# Computational Volumetric Error for Part Orientation in Rapid Prototyping

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**Abstract:**

This paper considers the problem of part build orientation and proposes a methodology to minimise the volumetric errors in parts caused by the staircase effect. The paper presents the development of a Generic Part Orientation System methodology for rapid prototyping by considering the issue of volumetric error encountered in parts during the layer by layer building process in a rapid prototyping system. A Generic Part Orientation System is developed for solid parts of any complexity. An algorithm is developed that slices the part with horizontal planes and computes the volumetric error of each layer using the complex shapes of the resulting contours of each layer. The system then determines the volumetric error at different orientation by rotation about user specified axes and recommends the best orientation based on the least amount of volumetric error in the part. The system has been tested with several examples of simple and complex parts. The system has been developed by considering the volume of material deposited in RP process such as Fused Deposition Modelling process, though the concept is applicable to any RP process. The methodology has been verified by volumetric error computation of several parts based on mathematical equations of solid primitives, a combination of basic primitives and also by direct experimental measurement of volumetric error on physical parts created on the Fused Deposition Modelling rapid prototyping system.

**Keywords:** Rapid prototyping, Fused Deposition Modelling, Part Orientation, Volumetric Error

## 1. Introduction

Commercial rapid prototyping systems, that build three-dimensional objects using the layer by layer building process, introduce an error in the part on the amount of material used compared to the volume specified by the computer aided design model. This affects the dimensional accuracy as well as surface finish for different part build orientations. Two common types of surface defects are normally encountered in parts built on a rapid prototyping system [1]. The first type, the chordal error, is caused by the STL format representation of the CAD model of the part, which consists of a web of triangles representing the surface. This format requires significant redundancy, and it is restricted to triangles. Thus, all curvilinear surfaces must be approximated as a series of linear segments leading to a non-smooth surface of a part. This type of error can be reduced to an extent by increasing the tessellation of triangles but this may cause very large size of STL file and computation problems because the valuable geometric and topological information are lost during the data transfer. The original curvature of the surface may never be recovered. The other type, the staircase error on the part, which is caused by the layer by layer nature of the rapid prototyping building process, varies with the nature of the surface of the part. Inclined and curved surfaces show staircase effects more predominantly than other surfaces. The orientation, at which the part is built, can have a significant effect on the quality of various surfaces of the part due to the stair-step effect. The orientation of the part also affects other factors such as the build time, the complexity of the support structure, shrinkage, curling, trapped volume, and material flow in some rapid prototyping processes [2]. The undesirable staircase feature is most often apparent on sloping or curved surface, and, it is impossible to eliminate it completely in most cases. The determination of proper orientation of part during the building process has therefore been an important issue in rapid prototyping .

The main objective of this research is to develop a generic part build orientation system for rapid prototyping based on the volumetric error encountered in parts during the layer by layer building process in a rapid prototyping system. The aim is to provide a set of techniques and methodologies that will help the RP user to select the best part orientation based on minimum volumetric error and hence enhance the quality of parts and improve the productivity of any rapid prototyping process.

## 2. Related Work

The determination of proper orientation of the part during the building process on a rapid prototyping has been an important problem to be resolved by the users of RP systems. There has been several works done on developing methods to determine an appropriate orientation based on different criteria. Allen and Dutta [3] developed a method for determining part orientation based on support requirements. In their algorithm, if two orientations require support structures with equal surface areas of contact, the orientation, in which the object has a lower centre of mass, is chosen. Practically, it is unlikely that any two orientations would have a support structure with the same contact area. Frank and Fedal [4] proposed an expert system tool that considers the various parameters that affect the production of the prototype and recommends the best direction of building the part based on both the user's input as well as on a decision matrix implemented within the expert system.

