CST020

# Study of Rail Vehicle Dynamic Response: A Computational Approach

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***Abstract***

"The continual rattling during the motion is principally produced by the fact that it is scarcely possible to retain the four points of the rails on which the wheels of the locomotive rests continually on one plane". This was suggested by an expert from an 1829 study of rail vehicle operation [1]. A significant number of texts and studies have presented different approaches and explanations to this problem some of these models are experimentally validated.

In KMITL, department of mechanical engineering, we are trying to develope a mathematical framework for calculating the dynamics of a rail vehicle. A major part of our study includes the study of rail wheel contact problem. Due to the above mentioned haunting motion of the train the contact plane changes continuously with time as such the contact forces vary too. There are number of contact theories existing based on the Hertz's static contact theory.

In this paper, the analysis of numerically investigates the significance of the wheel trade conicity in affecting the dynamics of railway vehicle. The railway vehicle, consisting of a coach, bogies, and wheel-sets, was modeled as multi-body system, 17 degrees of freedom. Lagrangian approach for obtaining the equations of motion for all the 17-degrees of freedom was presented. The wheel profile is considered to be of uniform conicity. The assumption of uniform conicity nullifies the effect of gravity as such an approximate expression for linearized gravitational stiffness is considered. Matlab numerical integration package for obtaining the responses of the equations of motion has been employed. The responses of each degree of freedom are analyzed for different wheel conicity.

***Keywords***: Lagrangian dynamics, Contact mechanics & Computational dynamics

## 1. Introduction

High speed trains on an average generate a lateral oscillating motion commonly known as haunting motion. This haunting motion becomes very prominent at higher speed and leads to ride discomfort. Kinematic studies reveal that the haunting oscillations grow proportionately with the circulation speed of the wheels. There are two sources of instability for the railway vehicle in general and they are:

- Bogie instability, induced by axles movement instability, this occurs due to the wheel conicity due to which the axles get transversely displaced.

- The instability due to the tendency of the vehicle to move along with the bogie at low frequency owing to the inertia of motion.

The dynamic study of railway vehicle includes the study of systems response under the influence of external stimuli.

For the method formulation we have used analytical dynamics approach in which the motion of various components of the system is studied based on applied forces and constraints in the form of first and second order differential equations. These second order differential equations also known as the system of equations represents the model and when solved numerically gives the system behavior or response. In this paper we have used Matlab's differential equation solver ode45 for numerical integrations [2]. A rigid body configuration for all the components of the system is considered.

## 2. Method Formulation

For the simulation and modeling purpose a specific vehicle model is being used. The track irregularities provide as an input to the system. The track irregularities are quantified as per FRA track safety standard [1].

An optimized design of suspension can reduce the vibration. The suspension contains dissipative elements which are modeled as linear spring and dumper model. All the rigid bodies in the system are considered to be point mass. Thus the system can have a total of 42 degrees of freedom for a total of 7 rigid bodies (6 DOF for each). Solving such a big system of equations is very challenging. It has been observed that for small amplitude vibrations there is no connection between lateral and transverse vibration [3].

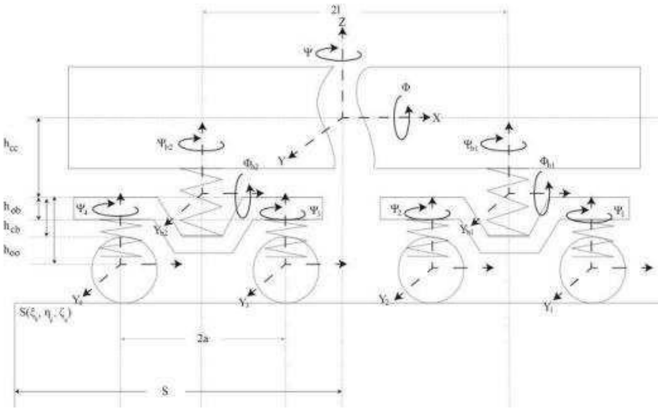

Fig. 1 Mechanical model of a typical passenger coach (Lateral view)

As illustrated in the Fig. 1 the coach's center of mass is located at $h_{cc}$ height from the separation plane between coach and the bogie. The transversal plane equally divides the central suspension is located at $h_{cb}$ distance from the plane of separation of the coach and the bogie. It is assumed that all elastic elements are weightless and linear in nature i.e. the damping force is directly proportional to the displacement. The multi-body model contains the following elements.

