Partial Similarity in Solar Tower Modeling

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Abstract

Dimensional analysis is employed to determine the similarity condition for physical models of a solar tower (a solar power plant for generating electricity). In a previous study, full geometrical similarity had led to using different solar heat fluxes for each model in order to achieve dynamic similarity: a condition difficult to realize in an actual practice. This study aims to create dynamic similarity while using the same solar flux for all models. The study reveals that, to achieve the desirable condition, the models cannot be fully geometrical similar, hence partial similarity. The functional relationship that provides the condition for this partial similarity is proposed and its validity is proved by using numerical solutions of flows in solar towers.

Keywords: Solar tower, Dimensional analysis, Partial similarity, Dynamic similarity, Natural convection.

Nomenclature

A	flow area, m^2
A_r	roof area, m^2
c_p	specific heat at constant pressure, J/(kg.K)
g	gravitational acceleration, m/s^2
h_c	tower height, m
h_r	roof height above the ground, m
$\dot{m}$	mass flow rate, kg/s
q''	insolation, W/m^2
q'''	solar heat absorption per unit volume, W/m^3
R	gas constant, J/kg K
r_c	tower radius, m
r_r	roof radius, m
S	source term in CFX
T	absolute temperature, K
V	flow velocity, m/s

Greek symbols

β	coefficient of volumetric thermal expansion, 1/K
γ	specific heats ratio
Π	dimensionless parameter
ρ	density of working fluid, kg/m^3

Subscripts

1	position at roof inlet (Figure 1)
2	position at tower inlet (Figure 1)
c	tower
m	model
p	prototype
r	roof

1. Introduction

Solar tower is a solar power plant proposed to generate electricity in large scale, by transforming solar energy into mechanical energy. In other words, it is an artificial wind generator, albeit a hot one. The schematic of a typical solar tower power plant is sketched in Figure 1. Solar radiation strikes the transparent roof surface, heating the air underneath as a result of the greenhouse effect. Due to buoyancy effect, the heated air flows up the tower and induces a continuous flow from the perimeter towards the middle of the roof where the tower is located. Shaft energy can be extracted from the thermal and kinetic energies of the flowing air to turn an electrical generator [1].

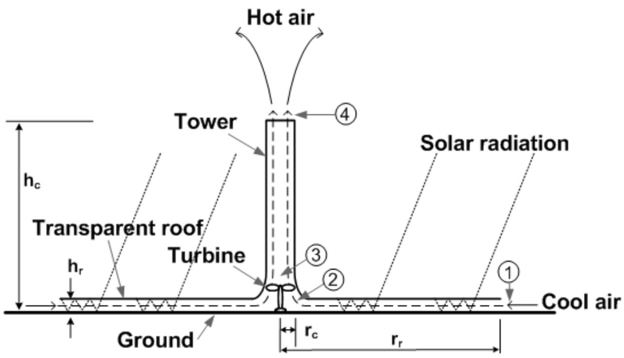

Figure 1 Schematic of a solar-tower power plant

Typical size of a solar-tower power plant is quite large. The 200 MW plant proposed in Australia is 1 km tall with a roof diameter of 7 km [2]. Experimental study in solar tower can therefore best be performed in a small scale model, but a similarity condition for the small scale model must first be established. In the previous study [3], it was found that for a geometrical similar model to be dynamical similar to the prototype the solar heat fluxes (insolation) must be different from that of the prototype's.

This is very inconvenient in an experimental setup since it suggests the use of a tinted glass roof of different shade than the one used in the prototype. It is therefore preferable that the same insolation be used in all models. Hence, this is the main aim of the investigation of this paper. After the investigation it is found that the same insolation could indeed be used, but at a price of not being able to use the full geometrical similarity; only a partial geometrical similarity is allowed.

2. Dimensional Analysis

In this study, the ratio of tower height to tower diameter is not very large. Therefore, the flow could be approximated as an inviscid flow without sacrificing much in accuracy [4]. In this study, a solar tower without a turbine is investigated.

