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A mathematical model to predict heat transmission through a glass window with a venetian blind installed

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Abstract

This article is about the development of a mathematical model for predicting heat transmission through a glass window with a curved venetian blind installed. A glass window was considered as a specular optical element. The venetian blind was considered as a non-specular optical element and treated as an effective layer. The predicted results from the mathematical model were the solar heat gain coefficient (SHGC) and the overall heat transfer coefficient. The study emphasized on developing more accurate models for the convective heat transfer coefficient used in various points in the fenestration. The predicted results were compared with the results from the experiments to verify the accuracy. It was found that the agreement between the predicted results and the experimental results was good. It was also found that the SHGC of the glass window with a venetian blind installed was dependent on the optical properties, slat angle and solar profile angle.

Keywords: Heat transmission, glass window, venetian blind, convective heat transfer coefficient, mathematical model.

1. Introduction

A large portion of the energy used in the air conditioning system for the commercial building is usually used to overcome the heat gain from the solar radiation passing through the building envelope in the part of the glass window. One way to reduce the heat gain through the window is to install a venetian blind as an indoor shading device. Much work has been done about heat transmission through a glass window with a venetian blind installed [1-6]. Chaiyapinunt and Worasinchai [7-8] have developed a mathematical model to calculate the shortwave and longwave optical properties for a glass window with a curved venetian blind installed. Chaiyapinunt and Khamporn [13, 14] have studied on the heat gain through the glass window with a curved venetian blind installed in term of solar heat gain coefficient in the shortwave part (ShW SHGC) and the solar heat gain coefficient in the longwave part (LoW SHGC). In this article, the study emphasized on developing more accurate models for the convective heat transfer coefficient used in various points in the fenestration. The predicted results were verified by comparing with the result from the experiments [9].

2. Mathematical model for a glass window with a venetian blind

The heat transmission through a glass window with a venetian blind installed can be written as

$$q = U \cdot (T_{out} - T_{in}) + SHGC \cdot I \tag{1}$$

where q is the heat gain, W/m^2. U is the overall heat transfer coefficient, $W/(m^2K)$. T_{out} is the outdoor air temperature, K. T_{in} is the indoor air temperature, K. $SHGC$ is the solar heat gain coefficient. I is the total incident radiation, W/m^2.

The first term on the right hand-side of Eq.(1) represents the heat gain caused by heat transfer in the part of conduction and convection. The last term represents the heat gain resulting from the solar radiation.

To calculate the heat gain from the solar radiation, the solar radiation can be separated into the direct solar radiation, the diffuse sky radiation, the diffuse reflected-ground radiation, and the direct reflected-ground solar radiation. The heat gain from the solar radiation can be written as

$$SHGC \cdot I = SHGC_D I_D + SHGC_{d,sky}\frac{I_d}{2} + SHGC_{d,ref}\,\rho\frac{I_d}{2} + SHGC_{D,r} I_{D,r} \tag{2}$$

where $SHGC_D$ is the SHGC for the direct solar radiation. I_D is the direct solar radiation, W/m^2. $SHGC_{d,sky}$ is the SHGC for sky diffuse solar radiation. I_d is the diffuse solar radiation, W/m^2. $SHGC_{d,ref}$ is the SHGC for reflected diffuse solar radiation. ρ is the ground reflectance. $SHGC_{D,r}$ is the SHGC for reflected direct solar radiation. and $I_{D,r}$ is the reflected-ground direct solar radiation, W/m^2.

A glass window is considered as a specular element, while a venetian blind is considered as a non-specular element. Therefore, determining the heat transmission through a glass window with a venetian blind installed is quite complex. The heat transmission for the glass window and a venetian blind depends on both the incident angle and the azimuth angle as shown in Fig.1. The solar heat gain coefficient for the direct solar radiation can be written as

$$SHGC(\theta,\varphi) = T^{fH}(\theta,\varphi) + \sum_{i=1}^{M} N_i A_i^f(\theta,\varphi) \tag{3}$$

where T^{fH} is the directional-hemispherical front transmittance. N_i is the inward-flowing fraction of the absorbed energy for ith layer in the system. A^f_i is the directional absorptance of the ith layer in the system. θ is the incident angle, ^{o}. φ is the azimuth angle, ^{o}.