- The coach case
- The bogies $b_j$ j=1,2
- The wheel sets $O_i$ i=1,…,4
- The center of mass of mechanical elements $O_c$, $O_{bj}$, $O_i$
- $X_c$, $Y_c$, $Z_c$, $\psi_c$, $\varphi_c$, $\theta_c$ – The coach's translational and rotational degrees of freedom
- $X_{bj}$, $Y_{bj}$, $Z_{bj}$, $\psi_{bj}$, $\varphi_{bj}$, $\theta_{bj}$– The bogie's translational and rotational degrees of freedom
- $X_{oi}$, $Y_{oi}$, $Z_{oi}$, $\psi_{oi}$, $\varphi_{oi}$, $\theta_{oi}$– The wheel's translational and rotational degrees of freedom
- $h_c$ – the distance between $O_c$ & $O_{bj}$
- $h_b$ – the distance between $O_{bi}$ & $O_i$

Considering the coach as a rigid body interconnected by spring and dampers. At the initial position the coach's center of mass coincides with the center of the line joining the two tracks. The yawing ($\psi$) motion is considered to be of small amplitude. The motion of the vehicle is considered to be in straight line. The wheel load is considered to be constant and the coupling between the lateral and vertical accelerations are neglected.

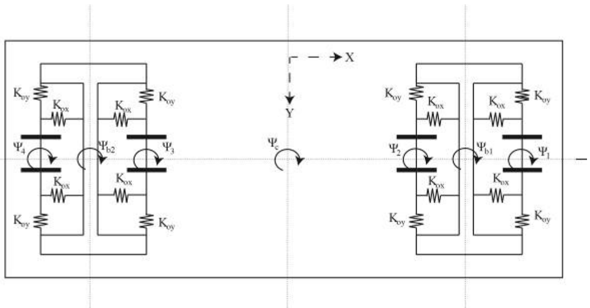

Fig. 2 Mechanical model of a typical passenger coach (Top view)

As such a total of 17 degrees of freedom is considered for study. These degrees of freedom includes: $y_c$, $\psi_c$, $\varphi_c$, $y_{bj}$, $\psi_{bj}$, $\varphi_{bj}$, $y_i$, $\psi_i$ with j= {1,2} & i={1,2,3,4}.

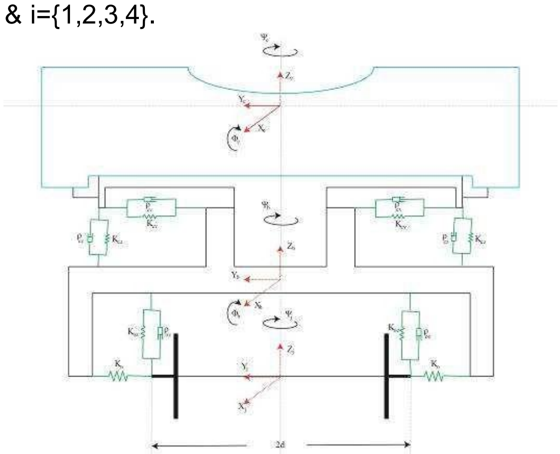

Fig. 3 Mechanical model of a typical passenger coach (Transverse view)

### 2.1 External forces to the system

There are two types existing external forces to the system:

1. Contact forces due to deformation of the wheel and rail contact.
2. Gravitational stiffness

The contact forces are also known as creep forces and they lie in the contact plane [4].

Kelkar's linear model is used for estimating the creep forces. The two tangential forces ($T_x$ & $T_y$) lies in the contact plane and the creep moment ($M_z$) around the axis perpendicular to the plane of the contact this is illustrated in the Fig. 4.