According to the previous work of Chitsomboon [5], the mathematical model of the flow in a solar tower was proposed as,

$$\frac{1}{2}\dot{m}V_1^2\left[\rho_1 - 2\rho_1 A_1^2\int_1^3\frac{dA}{A^3} + \frac{2A_1 q''}{V_1 c_p T_1}\int_1^3\frac{dA_r}{A^2} + \frac{2\rho_1 A_1^2 g h_c}{\gamma R T_1}\int_1^3\frac{dA}{A^3}\right] = \frac{\rho_1 g h_c q''}{c_p T_3}\int_1^3 dA_r \quad (1)$$

This mathematical model was a result of the synthesizing of the conservation equations of mass, momentum and energy, together with the ideal gas relation. Due to its reliable predictions in comparison with CFD, the forms of the terms appear in this equation could perhaps be used as a guide to form the list of variables in the dimensional analysis. To this end, the dependence of power, $(\rho AV)V^2/2$, on the independent parameters was proposed, in [3], as

$$\rho AV\frac{V^2}{2} = f^n(\rho, g, \frac{q'''}{c_p}, \beta, h_c) \quad (2)$$

By using $g, q'''/c_p, \beta$, and h_c as repeating variables, a straightforward dimensional analysis gives the dimensionless variables as

$$\frac{\rho AV\frac{V^2}{2}}{\frac{q'''\beta g}{c_p}h_c^4} = f^n\left(\frac{\rho}{\frac{q'''\beta}{c_p}\sqrt{\frac{h_c}{g}}}\right) \quad (3)$$

or $\Pi_1 = f^n(\Pi_2)$

The validity of Eq.(3) was proved by scaling the numerical results of the various models [3].

However, the proposed scheme also required, for dynamic similarity, that

$$q''_m = \sqrt{\frac{h_{r_m}}{h_{r_p}}}q''_p \quad (4)$$

thus for a small scale model the condition in (4) stipulates a lesser insolation for the small-scale model than that of the prototype's, an inconvenient condition as mentioned earlier. This leads to an idea to search for a similar condition that requires the same insolation for different models.

It is noted that the undesirable similar condition of the previous study might be due to the use of $q'''(=q''/h_r)$ instead of q''. Therefore, in this study the direct use of q'' is proposed. By examining Eq.(1) it is seen that this term always appear in combination as $q''A_r$, which is the total heat flux striking the entire roof. This quantity, rather that just q'', will be proposed in this study since it represent the well defined driving energy of the system.

The procedural steps in finding the dimensionless variables are listed as follows:

Step 1 Proposing the primitive-variable functional relationship:

$$\rho AV\frac{V^2}{2} = f^n(\rho, g, \frac{q''A_r}{c_p}, \beta, h_c) \quad (5)$$

Step 2 Using mass (M), length (L), time (t), and temperature (Θ) as the fundamental dimensions. The dimensions of various terms are listed in Table 1. (Note: The methodology used here is proposed in [6, 7]; it yields the same result as the familiar Buckingham's pi theorem with a little more convenience in algebra complexity.)

Table 1: Primitive variable's dimensions in terms of powers of the fundamental dimensions

	$\rho AV\frac{V^2}{2}$	ρ	g	$\frac{q''A_r}{c_p}$	β	h_c
M	1	1	0	1	0	0
L	2	-3	1	0	0	1
t	-3	0	-2	-1	0	0
Θ	0	0	0	1	-1	0

Step 3 Choose ρ, g, β, and h_c as the scaling variables. The fundamental dimensions can now be extracted as,

$$L = h_c \quad (6)$$

$$\Theta = \frac{1}{\beta} \quad (7)$$

$$t=\sqrt{\frac{h_c}{g}} \tag{8}$$

$$M=\rho h_c^{\ 3} \tag{9}$$

Step 4 Scale the remaining variables according to their dimensions in the powers of the fundamental dimensions, thus:

$$\Pi_1=\frac{\frac{1}{2}\dot{m}V^2}{\rho h_c^{\frac{7}{2}} g^{\frac{3}{2}}} \tag{10}$$

and

$$\Pi_2=\frac{q''A_r / c_p}{\rho h_c^{\frac{5}{2}} g^{\frac{1}{2}} \beta^{-1}} \tag{11}$$

Finally, the functional relationship is, $\prod_1 = f^n(\prod_2)$, or

$$\frac{\frac{1}{2}\dot{m}V^2}{\rho h_c^{\frac{7}{2}} g^{\frac{3}{2}}}=f^n\left(\frac{q''A_r\beta}{\rho c_p h_c^{\frac{5}{2}} g^{\frac{1}{2}}}\right) \tag{12}$$

3. Engineering Interpretation of the Dimensionless Variables

Engineering interpretation often helps in a deeper understanding of the physical phenomenon, which is being represented by the mathematics. The scaling variable of $\prod_1$, $\rho h_c^{\frac{7}{2}} g^{\frac{3}{2}}$, could be interpreted as the power required to lift the air up the tower height. To clarify this, first note that $\sqrt{h_c/g}$ is a time scale for the flow to climb from the base to the top of the tower. Accordingly,

$$\rho h_c^{\frac{7}{2}} g^{\frac{3}{2}}=\rho h_c^{\ 4} g\sqrt{g/h_c}=\rho h_c^{\ 4} g/\Delta t=\rho A\left(h_c/t\right) g h_c$$
$$=\rho A V g h_c=\dot{m} g h_c$$

The last term is readily revealed as the power required to lift the air up the tower. So, $\prod_1$ could be interpreted as the kinetic power of the system measured in the scaling unit that is proportional to the power required to lift the air up the tower.