For this study, the first term on the right hand-side of Eq.(3) is the value of SHGC in the part of shortwave radiation and the last term is the value of SHGC in the part of longwave radiation.

To simplify Eq.(3), the solar profile angle is defined. The relationship between the solar profile angle and the incident angle and azimuth angle can be written as

$$\phi_s = \tan^{-1}(\sin\varphi \tan\theta) \quad (4)$$

where ϕ_s is the solar profile angle, ^{o}.

The SHGC in Eq.(3) can now be written in the function of the solar profile angle instead of the incident angle and azimuth angle.

$$SHGC(\phi_s) = T^{fH}(\phi_s) + \sum_{i=1}^{M} N_i A^f_i(\phi_s) \quad (5)$$

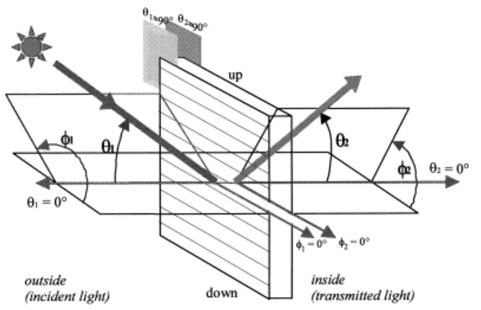

Fig.1 The incident angle and the azimuth angle of the fenestration system.

The SHGC for the diffuse solar radiation is determined by using the hemispherical average of SHGC for direct solar radiation which is divided into the SHGC for sky diffuse solar radiation and the SHGC for reflected diffuse solar radiation.

3. Heat transmission

The heat transmission through the glass window with a venetian blind installed can be calculated from the expression in Eqs.(1) and (5). The optical properties (transmittance, reflectance and absorptance) of the fenestration system can be calculated according to the method suggested by Chaiyapinunt and Worasinchai [7-8]. In the part of longwave radiation, the 1-D heat balance method is used for the analyzing. Fig. 2 shows the heat resistance network of the fenestration system having a double pane glass window with a venetian blind treated as an effective layer. The heat resistance network composes of the heat resistance in term of conduction, convection and radiation. The overall heat transfer coefficient can be written as

$$U_{total} = \frac{(R_{3-4} + R_{4-5} + R_{3-5})}{\left[R_{3-5}(R_{1-2} + R_{2-3} + R_{3-4} + R_{4-5}) + (R_{1-2} + R_{2-3})(R_{3-4} + R_{4-5})\right]} \quad (6)$$

where $R_{i,j}$ is the heat resistance between node i and node j. The heat resistance between node i and node j can be written as

$$R_{1-2} = \frac{1}{h_{c,out} + h_{r,out}} + \frac{0.5d_2}{k_2} \quad (7)$$

$$R_{2-3} = \frac{1}{h_{c,gap} + h_{r,gap}} + \left(\frac{0.5d_2}{k_2} + \frac{0.5d_3}{k_3}\right) \quad (8)$$

$$R_{3-4} = \frac{1}{h_{c,gapb} + h_{r,gapb}} + \left(\frac{0.5d_3}{k_3} + \frac{0.5d_4}{k_4}\right) \quad (9)$$

$$R_{4-5} = \frac{1}{h_{c,in} + h_{r,in}} + \frac{0.5d_4}{k_4} \quad (10)$$

$$R_{3-5} = \frac{1}{h_{c,across,3-5} + h_{r,across,3-5}} + \frac{0.5d_3}{k_3} \quad (11)$$

where $h_{c,x}$ is the convective heat transfer coefficient in the x space (out = outside air condition, gap = air in the gap between two glass layers, gapb = air in the gap between a glass layer and an effective layer, in = inside air condition, and across = air from node ith to node jth), $W/(m^2K)$. $h_{r,x}$ is the radiative heat transfer coefficient in the x space, $W/(m^2K)$. d_i is the thickness of the layer ith, m. k_i is the thermal conductivity of the layer ith, $W/(m\text{-}K)$.

