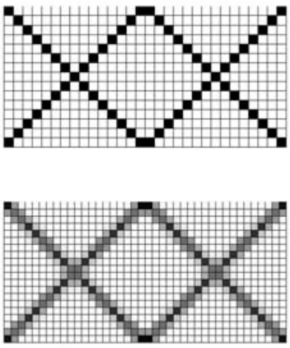

Figure 3 mapping from GEF domain to finite element domain

Implementing the GEF technique to the topology design problem (1) leads to a new design problem with the use of $\boldsymbol{\rho}^{GEF}$ instead of $\boldsymbol{\rho}$. The lower and upper bounds from the problem (1) are altered to be

$$\boldsymbol{\rho}^{\min} = \mathbf{T}^{\#}\boldsymbol{\rho}^{0} \qquad (3)$$
$$\boldsymbol{\rho}^{\max} = \mathbf{T}^{\#}\mathbf{1}$$

where $\mathbf{T}^{\#}$ denotes the pseudo-inverse of $\mathbf{T}$.

The gradient of a function f with respect to $\boldsymbol{\rho}^{GEF}$ can be expressed as [7]

$$\nabla f\big|_{\boldsymbol{\rho}^{GEF}} = \mathbf{T}^{T}\nabla f\big|_{\boldsymbol{\rho}} \qquad (4)$$

The procedure of the Optimality Criteria Method (OCM) with the use of GEF is presented in [7] and [8]. The advantage of the GEF technique is that it does not need sensitivity filtering technique or any additional constraint to deal with checkerboards as required by the classical approaches. The evaluation of function gradients can be carried out by using (4).

4. Design Test-Case

The numerical experiment is set up so as to demonstrate the effectiveness of the presented numerical technique on a structural mass minimisation problem with stress constraints. The mass minimisation is said to be unconventional for topological design as it is difficult to deal with. A few attempts in assigning the constrained mass minimisation to topology optimisation have been made but it seemed to be unsuccessful due to the checkerboard problem. For the implementation of GEF in dealing with checkerboards in the mass minimisation problem with stress constraints, the problem can be expressed as

$$\min_{\boldsymbol{\rho}^{GEF}} m(\boldsymbol{\rho}^{GEF}) \qquad (5)$$

subject to

$$\sigma_e(\boldsymbol{\rho}^{GEF}) \le \sigma_a$$
$$\mathbf{T}^{\#}\boldsymbol{\rho}^{e,\,min} = \boldsymbol{\rho}^{\min} \le \boldsymbol{\rho}^{GEF} \le \boldsymbol{\rho}^{\max} = \mathbf{T}^{\#}\boldsymbol{\rho}^{e,\,max}$$

where $\boldsymbol{\rho}^{e,\ min}$ is the vector of the lower limits of ground elements' thickness

$\boldsymbol{\rho}^{e,\ max}$ is the vector of the upper limits of ground elements' thickness

σ_e are the maximum shear stress at the centre point of the e^{th} elements

σ_a is the allowable stress.

The stress constraints are corresponding to the maximum shear stress failure theory while the safety factor is set to be 2.

Figure 4 displays the design domain of a cantilever plate under the applied load. The plate has the dimensions of L = 2 m and H = 1 m. The structure is made of the material with 200×10^{9} N/m^{2} Young modulus and 0.3 Poisson's ratio. The yield stress is set to be 200×10^{6} N/m^{2} for simplicity. The structure is meshed to have 20×10 elements while the GEF design variables have 16×8 elements. The ground element and GEF grids are depicted in Figure 5. The value of the upper bound of plate thickness, $\boldsymbol{\rho}^{e,\,max}$, is set to be 0.06, 0.04 and 0.03 m. Several maximum element thickness values are assigned in order to examine the effect of plate thickness on the resulting structural topologies.

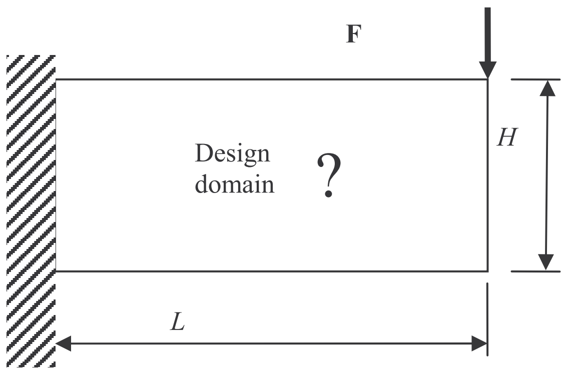

Figure 4 Design domain of a cantilever plate

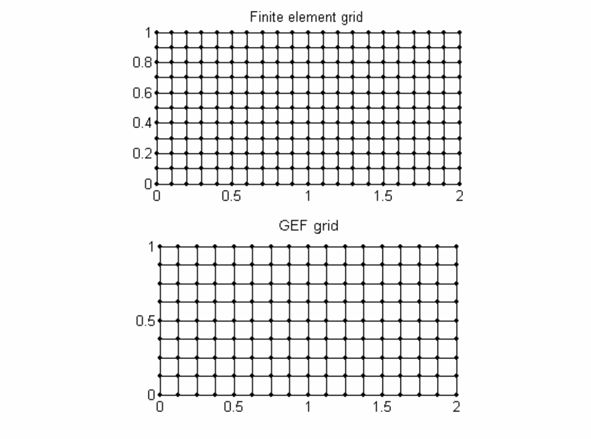

Figure 5 Ground element and design variable grids

5. Optimum Results

Sequential quadratic programming is used to solve the mass minimisation problem (5). The optimum topology obtained from solving the design problem without using GEF, where the maximum plate thickness is $\rho^{e,max}$ = 0.06 m, is displayed in Figure 6. It is found that the resulting topology is full of checkerboards. The optimum results in the cases of the maximum thickness $\rho^{e,max}$ being 0.04 and 0.03 m are illustrated in Figures 7 and 8 respectively. It can be seen that the optimum solutions are not reached.