Furthermore, $\prod_2$ could be interpreted as the buoyant force per unit weight. Note that $q''A_r\beta / c_p = \dot{m}\beta\Delta T$, and that the scaling variable

$$\rho h_c^{\frac{5}{2}} g^{\frac{1}{2}}=\rho h_c^{\ 3}\sqrt{g/h_c}=\rho h_c^{\ 2}\left(h_c/\Delta t\right)=\rho A V=\dot{m}\,.$$

$\prod_2$ now becomes $\beta\Delta T\approx \Delta\rho/\rho$, which could be interpreted as the buoyant force (created by $\Delta\rho$) scaled by the weight of the fluid of the same volume.

The entire relation of $\prod_1 = f^n(\prod_2)$ can be summarized as trying to find the relation of characteristic kinetic energy (the intuitive output) as a function of the principal characteristic driving force of the system (the intuitive input).

4. Similarity Requirements

To obtain similarity for the models, the stipulation required by $\prod_2$ between prototype and model is,

$$\left[\frac{q''\pi r_r^{\ 2}}{h_c^{\frac{5}{2}}}\right]_p=\left[\frac{q''\pi r_r^{\ 2}}{h_c^{\frac{5}{2}}}\right]_m \tag{13}$$

Substituting the conditions that the heat flux values are the same, it is obtained,

$$r_{r_m}=\left[\frac{h_{c_m}}{h_{c_p}}\right]^{\frac{5}{4}} r_{r_p} \tag{14}$$

It is shown in Eq. (14) that the roof radius does not scaled in a linear fashion with the tower height; this suggests a distorted model. In other words, this suggests a partial geometric similarity. The dimensions of partially geometrical similar models required for ensuring dynamic similarity among models are presented in Table 2.

Table 2 Specification of prototype and distorted model plants.

Case	Tower Height scale factor	Tower height (m)	Roof height (m)	Tower radius (m)	Roof radius (m)
Prototype	1	20	0.4	0.8	10
Model 1	0.125	2.5	0.05	0.1	0.74
Model 2	0.25	5	0.1	0.2	1.77
Model 3	0.5	10	0.2	0.4	4.21
Model 4	5	100	2	4	74.77
Model 5	10	200	4	8	177.83

5. CFD Modeling

Numerical calculations have been performed using CFX, one of the widely-accepted commercial CFD codes. For this purpose, CFX solves the conservation equations for mass, momenta, and energy using a finite volume method. Adaptive unstructured tetrahedral meshes were used in the present study. The plants studied were modeled as axis-symmetry where the centerline of the tower is the axis of symmetry. To simulate axis-symmetry, a 5 degree section of the plant is cut out from the entire periphery as shown in Figure 2. To make certain that similarity (or the lack thereof) was not affected by grid-topology irregularity, grid similarity for all the test cases was enforced. Grid similarity means that when the model is scaled to be the same size as the prototype, its grid is exactly the same as that of the prototype. Moreover, grid convergence was also ensured by varying the numbers of grid until no change in output parameter was noticed.

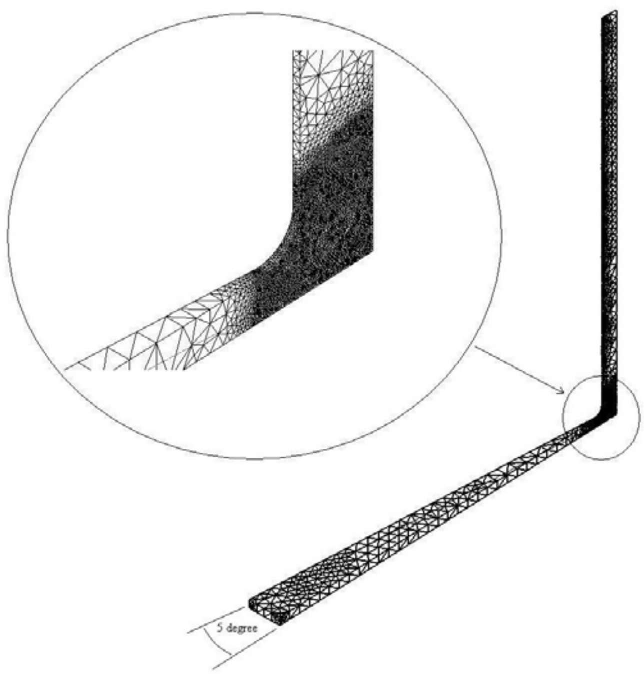

Figure 2 Unstructured grid used for the 5 degree axis-symmetric computational domain.