The inward-flowing fraction of the fenestration system can be written as

$$N_2 = U_{total} R_{1-2} \quad (12)$$

$$N_3 = U_{total}(R_{1-2} + R_{2-3}) \quad (13)$$

$$N_4 = U_{total}\left[(R_{1-2} + R_{2-3} + R_{3-4}) + \frac{R_{3-4}(R_{1-2} + R_{2-3})}{R_{3-5}}\right] - \frac{R_{3-4}}{R_{3-5}} \quad (14)$$

where N_i is the inward-flowing fraction of the layer ith.

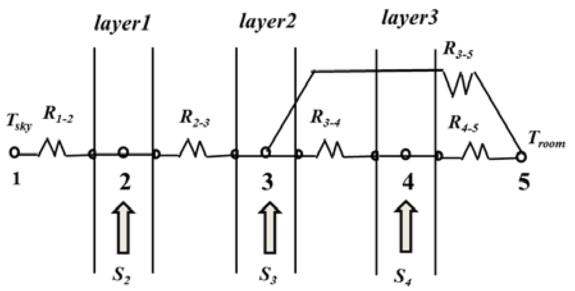

Fig. 2 The heat resistance network of the fenestration system having a double pane window with a venetian blind installed which considered as the effective layer.

The convective heat transfer coefficient is separated into five types; the convective heat transfer coefficient for the outside air condition, the convective heat transfer coefficient for the air between two glass layers, the convective heat transfer coefficient for the air between the glass layer and the effective layer, the convective heat transfer coefficient for the inside air condition, and the convective heat transfer coefficient air from node *i*th to *j*th layer (across the effective layer). In this study, the new correlations for the convective heat transfer coefficients are proposed.

The convective heat transfer coefficient for the outside air condition can be expressed by a correlation proposed by Yazdanian and Klems [10] which can be written as

$$h_{c,out} = \sqrt{\left[c_t\left[\Delta T\right]^{1/3}\right]^2 + \left[av^b\right]^2} \tag{15}$$

where c_t is the turbulent natural convection constant defined in Table.1, *W/(m²K⁴ᐟ³)*. ΔT is the temperature difference between the exterior glass layer and the outside air, *K*. *a* and *b* are constant defined in Table.1. *v* is the free steam wind speed at a 10-meter height, *m/s*.

Table. 1 MoWiTT constant [10].

	c_t, $W/(m^2K^{4/3})$	*a*, $W/(m^2K(m/s)^b)$	*b*
Windward	0.84±0.015	2.38±0.036	0.89±0.009
Leeward	0.84	2.86±0.098	0.617±0.017

The convective heat transfer coefficient for the air between two glass layers can be written as

$$h_{c,gap} = \frac{k_{air} \cdot Nu}{w} \tag{16}$$

$$Nu = \left[1 + \left(0.0303Ra^{0.402}\right)^{11}\right]^{0.091} \tag{17}$$

$$Ra = Gr \cdot \Pr \tag{18}$$

$$Gr = \frac{g\beta\rho^2 w^3 \Delta T}{\mu^2} \tag{19}$$

where k_{air} is the air thermal conductivity, *W/(m-K)*. *Nu* is the Nusselt number. *w* is the width of the gap, *mm*. *Ra* is the Rayleigh number. *Gr* is the Grashoff number. Pr is the Prandtl number. *g* is the gravitational acceleration, m/s^2. β is the thermal expansion coefficient of air. μ is the dynamic viscosity of air, Ns/m^2. ρ is the density of air, kg/m^3. ΔT is the temperature difference across the gap, *K*.

