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การสร้างรูปร่างคุณสมบัติของเปลวเพลิงเทอร์บิวเลนท์โดยวิธีแยกพิกเซลธรรมชาติ

Natural Pixel Decomposition Technique for Reconstructing Turbulent-Flame Property Profiles

จีระวรรณ เกตุนุ้ย, ปูมยศ วัลลิกุล, บัณฑิต ฟุ้งธรรมสาร
ศูนย์วิจัยการเผากากของเสีย ภาควิชาวิศวกรรมเครื่องกล คณะวิศวกรรมศาสตร์
สถาบันเทคโนโลยีพระจอมเกล้าพระนครเหนือ กรุงเทพฯ

Jeerawan Ketnuy, Pumyos Vallikul and Bundit Fungtammasan
The Waste Incineration Research Center (WIRC)
Department of Mechanical Engineering
King Mongkut's Institute of Technology North Bangkok

บทคัดย่อ

บทความกล่าวถึงการศึกษาขั้นตอนวิธีโทโมกราฟฟีแบบไม่ต่อเนื่องเพื่อสร้างภาพเสมือนของรูปร่างคุณสมบัติของเปลวเพลิงเทอร์บิวเลนท์ เปลวเพลิงที่ใช้ทดสอบในการสังเคราะห์ข้อมูลภาพฉายเป็นเปลวที่จำลองให้มีรูปร่างแบบทรงกระบอกสูงเยื้องศูนย์ ซึ่งแทนสนามของค่าการส่งผ่านเฉลี่ยแบบ 2 มิติ ได้นำวิธีการแยกพิกเซลธรรมชาติมาใช้ในขั้นตอนวิธีการสร้างภาพเสมือนเพื่อแก้ปัญหาที่มีตัวแปรมากกว่าสมการ โดยทำการศึกษาในรายละเอียดของเมตริกซ์พิกเซลธรรมชาติซึ่งแสดงคุณลักษณะของวิธีแยกพิกเซลธรรมชาติ รวมทั้งจำนวนมุมและจำนวนการเก็บข้อมูลแนวขวางที่เหมาะสมในการสร้างภาพเสมือน จากการศึกษาแสดงให้เห็นว่าวิธีการแยกพิกเซลธรรมชาติเป็นวิธีที่ทนทานต่อความไม่สมบูรณ์ของข้อมูลวิถีรวมในการสร้างภาพเสมือน และให้ผลที่สอดคล้องกับฟังก์ชันทดสอบ

Abstract

A discrete tomography reconstruction algorithm for reconstructing turbulent flame-property profiles has been studied. An artificial, off-centered "top-hat" profile representing a two-dimensional field of average value of transmittance of a turbulent flame is used as a test function. Limited projection data are generated from the test-function. The natural pixel (NP) decomposition technique for underdetermined problems has been used in this study with the matrix G, the characteristic matrix of the NP method, being examined in detail. The effects of the number of view angles and projection points on reconstruction results have also been investigated. It is found that the NP technique tolerates incomplete data and yield good reconstruction results, which are in good agreement with the test function.

1. Introduction

Two-dimensional functions, f(x,y), of moments of probability density functions for a particular region within turbulent flames can be represented by top-hat profiles [1,2,3,4]. An accurate reconstruction of the top-hat profiles from their tomographic data is a vital to the retrieval of thermodynamic probability density functions within turbulent flames[3,5]. Filter Back-projection (FBP) technique has been used successfully to reconstruct the moment functions for both symmetrical- and asymmetrical- turbulent flames [4]. The algorithm, however, needs excessive tomographic data in order to reconstruct accurately the moment functions.

Our previous work [6] suggested that reconstruction results from limited tomographic data can be obtained using either the Algebraic Reconstruction (ART) or Natural Pixel (NP) techniques. An off-centered gaussian profile was used as the test function. It also called for a more accurate numerical technique to treat matrix G, the characteristic matrix of the NP technique in order to obtain better reconstruction results.

In this paper, we review the definition of discrete tomography, NP technique and analyse G further and the pseudoinverse of the matrix G is calculated (see section 2). Reconstruction results of the off-centered tophat profiles are shown in section 3. At the end of section 3, a reconstruction from

incompleted data (view angles between 0-90°) is demonstrated through the NP and FBP techniques.