Proper boundary conditions are needed for a successful computational work. At the roof inlet, the total pressure and temperature are specified; whereas at the tower exit the 'outlet' condition with zero static pressure is prescribed. The 'symmetry' boundary conditions are applied at the two sides of the sector while the adiabatic free-slip conditions are prescribed to the remaining boundaries, consistent with the frictionless flow that are being modeled. All test cases had been computed until residuals of all equations reached their respective minima. Moreover, global conservation of mass had been rechecked to further ascertain the convergence of the test cases.

6. Results and Discussion

The insolations of the prototype are set at 600, 800, 1,000, and 1,200 W/m^2 which give Π_2, respectively, at 1.07×10^{-4}, 1.43×10^{-4}, 1.78×10^{-4}, and 2.14×10^{-4}. The equality of Π_2 were then used to compute the roof radiuses of the models required for the respective insolations (according to Eq.(14)).

The CFD results of the prototype and all distorted models are shown in figure 3-5. All plots to be subsequently presented are displayed along the scaled flow path, equaling zero at roof inlet and one at tower top).

Figure 3 shows the average velocity along the flow path; it can be seen that the velocities of the flow under the roof increase along the flow path and remains constant along the tower. The temperature profiles, shown in Figure 4, also demonstrate similar behaviors as the velocity profiles.

In Figure 5, the pressure distributions are seen to be nominally constant under the roof before falling linearly in the tower portion, to meet the hydrostatic pressure distribution at the tower top. Note that the ordinate is the gauge pressure scaled such that pressures at the top of tower are always zero.