The convective heat transfer coefficient for the air between the glass layer and the effective layer, the convective heat transfer coefficient for the inside air condition, and the convective heat transfer coefficient air across the effective layer are adopted from a correlation developed from Wright et al. [11] The resistance network used to model convective heat transfer for three points in the fenestration system is shown in Fig. 3.

The convective heat transfer coefficient for the air between the glass layer and the effective layer can be written as

$$h_{c,gapb} = \frac{k_{air}}{b} \tag{20}$$

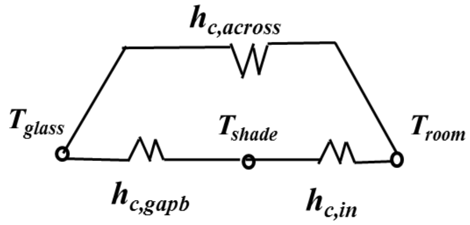

Fig. 3 The resistance network used to model convective heat transfer at surfaces exposed to the inside air condition.

where *b* is the space between the glass and effective layers, *m*.

The convective heat transfer coefficient for the inside air condition including the effect of slat angle can be written as

$$h_{c,in} = h_c\left(2 - \exp\left(-4.6\frac{b}{0.1}\right)\right) \cdot \left(1 + 0.2\left|\sin\left(2\phi_b\right)\right|\right) \tag{21}$$

where h_c is the convective heat transfer coefficient for the inside air condition expressed as Eq.(22), $W/(m^2K)$. ϕ_b is the blind slat angle, ^{o} as shown in Fig.4.

$$h_c = \frac{k_{air} \cdot Nu}{H} \tag{22}$$

$$Nu = 0.56\left(Ra_H \sin\gamma\right)^{1/4} \quad ; \; Ra_H \le Ra_{cv} \tag{23}$$

$$Nu = 0.13\left(Ra_H^{1/3} - Ra_{cv}^{1/3}\right) + 0.56\left(Ra_{cv}\sin\gamma\right)^{1/4} \quad ; \quad Ra_H > Ra_{cv} \tag{24}$$

$$Ra_{cv} = 2.5\times10^5\left(\frac{e^{0.72\gamma}}{\sin\gamma}\right) \tag{25}$$

$$Ra_H = \frac{\rho^2 H^3 g C_p \left|T_{b,n} - T_{in}\right|}{T_{m,f}\,\mu k_{air}} \tag{26}$$

$$T_{m,f} = T_{in} + \frac{1}{4}\left(T_{b,n} - T_{in}\right) \tag{27}$$

where *H* is the glass layer height, *m*. Ra_H is the Rayleigh number based on the height. γ is the tilt angle of window, ^{o}. C_p is the specific heat of air, *J/(kg-K)*. $T_{m,f}$ is the mean film temperature, *K*. $T_{b,n}$ is the internal glazing layer surface temperature, *K*.

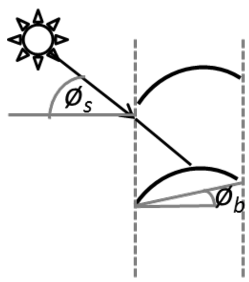

Fig.4 The slat angle of the curved venetian blind and the solar profile angle.

The convective heat transfer coefficient for the air across the effective layer can be written as

$$h_{c,across,t-j} = h_c\left(1-\exp\left(-4.6\frac{b}{0.1}\right)\right) \quad (28)$$

The radiative heat transfer coefficient can be determined by considering the venetian blind as the diathermanous layer and using the method developed by Collins and Wright [12] and Chaiyapinunt and Khamporn [13,14].

4. Verification

The predicted results from the developed mathematical model (using Eqs. (1)-(28)) are compared with the results from the experiment conducted by Collins and Harrison [9] to verify the accuracy of the mathematical model. The experiment was performed to measure net heat gain through glazing system using solar calorimeter. In this study, the verification is only performed on the value of solar heat gain coefficient.