There has been several works on the orientation problem specifically for the Stereolithography (SLA) rapid prototyping process. Kim, Lee and Park [5] developed an optimisation technique for optimal part orientation within the SLA process by considering an objective function related to the volume of trapped liquid resin, the total height of the part in the build direction and the area of surfaces with staircase protrusions. Cheng et al [6] developed a procedure for optimisation of part building orientation in SLA by assigning weights to various surface types affecting part accuracy.

It is observed that most of the work on part orientation problem has been related to the Stereolithography process. Not much work about orientation problems has been directed to Fused Deposition Modelling (FDM) process, in which, the accuracy and surface finish problems due to staircase effect are more dominant than in the SLA process. Furthermore, very little work seems to have been done on part orientation problem based on minimisation of volumetric error, which may well be applied not only to the FDM process but to other rapid prototyping processes as well. However, the volumetric error approach is especially relevant to the FDM process, where deposition of material from a nozzle creates the volumetric error more significantly compared to other RP processes.

## 3. Algorithm for Part Orientation Using Volumetric Error

The volumetric error in a rapid prototyping process in general and in FDM process in particular can be defined as the difference in the volume of material used in building the part compared to the volume specified by the CAD model. The total volumetric error in a part will be different at different orientation because of the layering building process. If the volumetric error in a layer in the RP part can be computed, then the volumetric error in the part will be the sum of all the volumetric errors in the layers used in the part. A generic algorithm has therefore been developed that computes the volumetric error in a part of any shape by summing up the volumetric error in each layer. The algorithm makes use of the STL model of the part, slices the part by infinite planes normal to the build direction, and then determines the best orientation based on the least amount of volumetric error in the part. The algorithm provides the user two options for orienting the part about different axes. In one option, the user can rotate the part about one axis only (either, x or y or z-axis) to determine the best orientation angle. In the second option, the user can rotate the part about two axes (first about z-axis, then about x or y-axis). We shall describe the algorithms and procedures for second options. Figure 1 shows the flow chart describing the algorithm for rotation about two axes. When two axes are selected by the user, the algorithm basically works in the same manner as one axis except that the part is rotated first about the *z*-axis at the starting angle ($i_1$=0), and then about the second axis (*x* or *y*) from 0° to 90° with the specified step increment of rotation ($d''$).