2. Descriptions

2.1 Discrete Tomography

In discrete tomography, the problem is derived in discrete form at the beginning of the reconstruction process. The two-dimensional domain is divided into pxp rectangular pixels and the function, f(x,y), that falls into each pixel, is approximated to have a constant value f_q, (q = 1...p^2), represented by a vector $\underline{f}$. The projection matrix ϕ, is then defined by the matrix that transforms the vector $\underline{f}$ into the projection vector $\underline{y}$ of length MxN where M and N represent the M view angles and the N number of projections at that view angle. Each element of $\underline{y}$ then represents a strip projection of $\underline{f}$. The graphical concept of the discrete tomography is shown in figure 1.

$$\phi \underline{f} = \underline{y} \tag{1}$$

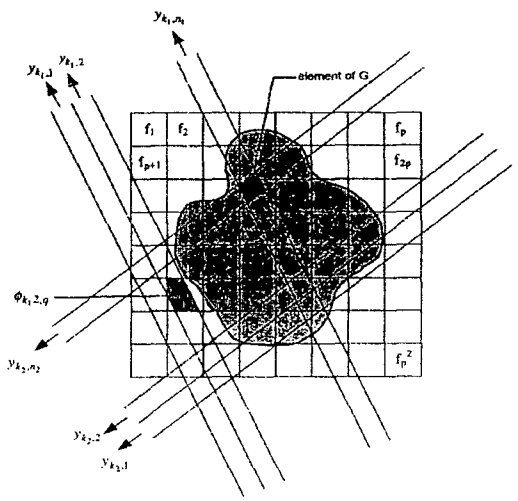

Figure 1 Determination of strip projection and projection matrix

2.2 Natural pixel decomposition and reconstruction

Natural pixel decomposition (NP) is a technique for reconstructing a square integratable function [7] from its incomplete tomographic data. With this technique, the vector $\underline{f}$ is written as linear combinations of the column space of ϕ^T:

$$\underline{f} = \phi^T \underline{x} \tag{2}$$

where the elements of $\underline{x}$ are unknown. The vector $\underline{x}$ is

$$\underline{x} = \begin{bmatrix} x^T(1) & x^T(2) & \cdots & x^T(k) & \cdots & x^T(M) \end{bmatrix}^T$$

and

$$x(k) = \begin{bmatrix} x_{k1} & x_{k2} & \cdots & x_{kn} & \cdots & x_{kN} \end{bmatrix}^T$$

The vectors $\underline{f}$ and $\underline{x}$ and the matrix ϕ^T can be written in terms of their elements as

$$\begin{aligned} f_1 &= \phi_{1,1}x_{11} + \phi_{12,1}x_{12} + \ldots + \phi_{kn,1}x_{kn} + \ldots + \phi_{MN,1}x_{MN} \\ f_2 &= \phi_{1,2}x_{11} + \phi_{12,2}x_{12} + \ldots + \phi_{kn,2}x_{kn} + \ldots + \phi_{MN,2}x_{MN} \\ &\vdots \\ f_q &= \phi_{1,q}x_{11} + \phi_{12,q}x_{12} + \ldots + \phi_{kn,q}x_{kn} + \ldots + \phi_{MN,q}x_{MN} \\ &\vdots \\ f_{p^2} &= \phi_{1,p^2}x_{11} + \phi_{12,p^2}x_{12} + \ldots + \phi_{kn,p^2}x_{kn} + \ldots + \phi_{MN,p^2}x_{MN} \end{aligned} \tag{3}$$

Substitute $\underline{f}$ from (2) into (1) and the result becomes

$$\begin{aligned} \underline{y} &= \phi\phi^T \underline{x} \\ \underline{y} &= G\underline{x} \qquad \text{where} \quad G = \phi\phi^T \end{aligned} \tag{4}$$

G is the natural pixels matrix. As can be seen from (4) as well as figure 1, the element G_{ij} is a correlation between the i^{th} and the j^{th} strips. And the component y_i of the projection vector $\underline{y}$ is the summation of all contribution of each correlation between the i^{th} and the j^{th} strips, starting from j = 1 to MN.

2.3 Evaluating the projection matrix: a simple example

Consider a domain consisting of 3×3 pixels and let each pixel have an area of unity. Two strips pass the pixels at the view angles of 0° and 90° respectively as shown in Figure 2.