The fenestration system chosen for this study were a single wood-frame window composed of the two 3 mm clear glass with a 13 mm air gap and the aluminum blind. The optical properties of 3 mm clear glass are shown in Table.2.

Table.2 The optical properties of 3 mm clear glass at normal solar incidence [9].

τ	α	ρ	ε
0.84	0.09	0.07	0.84

The venetian blind had a slat width of 25.4 mm, a slat thickness of 0.17 mm, a radius of curvature of 52.3 mm. The slats were separated by a pitch of 22.2 mm. Two different colors of the venetian blind; white and black blinds were selected to study. The white blind slat had a solar absorptance of 0.32 (reflectance of 0.68) and hemispherical emissivity of 0.75. The black blind slat had a solar absorptance of 0.90 (reflectance of 0.10) and hemispherical emissivity of 0.89. The conductivity of the slat was 120 W/m-K. The venetian blind was installed with a 30 mm distance from the inner glass surface to the center of the venetian blind.

Three different slat angles; -45, 0 ,and 45 degrees and two incident angles; 30 and 45 degrees were performed in this study.

To compare the results from the developed mathematical model with the results from the experiment performed by Collins and Harrison, the values of SHGC were used. For the developed mathematical model, the SHGC could be determined from Eq.(2).

The weather data composed of the direct incident radiation and the diffuse radiation are shown in Table. 3. The ground reflectance of 0.2 is chosen in this study.

Table. 4 shows the comparison between the predicted results from the developed mathematical model and the results from the experiment for a variety of blind colors, incident angles, slat angles and weather data. The agreement of the results is good. The discrepancy is less than 7% in all cases. The average deviation is around 0.2-3%.

Table.3 The direct incident radiation and the diffuse radiation used in the verification.[9,13]

blind color	incident angle,°	slat angle,°	I_D, W/m^2	I_d, W/m^2
white	30	0	701.61	164.58
white	45	0	504.48	163.42
white	30	45	759.53	166.73
white	45	45	606.44	181.14
white	30	-45	730.72	160.40
white	45	-45	592.67	167.16
black	30	0	754.73	177.03
black	30	45	650.11	240.45
black	45	45	643.21	160.82
black	30	-45	719.82	191.34

Table.4 The results from the developed mathematical model and the experiment.

blind color	incident angle,°	slat angle,°	$SHGC_e$	$SHGC_m$	deviation, %
white	30	0	0.59±0.01	0.610	3.34
white	45	0	0.56±0.01	0.525	-6.27
white	30	45	0.46±0.01	0.456	-0.89
white	45	45	0.44±0.01	0.439	-0.27
white	30	-45	0.65±0.01	0.634	-2.41
white	45	-45	0.65±0.01	0.655	0.78
black	30	0	0.65±0.01	0.654	0.66
black	30	45	0.64±0.01	0.607	-5.17
black	45	45	0.64±0.01	0.600	-6.22
black	30	-45	0.68±0.01	0.687	0.97

Note: $SHGC_e$ is the SHGC from the experiment.
$SHGC_m$ is the SHGC from the developed mathematical model.

5. The variation of the solar heat gain coefficient

To study the variation of the SHGC for the direct solar radiation with the related parameters, two 3 mm clear glass with a 13 mm air gap and the aluminum blinds with two different colors; black and white and three different slat angles; -45, 0, and 45 degrees were chosen. The optical properties of the glass windows and the venetian blinds are similar to the ones used in the experiment.

The predicted SHGC from the developed mathematical model for the plain double glass window and the glass window with the white and black blind are shown in Figs.4 and 5, respectively.

