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Distributed Parameter Multi-Model Predictive Control of Heat Conduction in Rod

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Abstract

In general, the rate of heat conduction in materials depends on a thermal conductivity of a material. The high value of thermal conductivity is mean the heat energy can pass through the materials faster than the low value of thermal conductivity materials. However, the thermal conductivity is a kind of a variable parameter due to the value temperature in materials. For this reason, the variation of thermal conductivity value is a main source of uncertainty in heat conduction control problem. This work proposed the method of distributed parameter system for separating the aluminum rod into semi dimension then, develop the state space model of low thermal conductivity value and high thermal conductivity case. Each state space models use for synthesizing the model predictive control algorithm in each of thermal conductivity value. The both of model predictive control in each thermal conductivity value are combined into the multi-model predictive controller by using weighting method. Simulation result shows the benefit of a multi-model predictive controller in the case of a wide range operating condition of the conduction process better than the conventional model predictive control.

***Keywords**:* Multi-model predictive control, Distributed parameter system, Heat equation

1. Introduction

In heat conduction process, the rate of heat transfer through a material depend on the value of thermal conductivity. Therefore, the high value of thermal conductivity is mean the heat energy can transfer through the material faster than the low value of thermal conductivity material. However, in reality the thermal conductivity is a kind of variation parameter. When the temperature of a material has reached the high temperature, the value of thermal conductivity has increased depend on material temperature. Therefore, in the control process of heat transfer through the material is a kind of an uncertain system due to thermal conductivity.

The heat equation problem is a kind of partial differential equation (PDE) sometimes known as parabolic or hyperbolic equation.

Several techniques have been proposed in the literature for controlling a heat conduction problem, for example, distributed parameter control, sliding mode control, PDE back stepping control, gain scheduling control and predictive control.

In [1] have proposed the distribute parameter control method and implement to a batch fluidized bed dryer. In this proposed, the researchers is developed a lumped model base on the heat and mass transfer between solid and gas in the bed. Then, the distributed parameter model is used to predict the behavior of the system.

In [2] study the design of feedback control of hyperbolic distributed parameter systems. The method based on the first order hyperbolic partial differential equation using the method of characteristics. Simulation results show that this method can provide effective control for the systems modeled by a scalar PDE as well as a system of PDE.

In [3] have been proposed the method of sliding mode boundary control of a one-dimensional unstable heat conduction system modeled by a parabolic partial differential equation. The simulation result of this paper shows the ability of sliding mode control for handling the uncertainty of boundary condition.

In [4] study the structure of a controller based on back stepping control for controlling the one-dimensional heat equation with time-varying domain. From the simulation result, the proposed method of the full state-feedback control law is provided for the application of temperature regulation in heat conduction process.

In [5] have been proposed the novel fuzzy mixing gain scheduling (FMGS) strategy that is based on the idea of fuzzy weighting of local model values of certain control parameters depend on thermal load in the process of heat transfer thought the buildings wall material. The experiment shows the excellent performance and effectiveness of the proposed control in a heat conduction in buildings material.

In [6] have proposed the predictive control of parabolic PDE with unsteady state and flux boundary control. From the simulation result, the proposed method has ability to enforce stability in the infinite-dimensional closed-loop system.

From the literature, many researchers developed control based on heat transfer model or heat equation such as, model predictive control, full state feedback and distribute parameter control. The advantages of model-based control are reduced process setting time, the controller can reduced the energy input to the system depend on the accuracy of mathematic model and the model of system can be developed into prediction form for calculating the future of output behavior.

However, the main drawback of model-based controller is a variation of a system parameter due to operating condition.

For improving the performance of model-based controller in a wide range operating region, this paper proposed the idea of multi-model predictive control by using the fuzzy weighting and applied that to the heat conduction process in aluminum rod.

The paper is organized as follows. Firstly, the mathematic model based on heat conduction is described. Secondly, the design of model predictive control from the information of the mathematic model. Thirdly, fuzzy weighting method of weight the local model predictive control into global controller. Next, the simulation result is given to confirm the proposed method by comparison between conventional and the proposed method. Finally, the conclusion of this work is presented in the final section.

2. Mathematic model

This paper uses a simple case of heat conduction in aluminum rod as shown in Fig. 1 and assume the problem is a one-dimension heat transfer.

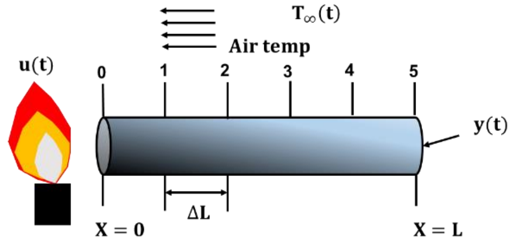

Fig. 1 One-dimension heat conduction problem.

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The heat source is applied to a rod at position $x=0$ and the output variable is a temperature at a tip of the rod $x=L$. The heat stored in aluminum rod is transfer to ambient according to convection mode. The thermal properties of aluminum can be seen in Table 1.