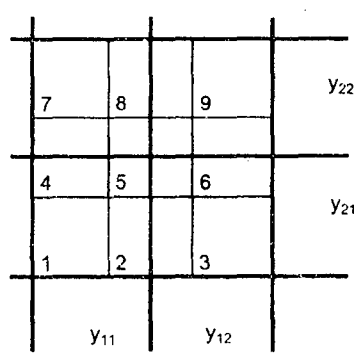

Figure 2 An example of function decomposition uses (3x3 pixels and 4 projection strips).

The matrix ϕ, according to (4), can simply be written as

$$\phi = \begin{bmatrix} 1 & 0.5 & 0 & 1 & 0.5 & 0 & 1 & 0.5 & 0 \\ 0 & 0.5 & 1 & 0 & 0.5 & 1 & 0 & 0.5 & 1 \\ 1 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0.5 & 1 & 1 & 1 \end{bmatrix} \quad (5)$$

As can be seen from (5), ϕ_{11} is the intersection area of the strip y_{11} and pixel 1, ϕ_{12} is the intersection area of strip y_{11} and pixel 2, and so on.

2.4 Analysis of the matrix G

The characteristic matrix of NP method is studied here in detail. We are solving

$$G\underline{x} = \underline{y}$$

for $\underline{x}$. The problem yields a unique solution if and only if G is nonsingular. Determination of the determinants of all sizes of the matrix G used in this work indicates that it is nearly singular, so G has many solutions. Since rank(G) is less than its columns, least square solution is not unique. A partial replacement for the inverse is provided by the pseudoinverse.

The concept is to find B, a matrix of the same dimensions as G^T so that

$$\begin{aligned} G*B*G &= G \\ B*G*B &= B \end{aligned} \quad (6)$$

and GB and BG are Hermitian. The computation is based on singular value decomposition of G and any singular values less than a given tolerance are treated as zero.

The problem becomes

$$\underline{x} = B\underline{y} \quad (7)$$

which is the minimal norm solution, and also minimizes norm of $\underline{x}$.

3 Results

3.1 Test function and its projections

The top-hat profile has been chosen as the test function, f(x,y) for this study. The function is of the form

$$f(x,y) = \begin{cases} \rho & for \quad (x-x_0)^2 + (y-y_0)^2 \le A^2 \\ 0 & Otherwise \end{cases} \quad (8)$$

where A, ρ, and (x_0,y_0) are set to be 0.35, 1, and (0.4,0.0) respectively. A surface plot of the function is shown in Figure 3.

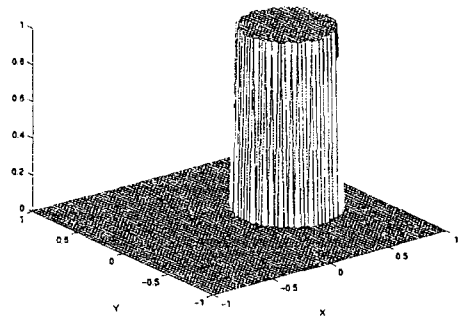

Figure 3 True function of Top-hat profile

The off-centered top hat profile has the analytical line integrated function at different angles of the form [8]

$$p_\theta(r) = \begin{cases} 2\rho\sqrt{(A^2 - r^2)} & for \quad |r| < A \\ 0 & for \quad |r| \ge A \end{cases} \quad (9)$$

The line integral (9) is used to calculate the analytical value of projection strips, that is

$$p_s(r_i) = \int_{r_i - \frac{1}{2}\Delta}^{r_i + \frac{1}{2}\Delta} p_\theta(r)dr = \begin{cases} \int_{r_i - \frac{1}{2}\Delta}^{r_i + \frac{1}{2}\Delta} 2\rho\sqrt{(A^2 - r^2)}dr, for |r| < A \\ 0 \qquad\qquad , for |r| \ge A \end{cases} \quad (10)$$

Figure 4 and Figure 5 show the line and strip (Δ =0.0317) projections of the top hat profile at view angle 0° respectively. The line and the projection functions are respectively used in the FBP and the NP reconstruction algorithms. In line projection (Figure 4), at the rim point of the upper edge of the hat, even though the absorption coefficient is set to be 1, there is no path of attenuation; therefore, the projection at this point is zero.