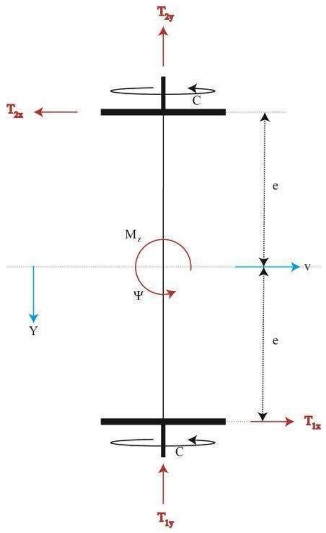

Fig. 4 External contact forces and the moment acting on the wheel sets

$$T_x = \chi_x v_x Q$$
$$T_y = \chi_y v_y Q + \chi_s r_0(\omega_s/v)Q$$
$$M_z = -\chi_s r_0 v_y Q + \chi_z r_0^2(\omega_s/v)Q \quad (1)$$

The spin creepage is given by:

$$\omega_{1s} = \omega_y sin\gamma_e$$
$$\omega_{2s} = \omega_y sin\gamma_e \quad (2)$$

The $\omega_y = \frac{v}{r}$ represents the angular velocity of the wheel. The creepage coefficients can be given as $\chi_x \approx \chi_y = \frac{300}{\sqrt{Q}} \cdots \frac{400}{\sqrt{Q}}$ the value of "Q" is expressed

in tons and the values depends on the ratio of the axis of the contact ellipse.

The spin coefficient is equal to .83 as suggested in literature [3]. This value depends upon the length of the axis of the contact ellipse. The creepage in the contact points can be given by:

$$v_{1x} = -v_{2x} = -[(\gamma/r_0)y + (e/v)\dot{\psi}]$$

$$v_{1y} = v_{2y} = \dot{y}/v - \psi$$

$$\omega_{1s} = -(v/r_0)\gamma_1$$

$$\omega_{2s} = -(v/r_0)\gamma_2 \quad (3)$$

Therefore the forces and the momentum at the contact point can be given by:

$$T_{1x} = -\chi Q(\frac{\gamma}{r_0} + \frac{e}{v}\dot{\psi})$$

$$T_{2x} = \chi Q(\frac{\gamma}{r_0} + \frac{e}{v}\dot{\psi})$$

$$T_{1y} = \chi Q(\frac{\dot{y}}{v} - \psi) - \chi_s Q \gamma_1$$

$$T_{1y} = \chi Q(\frac{\dot{y}}{v} - \psi) + \chi_s Q \gamma_1$$

$$M_{1z} = M_{2z} = -\chi_s Q r(\frac{\dot{y}}{v} - \psi)$$

(4)

Where, $\chi$ is the creepage coefficient, v is the coach's linear velocity, Q-is the wheel load, $\gamma$ is the effective conicity of the tread, $r_0$ is the wheel radius & $\chi_s$ is the spin creepage coefficient.

### 2.2Gravitational stiffness

When the loaded wheels of a railway vehicle rests over the tracks the force vector in the contact point can be decomposed in to two parts:

• Vertical component, which will support the weight of the vehicle

• Lateral component, which tends to get increasingly unbalanced with lateral displacement of the vehicle.

The lateral force component can be linearly approximated with lateral displacement to obtain linearized gravitational stiffness $c_g$. The direction of this vector is influenced by local radii of curvatures of both wheel and the rail. The concave wheel trade is made concave to provide a stable restoring force.

The normal to both surfaces at the contact point makes an angle equal to

$$\psi_0 = sin^{-1}\frac{y_0}{\Delta R}$$

(5)

with the gravity vector, here, $\Delta R = R_w - R_r$. For most of the practical cases the lateral displacement is assumed, not to be affect by the contact geometry.