Table 1. Thermal properties of aluminum

Thermal property	value	unit
k	215	$W/m^2 \cdot K$
C_p	897	$kJ/kg \cdot K$
ρ	2702	kg/m^3

The aluminum rod is divide into five-part, when $\Delta L = 0.2$ and $L=1$. The cross section area (A_c) of aluminum rod is assumed to 0.0314. The mathematic model in each part of aluminum rod can be developed by using heat balance equation [7] as shown in Eq. (1).

$$\Delta E_{st} = E_{in} - E_{out} \tag{1}$$

When ΔE_{st} an energy is stored in control volume in each part of aluminum rod, E_{in} is a heat energy input to the control volume and E_{out} is a heat energy output from the control volume. Fig. 2 is show a heat balance in the first section of aluminum rod.

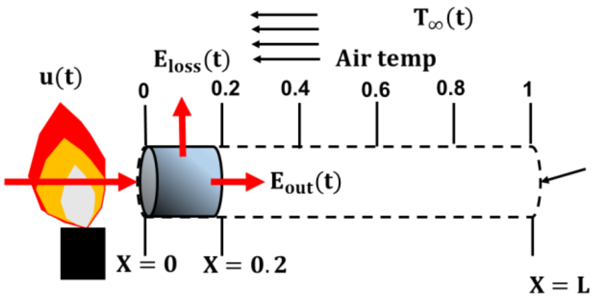

Fig. 2 Heat balance in the first part of aluminum rod.

The mathematic model of each part in aluminum rod can be written as Eqs. (2) – (6).

$$\frac{dT_1(t)}{dt} = -a_{11}T_1(t) + a_{12}T_2(t) + b_{11}u(t) + b_{12}T_\infty(t) \tag{2}$$

$$\frac{dT_2(t)}{dt} = a_{21}T_1(t) - a_{22}T_2(t) + a_{23}T_3(t) + b_{22}T_\infty(t) \tag{3}$$

$$\frac{dT_3(t)}{dt} = a_{32}T_2(t) - a_{33}T_3(t) + a_{34}T_4(t) + b_{32}T_\infty(t) \tag{4}$$

$$\frac{dT_4(t)}{dt} = a_{43}T_3(t) - a_{44}T_4(t) + a_{45}T_5(t) + b_{42}T_\infty(t) \tag{5}$$

$$\frac{dT_5(t)}{dt} = a_{54}T_4(t) - a_{55}T_5(t) + b_{52}T_\infty(t) \tag{6}$$

When all of the parameters in Eqs. (2) – (6) can be shown in Eqs. (7) – (24).

$$a_{11} = 1/C_P(L/kA_c + 1/hA_c) \tag{7}$$
$$a_{12} = 1/C_P(1/hA_c) \tag{8}$$
$$a_{21} = a_{12} \tag{9}$$
$$a_{22} = 1/C_P(2\,L/kA_c + 1/hA_c) \tag{10}$$
$$a_{23} = a_{12} \tag{11}$$
$$a_{32} = a_{12} \tag{12}$$
$$a_{33} = a_{22} \tag{13}$$
$$a_{34} = a_{12} \tag{14}$$
$$a_{43} = a_{12} \tag{15}$$
$$a_{44} = a_{22} \tag{16}$$
$$a_{54} = a_{12} \tag{17}$$
$$a_{55} = a_{22} \tag{18}$$
$$b_{11} = 1/C_P(L/kA_c) \tag{19}$$
$$b_{12} = 1/C_P(1/hA_c) \tag{20}$$
$$b_{22} = b_{12} \tag{21}$$
$$b_{32} = b_{12} \tag{22}$$
$$b_{42} = b_{12} \tag{23}$$
$$b_{52} = b_{12} \tag{24}$$

When h is convection heat transfer coefficient of ambient air. The convection coefficient in this work is set to 20 $W/m \cdot K$

The discrete system matrix (A_{kc1}) and control input matrix (B_{kc1}) in the case of a thermal conductivity coefficient in Table 1 can be written as Eq. (25) and Eq. (26), respectively.

$$A_{kc1} = \begin{bmatrix} 0.993 & 0.015 & 0.022 & 0.015 & 0.08 \\ 0.021 & 0.995 & 0.021 & 0.022 & 0.01 \\ 0.021 & 0.021 & 0.995 & 0.021 & 0.02 \\ 0.015 & 0.021 & 0.021 & 0.996 & 0.02 \\ 0.083 & 0.015 & 0.022 & 0.021 & 0.99 \end{bmatrix} \quad (25)$$

$$B_{kc1} = \begin{bmatrix} 0.04 & 0.01 \\ 0.04 & 0.01 \\ 0.03 & 0.01 \\ 0.01 & 0.01 \\ 0.07 & 0.02 \end{bmatrix}$$

(26)

When the temperature of the material has reach $100^{\circ}C$, the thermal conductivity of a material has increase to 245 $W/m^2 \cdot K$ [10]. Therefore, the discrete system matrix (A_{kc2}) and control input matrix (B_{kc2}) in the case of high thermal conductivity coefficient can be seen in Eq. (27) and Eq. (28), respectively.