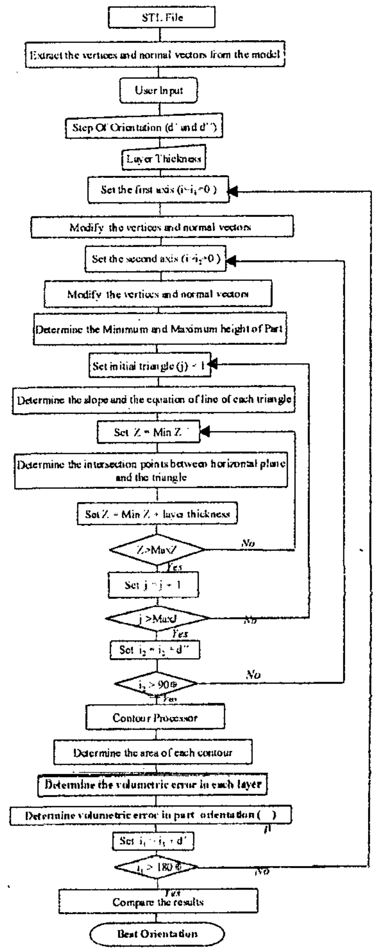


***Figure 1 Algorithm for determining the optimum orientation about two axis***

The procedure starts with the user providing the STL file of the part in the ASCII format. The ASCII STL file normally starts with the lower case key word ***solid*** followed by the file name and it ends with the word ***end solid*** followed by the file name. If the ASCII STL file has no file name, the user can skip the input stage. In case the ASCII STL file has a long file name and spacing between the words, the user should open STL file from the NOTEPAD in the Windows environment and then delete the long file name and input the short file name. Then, the user inputs the values of step angle of rotation and the layer thickness and selects the axis/axes of rotation. If the user provides smaller values of step angle of rotation and the layer thickness, a higher accuracy is expected for best orientation angle. In the beginning, when the STL file is loaded into the program, the model is placed in a normal position as determined by the user and set at 0° orientation. If it is not in the normal position, the user has to modify the model using the CAD software. The program displays the triangular facets of the STL model. At each orientation, the model is sliced from bottom to top by horizontal planes separated by a distance equal to the selected layer thickness. This creates a large number of intersection points (or vertices) between the facets and the plane. The vertices thus represent the part in that orientation and the unit normal vectors represent the direction of tessellated facet with respect to $z$-axis. Initially all the vertices are unordered collection of intersection points in the database. The program uses a sub-algorithm, called contour processor, which sorts out these vertices into groups with same z-coordinate values. Then the contours with the same z-values are created. The vertices in each contour must be re-ordered by the contour processor to determine the area bounded by the contour polygon. Thus, the contour processor used in the program automatically modifies and sorts out the vertices to generate horizontal contours. When the part is oriented about any axis in a direction specified by the user, the program first determines the minimum and maximum height of part with respect to the $x$-$y$ plane. Then the horizontal intersecting planes are generated to cut the part. Using the coordinate values of the vertices of each tessellated triangle, the program determines the slopes and the equations of the lines of each triangle. Then the coordinates of intersecting points between these lines and the horizontal planes are determined. Then these intersected points or vertices are sorted out by the contour processor in terms of same z-coordinate values from the minimum to the maximum height of part in $z$ direction. Sorting is unidirectional, either in descending or ascending order. Too many variations in sizes of triangles coupled with model complexity often complicate the procedure of sorting. Grouping of vertices is then performed considering the minimum and the maximum $z$ coordinate values as the grouping criteria. This creates horizontal contours. Each contour thus contains points with the same $z$ values. The contour processor re-orders these points in each contour on the basis of ascending order of angles subtended by each point with respect to the $x$-axis in the $x$-$y$ plane. Then these points are joined by straight lines and the program then determines the area bounded by the polygon of each contour. These contour areas are used to determine the volume difference generated by the two horizontal contours in a layer, which are then used to determine the volumetric error in that layer.

The algorithm described above works for any part of any complexity, with any layer thickness, and for rotation about selected axes. The computational working of the algorithm can be illustrated by considering the volumetric error computation in a simple example part such as a hemisphere part.

Consider a hemisphere to be built along the $z$-axis as shown in Figure 2. It is decomposed into layers that are the angular polyhedrons of decreasing cross-sections as shown in Figure 2 (a).

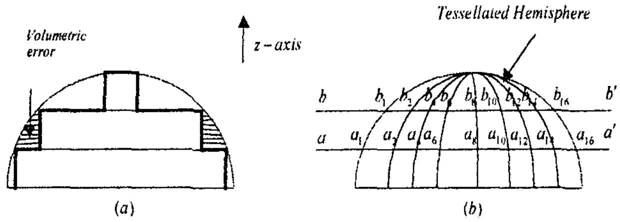


**Figure 2 Intersection points between the tessellated hemisphere with two horizontal planes.**

The angular polyhedron contains the tessellated hemisphere and two horizontal planes. The volumetric error of each layer is difference between the volumes of the polyhedron layer and horizontal rectangle layer. The total volumetric error is the sum of the volumetric error in each layer from bottom to top. Figure 2 (b) presents the intersection points between the tessellated hemisphere and two horizontal planes ($aa'$ and $bb'$). When a horizontal layer is to be built, plane $aa'$ intersects the facets of hemisphere at points $a_1$, to $a_{16}$ and plane $bb'$ also intersects the same set of facets at points $b_1$ to $b_{16}$ at a level higher than plane $aa'$.