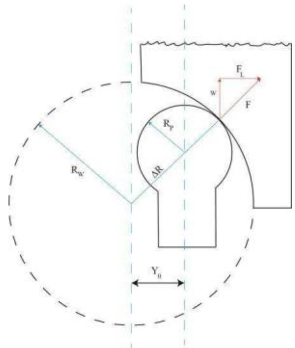

Fig. 5 Approximate wheel-rail geometry: at centered position

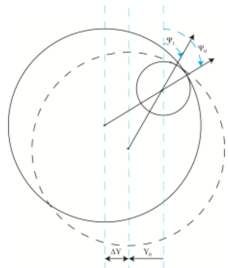

Fig. 6 Approximate wheel-rail geometry: at laterally displaced position

The wheel is assumed to experience a constant gravitational force (vertical) of "W" neglecting the quasi-static load transfer unbalances.

Now by the vector laws,

$$F_L = W tan\psi$$

$$F_L = W\frac{y}{\sqrt{\Delta R^2 - y^2}}$$

Differentiating with respect to y gives,

$$K_L = \frac{dF_L}{dy} = \frac{W}{\Delta R}\left(\frac{1}{1-(\frac{y_0}{\Delta R})^2}\right) \qquad (6)$$

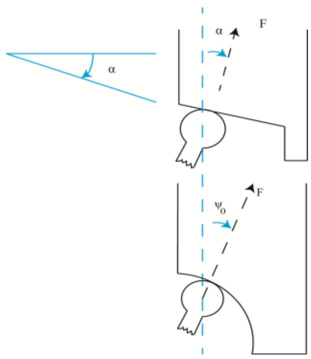

Fig. 7 Choice of the contact angle to conform to the conicity

The rails are laid at an angle of $\alpha$ this is done so that the wheel conicity will directly transfer the loads along the rail. These considerations demands $\alpha = \psi_0$ as,

$$K_L = \frac{W}{\Delta R} sec^3 \alpha \tag{7}$$

This value for the gravitational stiffness in the lateral direction [5].

A fixed frame of reference ( $\xi, \eta, \zeta$) as shown in the Fig. 1 is considered to be located at the wheelset plane, the track axis at a distance of $O_c$ (coach's center of mass) in order to determine the relative displacement of elements of the multi-body system and we can use the position vectors to represent component locations.

The central suspensions strokes in the three axis are given by:

$$\begin{aligned}
\Delta x_c &= -(\psi_c - \psi_{bj})(\pm d_s) \\
\Delta y_c &= y_c + h_{cc}\phi_c + (-1)^{j+1} l\psi_c - y_{bj} + h_{cb}\phi_{bj} \\
\Delta z_c &= (\phi_c - \phi_{bj})(\pm d_s)
\end{aligned} \tag{8}$$

The axle's stroke in the three axis are:

$$\begin{aligned}
\Delta x_0 &= -(\psi_{bj} - \psi_i)(\pm d_0) \\
\Delta y_0 &= y_{bj} + (-1)^{j+1} a\psi_{bj} + h_{ob}\phi_{bj} - y_i \\
\Delta z_0 &= (\pm d_0)\phi_{bj}
\end{aligned} \tag{9}$$

A Lagrange's method is applied to establish the equations of motion; the method can be stated as follows [6]:

$$\frac{d}{dt}\left[\frac{\partial(E-V)}{\partial \dot{q}_k}\right] - \frac{\partial(E-V)}{\partial q_k} + \frac{\partial D}{\partial \dot{q}_k} = Q_k$$
(10)

Where, $q_k$ - $\dot{q}_k$ represents the generalized co-ordinate and speed; $E$ represents the kinetic energy of the system; $V$ represents the potential energy of the system; $D$ is the energy dissipation function. These functions can be represented as:

$$E = \frac{1}{2}m_c\dot{y}^2 + \frac{1}{2}I_{cz}\dot{\psi}_c^2 + \frac{1}{2}I_{cx}\dot{\phi}_c^2 + \frac{1}{2}m_b\sum_{j=1}^{2}\dot{y}_{bj}^2 + \frac{1}{2}I_{bz}\sum_{j=1}^{2}\dot{\psi}_{bj}^2 + \frac{1}{2}I_{bx}\sum_{j=1}^{2}\dot{\phi}_{bj} + \frac{1}{2}m_0\sum_{i=1}^{4}\dot{y}_i^2 + \frac{1}{2}I_{oz}\sum_{i=1}^{4}\dot{\psi}_i^2$$

