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การคำนวณคลื่นของเรือแบบเชิงเส้นโดยใช้ฟังก์ชันของกรีน

Linear ship-wave computation using Green function

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Abstract

The purpose of this paper is to predict wave pattern generated by ship when a ship cruises with a constant speed in the *Fourier domain*. Boundary Element Method with Green function satisfying Neumann-Kelvin boundary condition at the mean position of free surface is used to compute free surface elevation. Fourier integral on the hull is replaced by Stokes' theorem, which transfers into contour integral around the surface. Numerical calculation is performed with Adaptive Simpson scheme in order to reduce the CPU time. The computational results are good agreement with the existing numerical result and experimental data.

1.Nomenclature

$E(x, y; t)$	Wave elevation
$E_1(\chi)$	Complex exponential integral
F	Froude number
$G(M,M')$	Green function
K	Pole
l	Characteristic length
$M(x, y, z)$	Field point
$M'(x', y', z')$	Unit singularity point
$\vec{n}$	Unit vector
$\Re$	Real part of complex number
S_b	Body surface
S_f	Free surface
S_i	Enclosing contour surface
S_w	Wake surface
U	Steam velocity
Φ	Total velocity potential
ϕ	Perturbation potential
σ	Source
μ	Doublet

2.Introduction

Solving the hydrodynamics problem with free surface effect by numerical method has been studied for more than thirty years. Chapman [1] use finite element method to solve piercing plate problem. Salvesen [2] uses strip theory with Green function presented by *Wehausen and Laitone* [3] *to solve ship hull* problem. The last method is developed for solving many problem such as Ba, Guilband, and Coisier [4] used to solve steady flow problem, Inglis and Price [5], Nontakeaw [6] used to solved unsteady flow problem. B.Ponizy [7] used Kelvin singularity with the Green function presented by Noblesse [8]. This Green function composes of 2 parts, Rankine term and wave generated term. He uses tabulation technique for Green function and uses Gauss method for surface integration solving anti-symmetric flow *problem*.

In this papers, The present study uses Kelvin singularity with the Green function in the analytic form presented by Guevel and Bougis [9] and developed by Nontakeaw [6] using Adaptive Simpson method for the function. The surface integration can be transfer into a contour one by using the Stokes' theorem to solve symmetric steady flow problem and compute wave pattern.

3. Numerical Method

Consider a ship moving with steady forward speed U in the presence of a free surface. We assume that the fluid is *incompressible and inviscid that the flow is irrotational so that a* velocity potential exists in the fluid domain. An $Oxyz$ cartesian coordinate is chosen such that z=0 plane corresponds to the calm water level and z is positive upwards. This coordinate system for the forward speed problem is translating in the positive x direction relative to a space fixed frame and x-z plane is coincident with the center-plane of the body. The total velocity potential can be expressed as

$$\Phi = -Ux + \phi \quad (1)$$

Where ϕ is the perturbation potential. The boundary value problem is governed by the Laplace equation and both Φ and ϕ satisfy the Laplace equation, The fluid domain is bounded on top by the free surface S_f, internally by the hull of surface S_b, the wake surface Sw, and enclosing contour surface Si (see Fig.1)

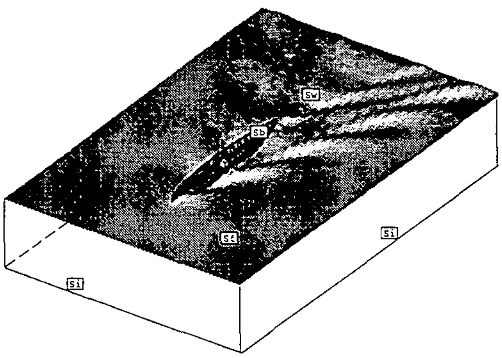

Figure 1 : Coordinate system

On the hull surface, we require that flow be tangential to the hull surface (Neumann Condition)

$$\frac{\partial\phi}{\partial n} = \vec{U}.\vec{n} \qquad \text{on body} \quad (2)$$