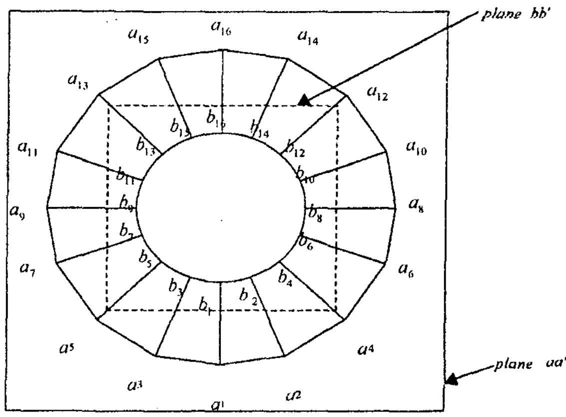


**Figure 3 The top view of the tessellated hemisphere and two horizontal planes.**

Figure 3 shows the top view of the tessellated hemisphere cut by the horizontal planes. The contour of right angular polyhedron, $b_1$ to $b_{16}$, on plane $bb'$ is projected along $z$-axis into plane $aa'$ at points $a_1'$ to $a_{16}'$. The volumetric difference between the projected layer along $z$-axis and tessellated part is then the three-dimensional error in each layer. Considering the horizontal layers intersecting the tessellated hemisphere as shown in Figure 3, the horizontal area in each layer is given by

$$A_i = (1/2)\left(\sum_{i=1}^{n-1} x_i y_{i+1} - x_{i+1} y_i\right) + (1/2)[x_n y_1 - x_1 y_n] \qquad (1)$$

where $n$ = the number of intersection points in each layer,

$i = 1 \le i \le n$ and

$x_i, y_i$ = the coordinates of intersection points

The volumetric error (VE) is given by

$$VE = \left|\sum_{i=1}^{k} (A_{i+1} - A_i) * t/2\right| \qquad (2)$$

where $k$= the number of layers, and

$t$= layer thickness

The volumetric error is computed at each angle. When the orientation about the second axis is completed at 90°, the orientation about first axis is incremented by the step of rotation $d'$, and this is again followed by all the steps of rotation about the second axis. This process continues until the end angle of rotation about the first axis is reached to 180°. Thus each step rotation about $z$-axis is followed by the step rotations about the second axis ($x$ or $y$) from 0° to 90°. When a part, which is symmetrical about $z$-axis, is rotated about $z$-axis, the components of unit normal vector along $z$-axis do not change, and this keeps the volumetric error constant. But this is not the case if the part is of non-symmetric geometry about the $z$-axis. During the orientation processes, the volumetric errors are compared with previous values at each step of orientation. The algorithm then determines the minimum of all volumetric errors and then recommends the best orientation based on the least amount of volumetric error.

## 4. Verification Generic Part Orientation System

The accuracy and proper functioning of the Generic Part Orientation System was verified by comparing the volumetric errors of standard primitives and parts obtained analytically from the part orientation system with the results obtained from the generic system for the same primitives and parts. The verification was also carried out by comparing the volumetric error values obtained from the generic system with those determined experimentally on physical primitives and parts [7]. For verification of the generic part orientation system, the following types of primitives and parts were rotated about the $x$-axis and the relative volumetric errors were computed and compared with the analytical. In this paper, the Part Orientation System for volumetric error for the case of pyramid is illustrated.

Figure 4 shows the solid model and tessellated model of a pyramid at initial angle of orientation. The STL file of the pyramid was found to have 6 facets. The percentage relative volumetric error of the pyramid were computed by the generic part orientation system and the analytical part orientation system for orientation about $x$-axis in steps of 5° ranging from 0° to 90° at a layer thickness 1 mm. Figure 5 shows the resulting graphs of variation of relative volumetric errors with the angle of orientation. For the case of the pyramid, the recommended part build orientation is found to be at 64° at which the analytical and generic values of percentage relative volumetric error are found to be the minimum.