$$\begin{aligned}
V &= k_{cy}\sum_{j=1}^{2}(y_c + h_{cc} + (-1)^{i+1} l\psi_c - y_{bj} + h_{cb}\phi_{bj})^2 \\
&k_{cx}\sum_{j=1}^{2}[(\psi_c - \psi_{bj})(\pm d_c)]^2 + k_{cz}\sum_{j=1}^{2}[(\phi_c - \phi_{bj})(\pm d_c)^2] \\
&+k_{oy}\sum_{j=1}^{2}\sum_{i=1}^{4}(y_{bj} + (-1)^{i+1} a\psi_{bj} - h_{ob}\phi_{bj} - y_i)^2 + \\
&k_{ox}\sum_{j=1}^{2}\sum_{i=1}^{4}[(\psi_{bj} - \psi_i)(\pm d_0)]^2 + k_{oz}\sum_{j=1}^{2}[\phi_{bj}(\pm d_0)]^2 \\
D &= \rho_{cy}\sum_{j=1}^{2}(\dot{y}_c + h_{cc}\dot{\phi}_c + (-1)^{j+1} l\dot{\psi}_c - \dot{y}_{bj} + h_{cb}\dot{\phi}_{bj})^2 \\
&+\rho_{cx}\sum_{j=1}^{2}[(\dot{\psi}_c - \dot{\psi}_{bj})(\pm d_c)]^2 + \rho_{cz}\sum_{j=1}^{2}[(\dot{\phi}_c - \dot{\phi}_{bj}(\pm d_c)]^2 \\
&+\rho_{oz}\sum_{j=1}^{2}\sum_{i=1}^{2}[\dot{\phi}_{bj}(\pm d_0)]^2
\end{aligned} \tag{11}$$

The generalized contact forces in the $y_i$ and $\psi_i$ direction can be given as:

$$\begin{aligned}
Q_{yi} &= -2\chi Q\left(\frac{\dot{y}}{v} - \psi_i\right) - c_g(1-\chi_s)(y_i - \eta_i) \\
Q_{\psi i} &= -2\chi Q_e\left[\frac{\gamma}{r_0}(y_i - \eta_i) + \frac{e}{v}\dot{\psi}_i\right] + 2\chi_s Q r_0\left(\frac{\dot{y}_i}{v} - \psi_i\right)
\end{aligned} \tag{12}$$

Where, $C_g$ is the lateral gravitational stiffness & $\eta_i$ is the track deviation in the transverse direction.

The input to the model consists of irregularities of the track in the transverse direction, text shows that these irregularities can be represented by periodic function [1], the exact representation of the irregularity is impossible to determine, as such statistical method can be used to determine the track irregularity. In our case it is represented as

$$\eta_{1,2} = \eta_0 cos[2\pi(vt + l \pm a)/L]$$
$$\eta_{3,4} = \eta_0 cos[2\pi(vt - l \pm a)/L]$$

for all the four wheels.

Here $v$ is the coach's circulations velocity, $L$ is the wavelength of periodic irregularity and $\eta_0$ is the track deviation in the transverse direction. Texts show that short wavelength periodic irregularities of a track is normally of the range of 30-300mm, such irregularities are known as corrugation. The amplitude varies from a few hundredth of mm to 1mm [7].

The complete physical parameters of the model are tabulated in the appendix of this paper.

### 3. Railway Vehicle Response

The vehicle is modeled with data as provided in the appendix.The EOM of the system can be represented in a more general form as:

$$[M]_{17\times17}\{\ddot{\mathbf{q}}\} + [C]_{17\times17}\{\dot{\mathbf{q}}\} + [K]_{17\times17}\{\mathbf{q}\} = \{F(t)\} \quad (13)$$

Where $[M]$ is the mass matrix diagonal and positive definite, $[C]$ and $[K]$ are the damping and stiffness matrices respectively, $\{\ddot{\mathbf{q}}\}$ – acceleration vector, $\{\dot{\mathbf{q}}\}$ velocity vector & $\{\mathbf{q}\}$ is the generalized co-ordinate and $\{F(t)\}$ is the external force vector.