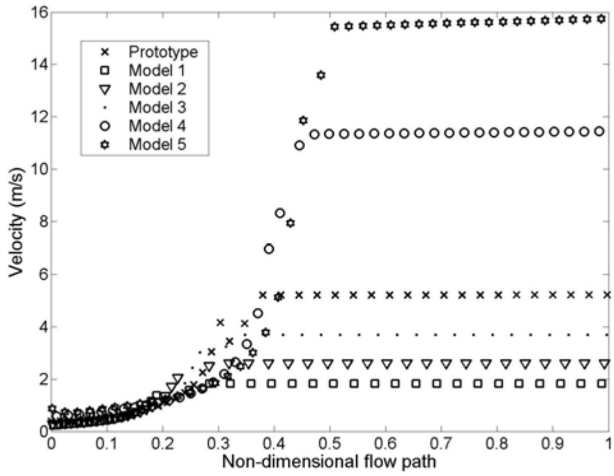

Figure 3 Numerical prediction of velocity profiles for insolation = 800 W/m^2

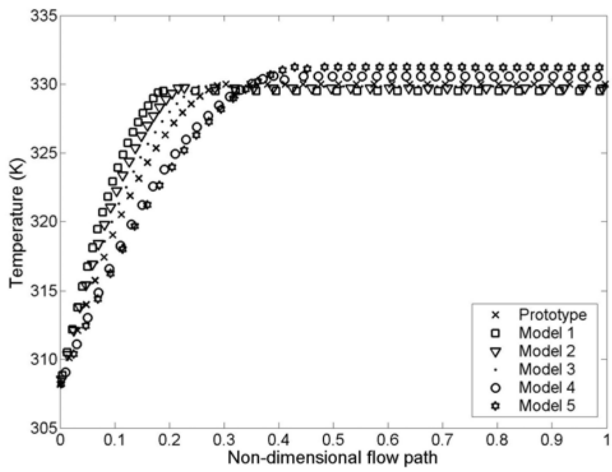

Figure 4 Numerical prediction of temperature profiles for insolation = 800 W/m^2

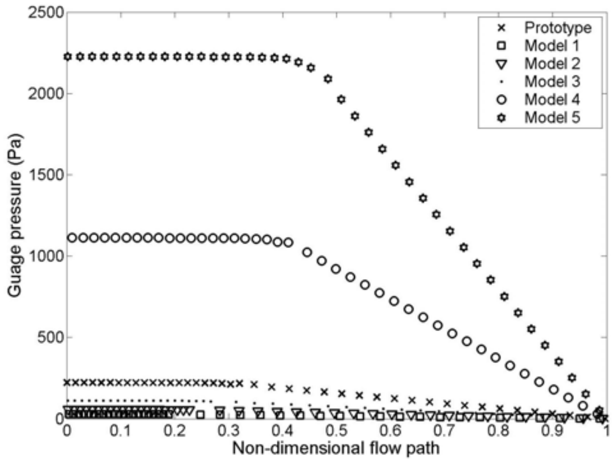

Figure 5 Numerical prediction of pressure profiles for insolation = 800 W/m^2

Figures 3-5 showed the huge discrepancies of the primitive flow variables between the prototype and the different-size models, which are partially geometrically similar and Π_2 -similar to the prototype.

If dynamic similarity existed as proposed, scaled data of the various cases, according to the forms of the dimensionless variables, must collapse. To this end, Figure 6 presents the distributions of the values of the dimensionless variable, Π_1, computed from the involved primitive variables. The figure displays the clusters of values of Π_1 according to various values of Π_2 that were used in the computations. It is clearly seen that the widely scattered data now almost collapse, suggesting similarity or so it appears.

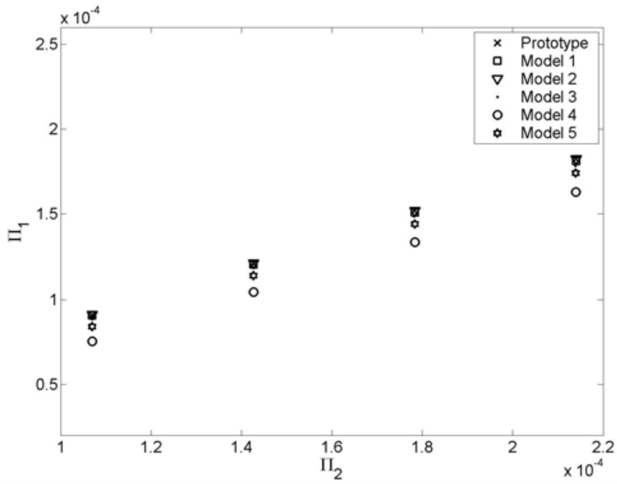

Figure 6 Π_1 - Π_2 characteristics.

To quantify similarity, the percentage difference of Π_1 between the prototype and all the distorted models is defined as follows:

$$\text{Percentage difference of } \Pi_1 = \frac{\left|\Pi_{1\text{model}} - \Pi_{1\text{prototype}}\right|}{\Pi_{1\text{prototype}}} \times 100$$

By using the above equation, the maximum percentage difference of Π_1 between the partial geometrical similar models and the prototype is presented in Table 3.

Table 3 The maximum percentage difference of Π_1 (MPDP) between prototype and all distorted models.

Case	h_{c_m} / h_{c_p}	MPDP (%)
Prototype	1	N/A
Model 1	0.125	2.18
Model 2	0.25	0.65
Model 3	0.5	0.49
Model 4	5	7.27
Model 5	10	16.81

Table 3 shows interesting results in that when the size is made 4 folds smaller, the error is 0.65%; but when it is made 5 folds larger, the error is much higher at 7.27%. It seems, then, that when the physical sizes are larger the percentage errors are higher, and vice versa. In other words, the errors are not scaled linearly. This might be attributable to the effect of artificial viscosity, which is an integral part of most numerical schemes.

Noted that similarity here is proposed on an inviscid platform. The artificial viscosity renders the inviscid flow to be viscous-liked. The larger (thus taller) plants tend to create higher flow velocities, resulting in much higher artificial Reynolds numbers than the smaller plants. The relative differences in artificial Reynolds numbers are thus believed to be the reason behind the discrepancies in the ‘inviscid’ similarity for all these cases. Physical experimentation is the only way to prove the validity of the ‘inviscid’ similarity condition as proposed in this study.

7. Conclusions

Dimensionless variables are proposed to guide an experimental study of the flow in a physical solar tower model. Distorted geometry (partial similarity) is required for the model in order to be dynamical similar to the prototype while holding the insolation to be the same for all cases. Computational fluid dynamics (CFD) is employed to obtain numerical results that are used to prove the similarity of the proposed dimensionless variables. Analyses of the results from CFD show that the partially geometrical similar models are dynamically similar to the prototype, to within an, perhaps acceptable, error band which is still uncertain as to what causes it.

Acknowledgments

The authors are thankful to the Royal Golden Jubilee (RGJ) Ph.D. Program of the Thailand Research Fund (TRF): the sponsor of this research project.

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