Where n is a unit outer normal vector. On the free surface $E(x, y)$, the perturbation potential must satisfy the kinematic free surface boundary condition

$$\frac{\partial\phi}{\partial z} = (U + \phi_x)E_x + \phi_x E_y \qquad \text{on } z = E(x, y; t) \quad (3)$$

and the dynamics free surface boundary condition

$$gE + U\phi_x + \frac{1}{2}(\overrightarrow{\text{grad}}\phi)^2 = 0 \quad \text{on } z = E(x, y; t) \quad (4)$$

Since the elevation of wave is assumed small, the kinematic and dynamic boundary conditions on the free surface may be linearlized, respectively

$$\phi_z = UE_x \quad (5)$$

$$E = -\frac{1}{g}U\frac{\partial\phi}{\partial x} \quad (6)$$

and combined to be satisfied Neumann-Kelvin free surface condition

$$g\phi_z + U^2\phi_{xx} = 0 \qquad \text{on } z = 0 \quad (7)$$

The Green's Theorem is used to define perturbation potential

$$\phi(M) = \frac{1}{4\pi}\iint_s \left(\frac{\partial\phi(M')}{\partial n_{M'}}G(M,M') - \phi(M')\frac{\partial G(M,M')}{\partial n_{M'}}\right)ds(M') \quad (8)$$

Where $M = (x, y, z)$ is field point, $M' = (x', y', z')$ is source point, $S = S_b + S_f + S_w + S_i$ is the boundary surface surrounding the fluid domain. σ is the constant source strength ($\sigma = \partial\phi/\partial n$), μ is the constant doublet strength distributed on the body, and G is the Green function. Mathematically it is the solution of Poisson equation ($\nabla^2 G = \delta(x-x')\delta(y-y')\delta(z-z')$), physically it is the response of a system when a unit point source is applied to the system. For this problem, The Green function, Nontakeaw [6], satisfy Neumann-Kelvin free surface condition (7)

$$G(M,M') = g_0 + g_1 \quad (9)$$

$$g_0 = \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} - \frac{1}{\sqrt{(x-x')^2 + (y-y')^2 + (z+z')^2}} \quad (10)$$

$$g_1 = -\frac{2}{\pi l}\Re\left(\int_0^{\frac{\pi}{2}} K[G_1(K\xi) + G_1(K\xi')]d\theta\right) \quad (11)$$

$$\xi = \frac{1}{l}[z + z' + i((x-x')\cos\theta + (y-y')\sin\theta)] \quad (12)$$

$$\xi' = \frac{1}{l}[z + z' + i((x-x')\cos\theta - (y-y')\sin\theta)] \quad (13)$$

$$K = \frac{1}{F^2\cos^2\theta} \quad (14)$$

$$G_1(\chi) = \begin{cases} \exp(\chi)E_1(\chi) & \text{if } \Im(\chi) \geq 0 \\ \exp(\chi)[E_1(\chi) - i2\pi] & \text{if } \Im(\chi) < 0 \end{cases} \quad (15)$$

$$E_1(\chi) = \int_x^\infty \frac{\exp(-t)}{t}dt \text{ ; } 1^{st} \text{ order complex exponential integral}$$

Where l is characteristic length, $\Re$ is real part of function. The term g_0, often studied in aerodynamics or hydrodynamics using Rankine's singularities. We use Neumann condition to be solved for the unknown function (doublet, μ). Replace equation (8) into equation (2)

$$\iint_s \sigma(M')\frac{\partial G}{\partial n_M}(M,M')ds - \iint_s \mu(M')\frac{\partial^2 G}{\partial n_M \partial n_{M'}}(M,M')ds = 4\pi\vec{U}.\vec{n} \quad (16)$$

For a polygonal panel, the surface integral can be transfer into a contour one by using the Stokes' theorem (see for example Y. Tiaple[10])

4. Numerical results

Submerged ellipsoid body :