$$q = [y_c \psi_c \phi_c y_{b1} \psi_{b1} \phi_{b1} y_{b2} \psi_{b2} \phi_{b2} y_1 \psi_1 y_2 \psi_2 y_3 \psi_3 y_4 \psi_4] \quad (14)$$

An initial condition of $\{q\} = \{0\}_{1\times17}$ is considered. The vehicle critical velocity in the longitudinal direction is arbitrarily chosen at 50 m/sec (180 Km/hr). The plots for the coach bogie and the wheel set's degree of freedom are illustrated in the following figures.

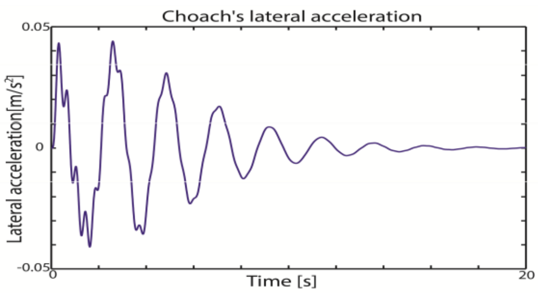


Fig. 8 Coach's lateral acceleration ($y_c$)

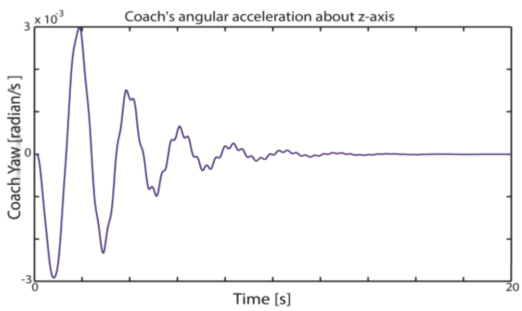


Fig. 9 Coach's angular acceleration ($\psi_c$) about Z-axis

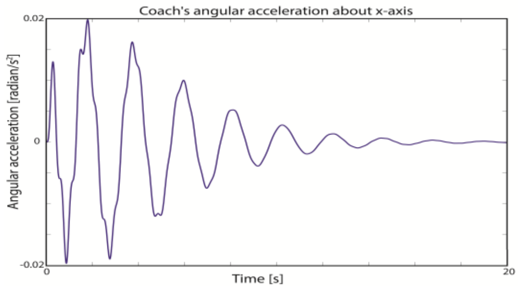


Fig. 10 Coach's angular acceleration ($\phi_c$) about X-axis

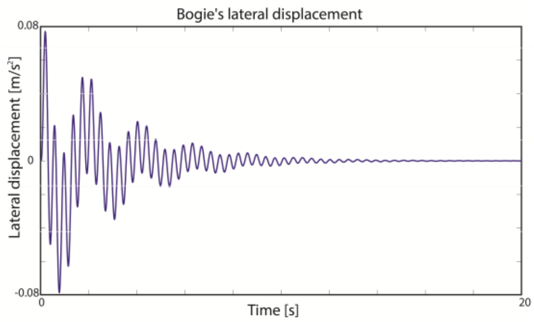


Fig. 11 Bogie's lateral acceleration ($y_{b1}$)