From Figs. 4 and 5, it can be seen that the SHGC is dependent on the solar profile angle and slat angle. The SHGC of the plain glass window has the largest value at any solar profile angle compared to the glass window with a venetian blind installed because a venetian blind can block some part of solar radiation. The incident angle is 90 degree at noon and then is lower until the position of the sunbeam acting on the glass window closed to horizon which the solar profile angle is nearly 0 degree. For the blind slat angle of -45 degree, the value of SHGC is slightly increased to the maximum value at the solar profile angle about 45 degree and then is decreased as the solar profile angle are increased till 90 degree. For the blind slat angle of 0 and 45 degrees, the value of SHGC is slightly decreased as the solar profile angle is increased at any considered solar profile angle. It is obviously seen that the value of SHGC at the blind slat angle 45 degree is the smallest compared to the other two slat angles because the blind can block a large amount of the solar radiation.

It is also found that all values of SHGC of the white venetian blind are smaller than the black venetian blind because of having the lower value of slat absorptance which leading to the higher value of slat reflectance.

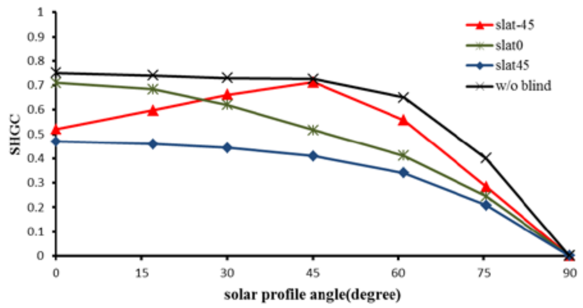

Fig. 4 The predicted solar heat gain coefficient of the fenestration system using the double clear glass and the white venetian blind in different slat angles.

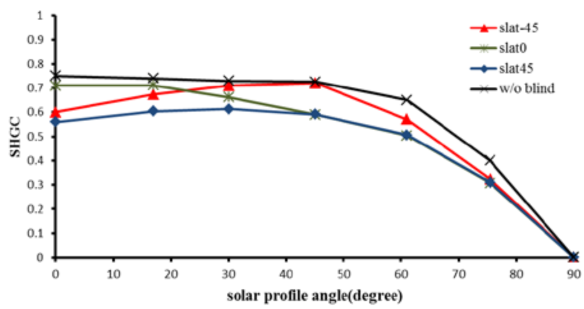

Fig. 5 The predicted solar heat gain coefficient of the fenestration system using the double clear glass and the black venetian blind in different slat angles.

The value of SHGC is considered separately as ShW SHGC and LoW SHGC and are shown in Figs. 6 to 9.

From Figs. 6 and 7, setting the blind slat angle at 45 degree gives the smallest value of ShW SHGC compared to ShW SHGC of the other two slat angles. For the blind slat angle of 0 degree, a portion of solar radiation can pass through the fenestration system at the range of solar profile angle of 0 to 45 degrees leading to the large value of ShW SHGC. Setting the blind slat angle at -45 degree, the value of ShW SHGC is increased to the maximum value at the solar profile angle of 45 degree and then is decreased as the solar profile angle is increased. Unlike the value of SHGC, it is found that the value of ShW SHGC of black venetian blind are all smaller than the white venetian blind since the black venetian blind has the value of reflectance lower than the white venetian blind. Moreover, it is obviously seen the difference of the value of ShW SHGC between the glass window with the blind of different color when the blind slat angle is at 45 degree.

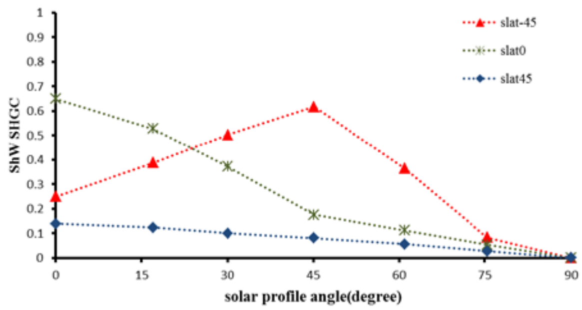

Fig. 6 The predicted shortwave SHGC of the fenestration system using the double clear glass and the white venetian blind in different slat angles.