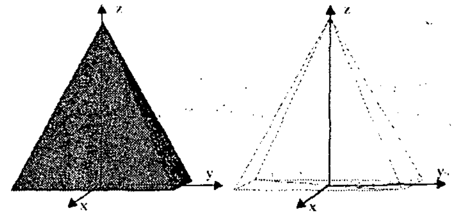


*Figure 4 The solid (a) and tessellated (b) model of a pyramid*

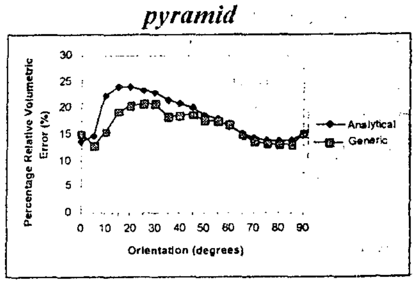


*Figure 5 Comparison of variation of relative volumetric errors for a pyramid*

Using the algorithm for rotation about two axes, results were obtained for orienting the pyramid first about $z$-axis then at different step angles and then rotation about the $x$-axis ranging from 0° to 90° for the given $z$-axis. Figure 6 shows the plots of the variation of relative volumetric errors for the pyramid, which is rotated first about the $z$-axis at 0°, 45°and 60° and then about the $x$-axis. For the case when the pyramid was oriented at 0°, 45° and 60° then rotated about the $x$-axis, the best part orientation angles were found to be at 64°, 25° and 5° respectively at which the percentage relative volumetric error is the least.

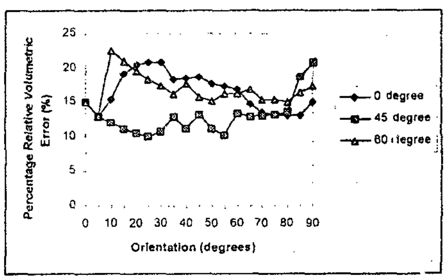


*Figure 6 Variation of relative volumetric error about z-axis at 0°, 45° and 60° then about x-axis for a pyramid*

It is observed that the pyramid, which has a rectangular base, is symmetric about two vertical planes, but the slope surface of pyramid in each face has different values. Therefore, when the pyramid is rotated first about $z$-axis at any orientation, the coordinates in each facet will be changed. Thus, the slope and line equations of pyramid associated with the facets with respect to the $z$-axis will also be changed with the rotation. It directly affects the best part orientation. It is therefore recommended that the user should build the part such that it is first rotated about the $z$-axis at 45° and then rotated about the $x$-axis at 25° because the percentage relative volumetric error is the least. Thus, the generic part orientation system developed with this algorithm will provide the RP user to make a better decision in fabricating RP parts with higher degree of accuracy and surface finish. However, it is found that it also normally depends upon the dimensions of the pyramid (the ratio of the height to the base of the pyramid).

## 5. Experimental Verification of Part Orientation System

The computed values of volumetric errors obtained from the Generic Part Orientation System can also be verified by comparison with the experimentally determined volumetric errors for each primitive and the Example Parts. Physical plastic prototypes of a pyramid, a cone, a cylinder, a cube, a hemisphere and one Example Part were built at different orientations ranging from 0° to 90° at 15° intervals on the Fused Deposition Modelling system. Thus seven identical parts for each object were produced at seven different orientations. The layer thickness used in the FDM machine was taken as 0.25 mm. The dimensional of the pyramid produced on the FDM machine was length l=30 mm, width w=30 mm and height h=30 mm. For further illustration and comparison purpose, the results of percentage relative volumetric error were also obtained using the mathematically derived analytical part orientation system. In this paper, the pyramid is illustrated and compared below. Figure 7 shows the same results as plots of variation of the relative volumetric errors for the pyramid. It is noted that the generic values compare well with analytically and experimentally determined values for the pyramid.