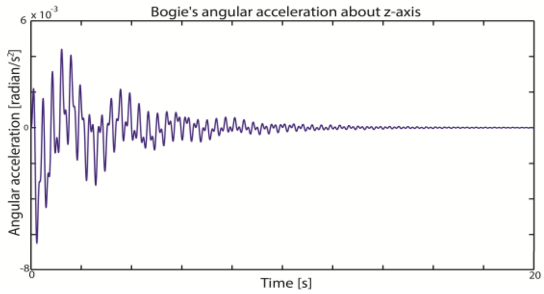


Fig. 12 Bogie's angular acceleration ($\psi_{b1}$) about Z-axis

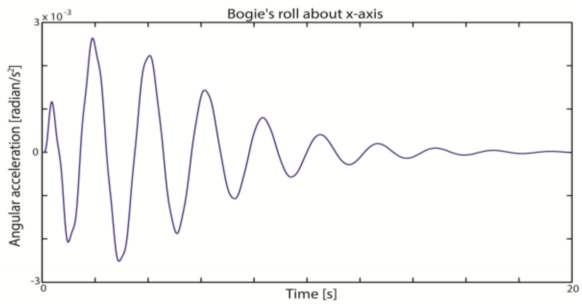


Fig. 13 Bogie's angular acceleration ($\phi_{b1}$) about X-axis

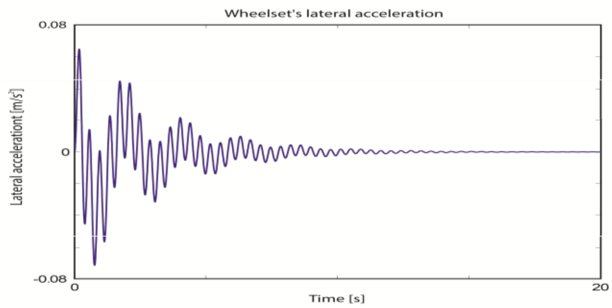


Fig. 14 Wheel set's lateral acceleration ($y_1$)

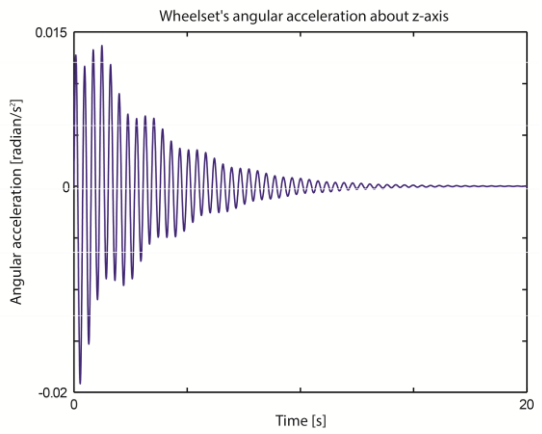


Fig. 15 Wheel set's angular acceleration ($\psi_1$) about Z-axis.

## 4. The vehicle's critical speed

The vehicle's critical speed is calculated by calculating the eigenvalues of the characteristic matrix. The system tends to be stable as long as all the eigenvalues of the characteristic matrix has a negative real part; asymptotically stable for eigenvalues with real part equals to zero, & tends to be unstable for positive real part of the eigenvalues. We have developed MATLAB program for calculation of eigenvalues at different circulation speeds and the critical velocity comes out to be 61 m/sec i.e. 219.5 Km/hr. We have plotted the wheel set's response for different velocity levels.