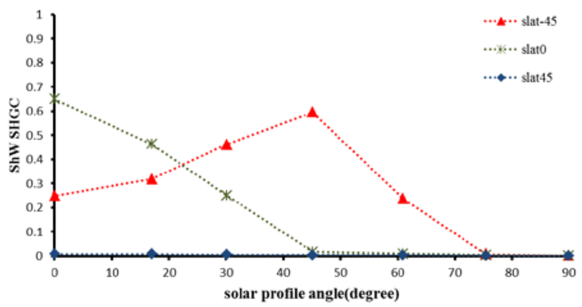

Fig. 7 The predicted shortwave SHGC of the fenestration system using the double clear glass and the black venetian blind in different slat angles.

Figs. 8 and 9 show the predicted LoW SHGC of the fenestration system. When the blind is set in the position that block a large amount of solar radiation, the solar radiation is then absorbed and changes into the longwave radiation. For this reason, it is found that the glass window with blind slat angle of 45 degree gives the largest value of LoW SHGC.

Setting the blind slat angle at 0 degree, the LoW SHGC has the largest value at the solar profile angle of 45 degree because at this sun position there is the largest amount of solar radiation strike on the blind. On the other hand, setting the blind slat angle at -45 degree, the LoW SHGC has the smallest value at the solar profile angle of 45 degree because the large amount of solar radiation can pass through the blind therefore there is a small amount of solar radiation absorbed on the blind.

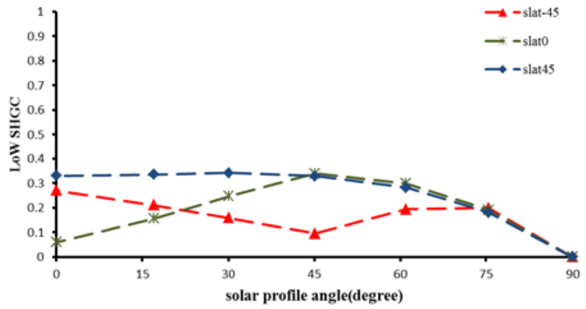

Fig. 8 The predicted longwave SHGC of the fenestration system using the double clear glass and the white venetian blind in different slat angles.

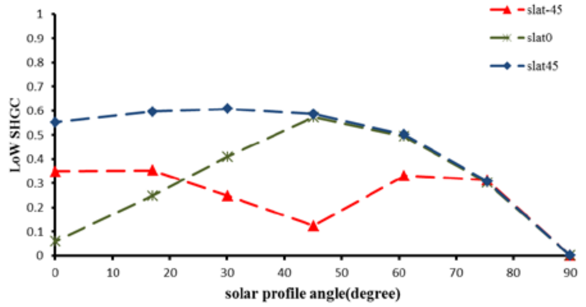

Fig. 9 The predicted longwave SHGC of the fenestration system using the double clear glass and the black venetian blind in different slat angles.

It is found that LoW SHGC is directly proportional to the slat absorptance (the fenestration system using the black venetian blind gives the higher value of LoW SHGC than the white venetian blind).

6. Conclusion

In this study, it was found that using the new proposed expressions for the convective heat transfer coefficient in the developed mathematical model gave the accurate predicted results of the SHGC.

Installing a venetian blind to the glass window could reduce heat transmission in the term of SHGC. It was also found that the value of SHGC depended on the slat angle, solar profile angle and optical properties of a blind slat.

Among three different slat angles, setting the blind slat angle at 45 degree could reduce the largest value of SHGC because the blind could block the largest amount of solar radiation. Moreover, a white venetian blind could reduce SHGC more than the black venetian blind. Setting the blind slat angle at 45 degree and using a white venetian blind would result in the largest value of LoW SHGC and the smallest value of ShW SHGC.

7. Acknowledgement

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TSF009

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