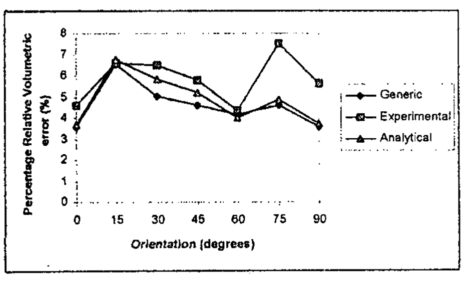


*Figure 7 Comparison of generic, experimental and analytical percentage relative volumetric error in a pyramid*

The generic system has been verified by using different types of primitives and example parts with the system, and comparing them with mathematically derived values of volumetric error for the same primitives and parts. The trend of the graphs and the magnitudes of the volumetric errors agreed well in almost all the cases. The generic system was further validated by comparing the results with experimentally determined values of volumetric errors on actual parts.

## 6. Application to Complex Parts

This research presents the application and usefulness of the Generic Part Orientation System to some selected real life parts, which have varying degree of complexity of shape and geometry. The shapes of these parts were so selected that they exhibited appreciable staircase effect when they were fabricated using a uniformly sliced model. Some of the parts were built on the FDM machine. The parts have been selected from a range of applications including toy industry and solid geometry. For each part the best orientation has been determined for rotation about one axis as well as rotation about a combination of two axes, and using different layer thicknesses. The application of the Generic Part Orientation System in this paper will be illustrated the Disc Cam. Figure 8 shows the solid model and the tessellated model of a disc cam having a curved profile and which is $2\frac{1}{2}$ dimensional part of uniform thickness with a complex contour. The STL file of the part was found to have 104 facets. The percentage relative volumetric errors of the part were computed and displayed by the system for orientation about $x$-axis in steps of 5° ranging from 0° to 90° for the three layer thicknesses.

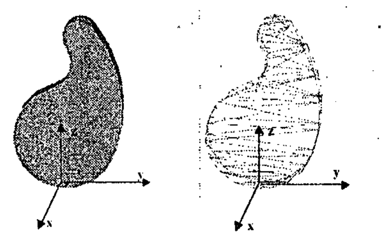

*Figure 8 The solid (a) and tessellated (b) models of the disc cam*

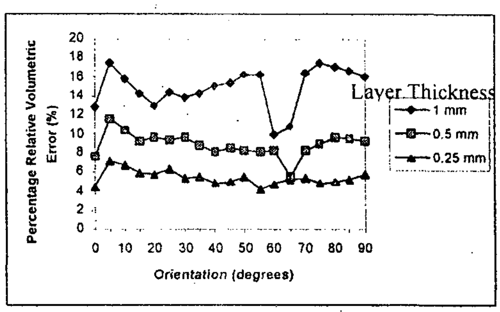


*Figure 9 Variation of relative volumetric errors for the disc cam*

Figure 9 shows the resulting graphs of variation of relative volumetric errors with the angle of orientation for the three layer thickness 1, 0.5 and 0.25 mm. The best orientation angle recommended by the system is at 60°, 65° and 55° for the given layer thicknesses respectively. It is found that the percentage volumetric error values at the layer thickness 1 mm follow the same pattern as at the layer thickness 0.5 and 0.25 respectively.

Using the option for rotation about two axes, results were obtained for orienting the disc cam first about *z*-axis at selected step angles and then about the *x*-axis ranging from 0° to 90° for the given z-axis angle. Figure 10 and 11 show the plots of the variation of relative volumetric errors for the disc cam, which is rotated first about *z*-axis at 45°and 60° and then about the *x*-axis for the three layer thickness of 1, 0.5 and 0.25. When the part was oriented at 45° and then rotated about the *x*-axis at three layer thicknesses, the recommended part build orientations are found to be at 25°, 0° and 0° respectively. In additional, the percentage volumetric error is rapidly increased at layer thickness 1 mm. It shows that the slice will miss some small features in part orientations. Therefore the user should choose lesser value of layer thickness. In case of orientation first about *z*-axis at 60° and then about *x*-axis for the three layer thicknesses, the recommended part build orientations are found to be only at 0° at which the percentage relative volumetric error is found to be the minimum. The percentage volumetric error is rapidly increasing and has the same pattern as the orientation about *z*-axis at 45°.