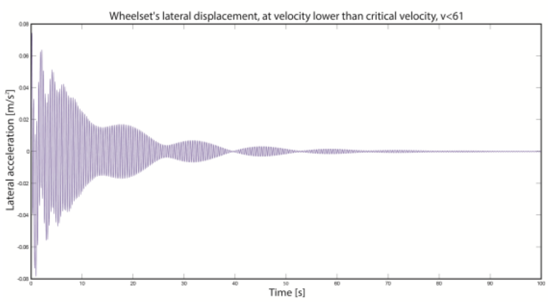


Fig. 16 Wheel set's lateral acceleration about Z-axis at $v < v_{critical}$

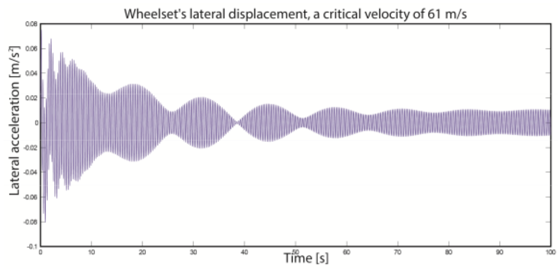


Fig. 17 Wheel set's lateral acceleration about Z-axis at $v = v_{critical}$

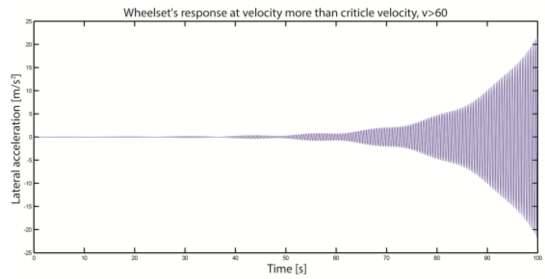


Fig. 18 Wheel set's lateral acceleration about Z-axis at $v > v_{critical}$

## 5. Critical velocity with varying wheel trade conicity

The vehicle's critical speed depends significantly on the wheel rail profile. Although the wheel rail profile approximation in our case is a crude one, we can still estimate the effect of wheel wear on the critical velocity of the vehicle as illustrated in the following plot.

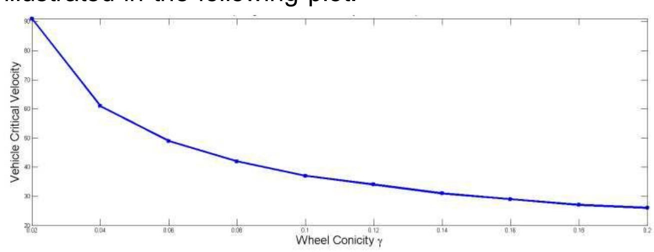


Fig. 19 Variation of critical velocity with the wheel conicity

## 6. Conclusion

This paper presents a computational dynamics model for rail vehicle behavior and vehicle behavior at different velocities. The model can be used for study of railway vehicle response in different scenarios. A study of railway vehicle's critical velocity based on wheel conicity has ben done. The effect wheel conicity is found out to be very significant as such a detailed study of wearing behavior of the wheel profile is very important and can be a future scope. We are working towards more detailed mathematical model for wheel rail contact and track irregularities model. Another aspect we are currently working is towards generalization of this model to fit it in a generalized mathematical framework.

## 8. References

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## 9. Appendix: Vehicle's modeling data

| | |
|---|---|
| Body case mass | $m_c = 30760$ kg |
| Bogie mass | $m_b = 23000$ kg |
| Wheelset mass | $m_o = 1410$ kg |
| Body case moment of inertia | $I_{cx} = 5396$ kg$m^2$; $I_{cz} = 1661732$ kg$m^2$ |
| Bogie moment of inertia | $I_{bx} = 2240$ kg$m^2$; $I_{bz} = 2965$ kg$m^2$ |
| Axles moment of inertia | $I_{oy} = 980$ kg$m^2$; $I_{ox} = 100$ kg$m^2$ |
| Central suspension stiffness | $k_{cx} = 133$ kN/m; $k_{cy} = 133$ kN/m; $k_{cz} = 473$ kN/m |
| Axles suspension stiffness | $k_{ox} = 0$ kN/m; $k_{oy} = 25$ kN/m; $k_{oz} = 18$ kN/m |
| Central suspension damping | $\rho_{cx} = 0$ kN/m/s; $\rho_{cy} = 25$ kN/m/s; $\rho_{cz} = 18$ kN/m/s |
| Damping of the axles suspension | $\rho_{oz} = 367$ kN/m/s |
| Wheel tread radius | $r_0 = 0.460$ m |
| The track's gauge | $2e = 1435$ m |
| The distance between wheel base | $2a = 2560$ m |
| The distance between bogies | $2l = 17.2$ m |
| The distance between the central suspension's springs | $2d_c = 2$ m |
| The distance between the axle's suspension springs | $2d_0 = 2$ m |
| The distance case centre- central suspension | $h_{cc} = 1.24$ m |
| The distance axles suspension- bogie center | $h_{ob} = .01$ m |
| The distance central suspension- bogie centre | $h_cb = .06$ m |
| Load on the wheel | $Q = 51250$ N |
| The creepage coefficient | $\chi = 190$ |
| The spin creepage coefficient | $\chi_s = .83$ |
| The effective wheel conicity | $\gamma = 0.04$ |
| The maximum testing speed | $v_{max} = 50$ m/s |