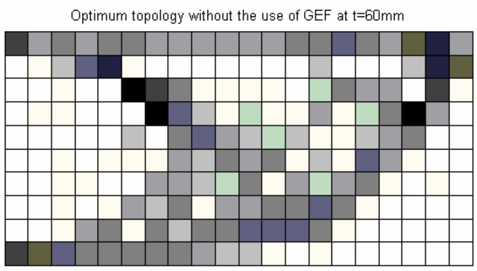

Figure 6 Optimum solution without the use of GEF at $\rho^{e,max}$ = 0.06 m

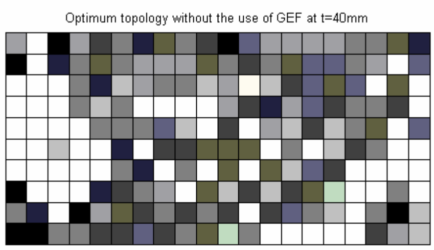

Figure 7 Optimum solution without the use of GEF at $\rho^{e,max}$ = 0.04 m

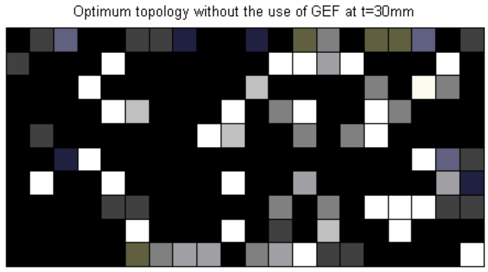

Figure 8 Optimum solution without the use of GEF at $\rho^{e,max}$ = 0.03 m

The optimum topology obtained from solving the design test-case with the use of GEF, where the maximum plate thickness is $\rho^{e,max}$ = 0.06 m, is displayed in Figure 9. The resulting topology is said to be checkerboard-free. The optimum results in the cases of maximum thickness being 0.04 and 0.03 m are illustrated in Figures 10 and 11 respectively. It is shown that the resulting topologies with various maximum plate thickness values being assigned are slightly different. The optimum topology from solving the compliance minimisation (1) by using the GEF technique with 70% mass reduction and the OCM method [7] is displayed in Figure 12. The topology is similar to those displayed in Figures 9 10 and 11. However, the plate thickness is still unknown, which is the disadvantage of this approach compared to solving the problem (5).

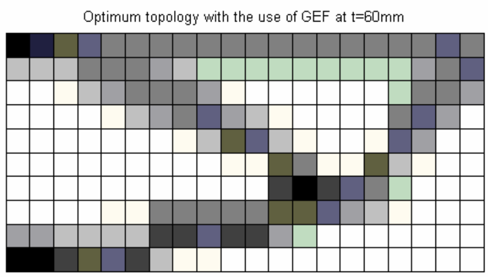

Figure 9 Optimum solution with the use of GEF at $\rho^{e,max}$ = 0.06 m

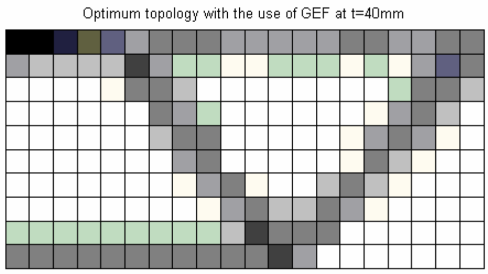

Figure 10 Optimum solution without the use of GEF at $\rho^{e,max} = 0.04$ m

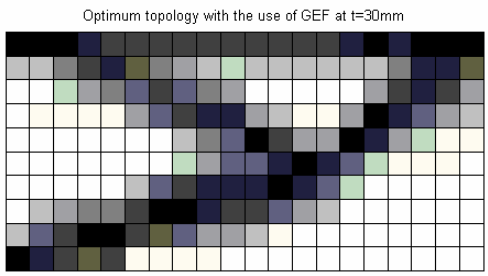

Figure 11 Optimum solution without the use of GEF at $\rho^{e,max} = 0.03$ m

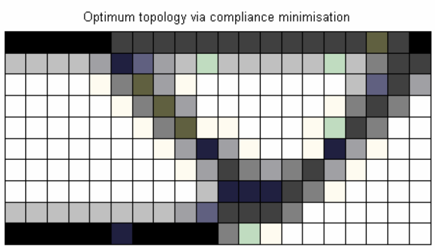

Figure 12 Optimum solution of compliance minimisation

6. Conclusions and discussion

The mathematical problem of topology optimisation is introduced. The numerical technique named ground element filtering is detailed and illustrated. The technique is implemented on the mass minimisation with stress constraints where design variables represent a structural topology. The optimum results show that the GEF approach is effective and powerful for mass minimisation topological design problem. It can be used to suppress the undesirable checkerboards on a structural topology. The introduction of the GEF technique also enables an unconventional topological design problem being accomplished.

Acknowledgement

The authors are grateful for the support from the Thailand Research Fund (TRF).

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