In order to evaluate the resemblance between the test profile and its reconstruction results, we used the concept of "Picture distance" (PD) [9]: it is the normalized root-mean-square distance, d ,as defined by

$$d = \sqrt{\frac{\sum_{i=k}^{M}\sum_{j=n}^{N}(T_{i,j} - R_{i,j})^2}{\sum_{i=k}^{M}\sum_{j=n}^{N}(T_{i,j} - T_{mean})^2}} \qquad (11)$$

where $T_{i,j}$ is the value of original profile, $R_{i,j}$ is the reconstruction, and T_{mean} the average value of $T_{i,j}$ over the region of interest.

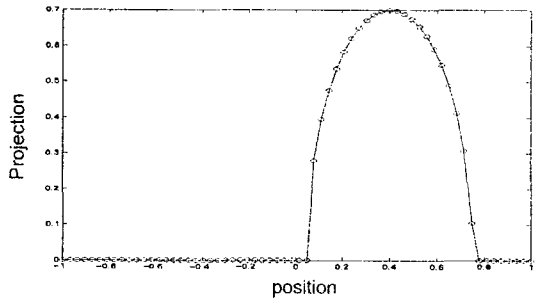

Figure 4 Projection of a top hat profile (64 sampling points)

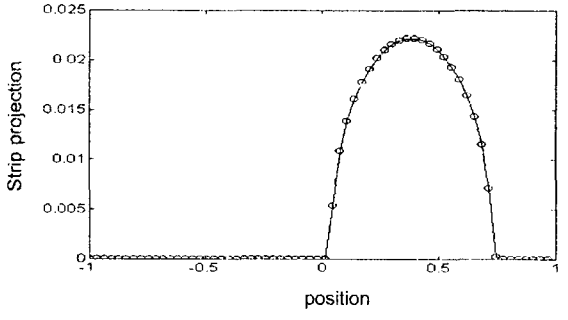

Figure 5 Strip projection of a top hat profile (64 sampling points)

3.2 Reconstruction from completed view angles

Effects of the number of angular samplings is studied at the appropriate number of pixels and number of lateral projections of 64x64 pixels and 64 points respectively.

It is shown in Figure 6 that most of the values of PD calculated from the FBP reconstruction results are higher than that from the NP reconstruction results. Figure 7 and 8 show the reconstruction results of NP and FBP techniques, respectively. It is clearly demonstrated that for a given number of the pixels and projections, a more accurate approximation function is obtained when more angular samplings are used. For FBP technique, noise like patterns [4,9] due to the under sampling of angular projections are notable. Conversely, NP technique tolerates the kind of error.

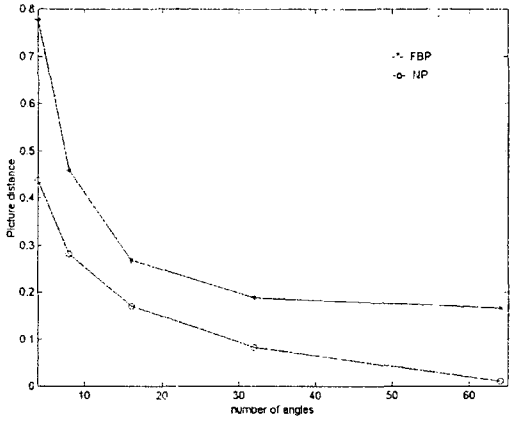

Figure 6 Picture distance : FBP and NP

3.3 Reconstruction from incompleted view angles

Reconstruction from incompleted data where only 90 degree projection are available (see Figure 9 and Figure 10) clearly demonstrated that the NP technique has the capability to reconstruct the test function from incompleted data. It is obvious that the FBP technique can only reconstruct a function from its complete projection data.

4. Conclusions

Discrete tomographic reconstruction of an off-centered top-hat function from its completed and incomplete data has been studied. With NP technique, its characteristic matrix, G, is analysed mathematically and found that it is close to singular. Pseudoinverse, then is introduced to solve this problem. Comparison the reconstruction results between two methods, FBP and NP, shows that NP gives more accurate reconstruction results than FBP, distinctively in the case of incompleted data. At the same time, it is found that the more view angles are used the more accurate reconstruction results can be obtained.

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The computation parts of this work were performed using Matlab® software at the Faculty of Engineering Computing Facility, King Mongkut's Institute of Technology North Bangkok.