For the investigated numerical method, we initially examined a fully submerged body since this would avoid any difficulties associated with the intersection of the hull with the free surface. We chose submerged ellipsoid compared, to results presented by B. Ponizy [7]. The dimension of ellipsoid is a / b = 5, l= 2c and $c = \sqrt{a^2 - b^2}$. Where a is semi-major axis, b is semi-minor axis, l is the length. The hull centerline was located at a depth of z = 0.5c. The panels on the body are discretized by using a cosine spacing in all directions. In this case, the surface integral has only the hull of surface S_b because of immerged symmetric flow. Computed wave pattern for ellisoid is compared with numerical in B. Ponizy [7]. Fig.(2) shows wave profile along the body calculated from Eq. (6) at *Froude* = 0.6 . Fig.(3) shows wave contour on the free surface. Fig. (4) shows ellipsoid generated waves.

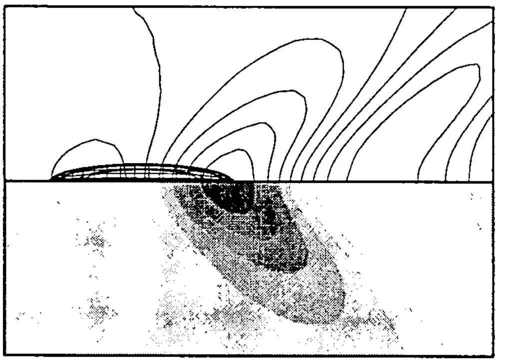

Figure3 : Wave contour on the free surface

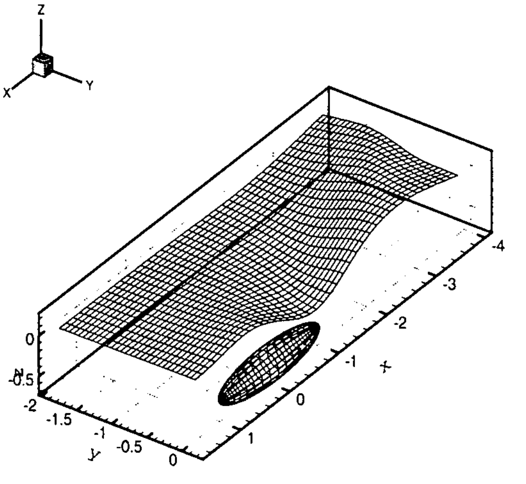

Figure 4 : Ellipsoid generated waves, *Froude* = 0.6

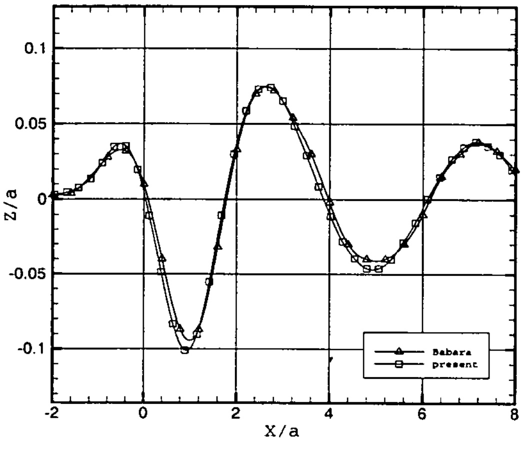

Figure 2 : Wave pattern for *Froude* = 0.6

Wigley hull :

The wigley hull is a mathematical hull form its offset are defined by

$$y = \frac{1}{2}B[\ 1 - (\frac{2x}{L})^2\ ][\ 1 - (\frac{z}{H})^2\ ] \quad (17)$$

Where B is beam = 2, H is draft = 5, and L is length = 80 . The panels on the body are discretized by using a cosine spacing in all directions (see Fig.5) and the mesh on the free surface are discretized by a hyperbolic tangent spacing in the surface hull (see Fig.6).