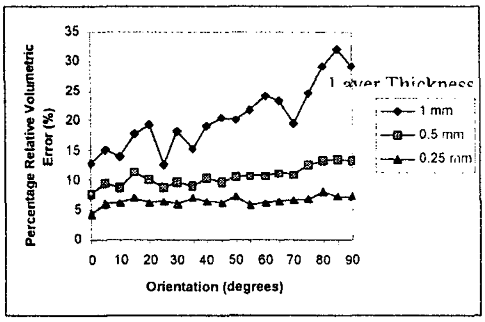


***Figure 10 Variation of relative volumetric error about z-axis at 45° and then about x-axis for the disc cam***

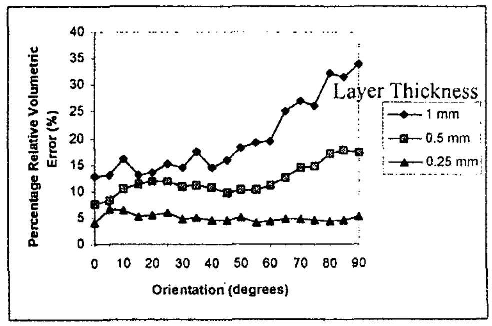


***Figure 11 Variation of relative volumetric error about z-axis at 60° and then about x-axis for the disc cam***

## 7. Conclusion

A generic algorithm has been developed to determine the best part orientation in rapid prototyping processes based on minimum volumetric error in the part due to staircase effect. The part orientation system based on this algorithm is shown to work for a range of complex parts and a range of layer thicknesses. The system allows the part to be orientated in space by manipulation of rotation about any of the three axes individually or by rotation with a combination of two axes. The algorithm has been verified experimentally and mathematically by considering the volumetric errors in primitive shapes and by actual parts built on the FDM rapid prototyping system. The part orientation system developed with this algorithm will provide the RP user to make a better decision in fabricating RP parts with higher degree of accuracy and surface finish

## References

[1]. Weeren, R.V., Agarwala, M., Jamalabed, V. R. and Bandyophadyay, A., "Quality of Parts Processed by Fused Deposition", Proceeding of SFF Symposium, 1995, Pages 314-325

[2]. Masood, S.H. and Lim, B.S., "Concurrent intelligent rapid prototyping environment", *Journal of Intelligent Manufacturing*, Volume 6, No. 5, pp 291-310, 1995.

[3]. Allen, S. and Dutta, D., "On the computation of part orientation using support structures in layered manufacturing." *Technical Report UM-MEAM-TR-94-15 Department of Mechanical Engineering.*, University of Michigan, Ann Arbour., 1994.

[4]. Frank, D., and Fadel, G., "Expert system-based selection of the preferred direction of build for rapid prototyping Processes", *Journal of Intelligent Manufacturing*, Volume 6, No. 5, pp 339-345, 1995.

[5]. Kim, J. Y., Lee, K. and Park, J.C., "Determination of Optimal Part Orientation in Stereolithographic Rapid Prototyping", Technical Report, Department of Mechanical Design and Production Engineering, Seoul National University, Seoul, July, 1994.

[6]. Cheng, W., Fuh, J. Y. H., Nee, A. Y. C., Wong, Y. S., Loh, H. T. and Miyazawa, T., "Multi-Objective Optimisation of Part-Building Orientation in Stereolithography", Rapid Prototyping Journal, Volume 1, No. 4, 1995, Pages 12-23.

[7]. Masood, S.H., **Rattanawong, W.** and Iovenitti, P., "Part Build Orientation Based on Volumetric Error in Fused Deposition Modelling", International Journal of Advanced Manufacturing Technology, Volume 16, No. 3, 2000, Pages 162-168.