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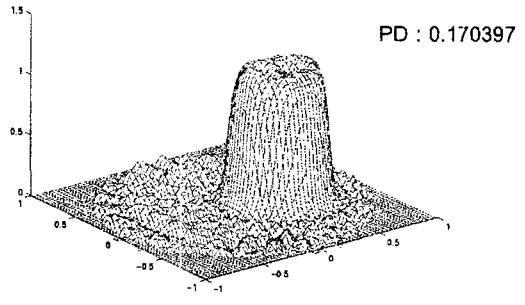

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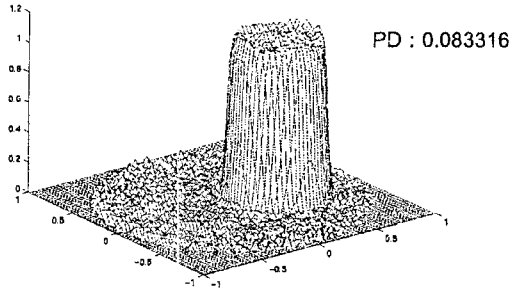

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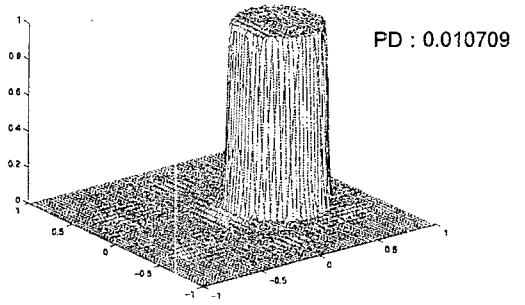

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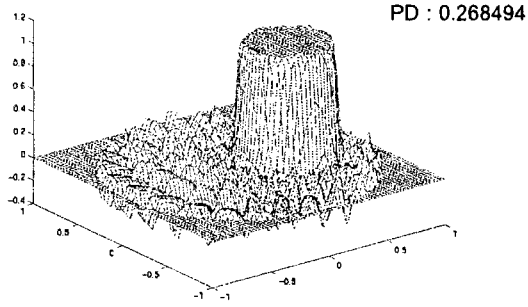
Figure 7 Reconstruction results using NP technique with 16, 32, 64 sampling angles respectively



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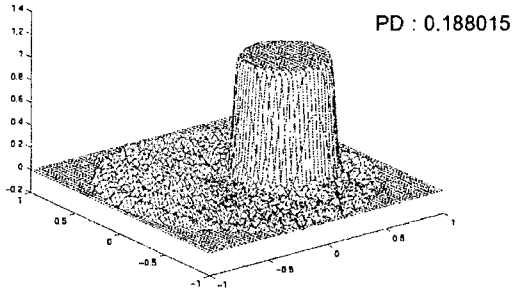

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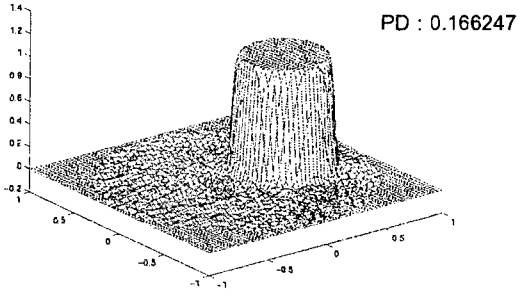

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Figure 8 Reconstruction results using FBP technique with 16, 32, 64 sampling angles respectively

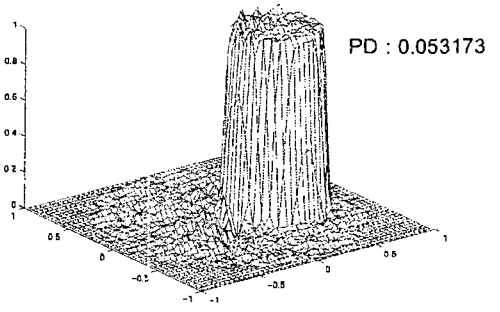

Figure 9 Reconstruction result from incompleted data using NP technique

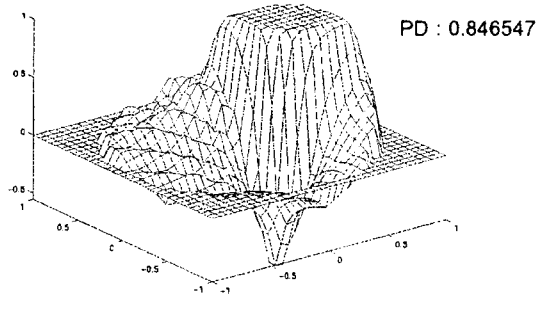

Figure 10 Reconstruction result from incompleted data using FBP technique