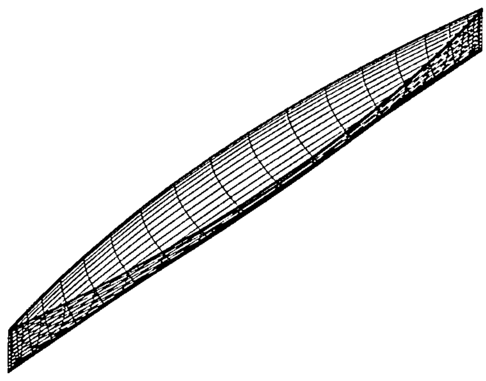

Figure 5 : Wigley model

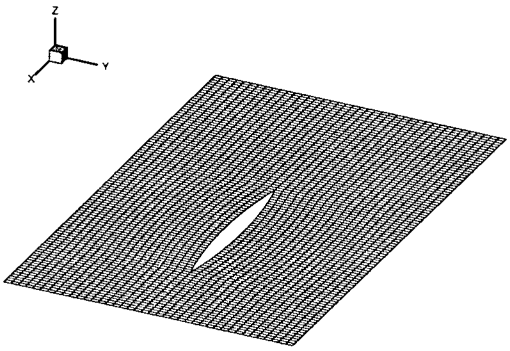

Figure 6 : mesh on free surface

In this case, the surface integral has 2 parts, the free surface S_f and the surface hull S_b . The free surface can be transfer into contour one around the hull across free surface by using Stokes' theorem (see Y. Tiaple[10]). Computed wave pattern of the wigley hull is compared with numerical in B. Ponizy [7]. Fig.(7) shows wave profile along the hull at *Froude = 0.266* calculated from Eq. (6) . Fig. (8) shows Wigley generated waves.

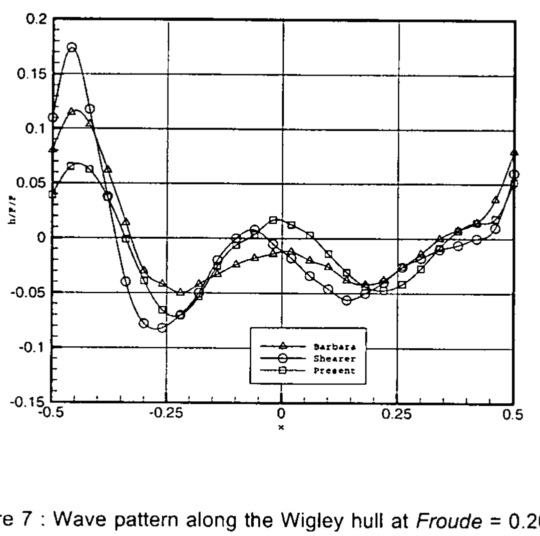

Figure 7 : Wave pattern along the Wigley hull at *Froude* = 0.266

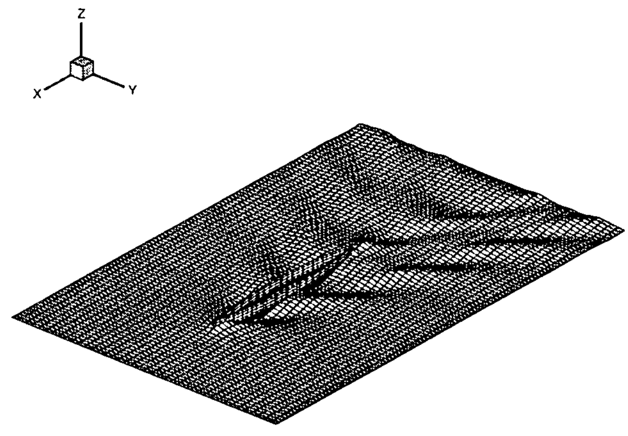

Figure 8 : Wigley generated waves, *Froude* = 0.266

5. Conclusion

In this paper, the authors have described a boundary element method (or panel method) with Green function satisfying Neumann-Kelvin boundary. This method uses Neumann condition (actual condition) to solve strength of constant doublets on the body. Consequently, the free surface elevation is computed from linear dynamic free surface condition. The computational results obtained are in a good agreement B. Ponizy data [7] in Fig.(2) and Fig.(7) . The surface integral computed by contour one using Stokes' theorem. Actually, this method uses a number of panel on surface less than Gauss method and spend a little numerical time.

6. References

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