$$A_{kc2} = \begin{bmatrix} 0.992 & 0.025 & 0.032 & 0.026 & 0.02 \\ 0.025 & 0.994 & 0.025 & 0.032 & 0.03 \\ 0.031 & 0.025 & 0.995 & 0.025 & 0.03 \\ 0.026 & 0.032 & 0.025 & 0.995 & 0.03 \\ 0.016 & 0.026 & 0.032 & 0.025 & 0.99 \end{bmatrix} \quad (27)$$

$$B_{kc2} = \begin{bmatrix} 0.005 & 0.0165 \\ 0.063 & 0.0165 \\ 0.053 & 0.0165 \\ 0.033 & 0.0165 \\ 0.017 & 0.0165 \end{bmatrix} \quad (28)$$

The output matrix (C) and distribute matrix (D) can be seen in Eq. (29) and Eq. (30), respectively.

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (29)$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (30)$$

The model predictive controller is designed based on mathematic model in the case of low thermal conductivity and high thermal conductivity.

3. Controller design

In this section, we present the basic of model predictive control for the unfamiliar readers. The step of model predictive control in this section was followed [8] in which the readers will find more details.

Based on a measurement signal obtained at current time step k, the controller will predict the sequence of control input from current time step k to future control input time step $k+N_C$ as shown in Fig. 3(b). The sequence of control input is used to bring the prediction output $\hat{y}(t|k)$ from the current time step k to meet the reference trajectory $r(t|k)$ at coincidence point in future perdition output time step $k+N_P$ as shown in Fig. 3(a). In the real process, only the first control input from the control sequence is applied to making a new measurement signal.

The algorithm cycle is repeated for step $k+1$ and so on until the measurement signal meets set-point trajectory $s(t|k)$. Since the future perdition output time step N_P is remains the same length, but slides along by one sampling interval at each time step. This algorithm known as receding horizon. The signal $\hat{y}_f(t|k)$ is the predicted free response, which is the response of the prediction model if the future control input trajectory remains at the latest value $u(k-1)$.

The linear model of model predictive control can be written in state-space form as

$$x(k+1)=A_{kci}x(k)+B_{kci}u(k) \tag{31}$$

When $i=1$ is mean the model of heat rod at low thermal conductivity and $i=2$ is mean the model at high thermal conductivity.

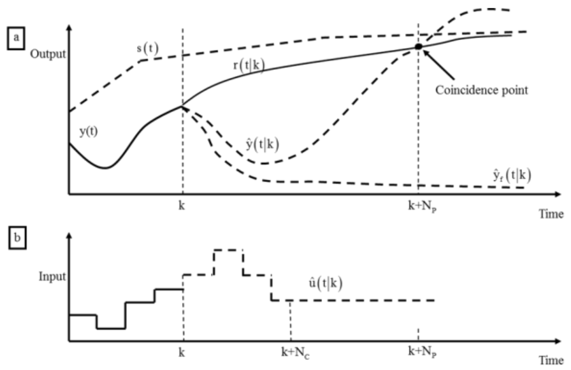

Fig. 3 Model predictive control basics: (a) The prediction output, (b) The sequence of control input

The variable x and u are state vector and control input. The parameters A_{kci} and B_{kci} are plant matrix and control input matrix, respectively.

The output equation can be written as

$$y(k)=Cx(k)+Du(k) \tag{32}$$

Where y is output vector and C and D are output matrix and disturbance matrix, respectively.

The designs of a model predictive control using the difference of the state-space form so, taking a difference operation on both sides of state-space equation. The difference of state-space form can be written as

$$\Delta x(k+1)=A_{kci}\Delta x(k)+B_{kci}\Delta u(k) \tag{33}$$

The input to state-space model is $\Delta u(k)$. The next step is to connect $\Delta x(k)$ to the output equation $y(k)$. Therefore, define a new state variable vector is chosen be

$$x(k)=\left[\Delta x(k)^T \quad y(k)^T\right] \tag{34}$$

The difference of an output equation can be shown as

$$y(k+1)-y(k)=C\Delta x(k+1) \tag{35}$$

The augmented model, which is a state-space model with embedded integrator, will be used in the design of model predictive control by putted the difference of the state-space and the difference of an output equation together leads to following state-space model as

$$\left\{\begin{matrix} \overbrace{\begin{bmatrix}\Delta x(k+1)\\ y(k+1)\end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix}A_{ki} & 0^T\\ CA_{ki} & 1\end{bmatrix}}^{A}\overbrace{\begin{bmatrix}\Delta x(k)\\ y(k)\end{bmatrix}}^{x(k)} + \overbrace{\begin{bmatrix}B_{ki}\\ CB_{ki}\end{bmatrix}}^{B}\Delta u(k) \\ y(k)=\overbrace{[0 \quad 1]}^{C}\begin{bmatrix}\Delta x(k)\\ y(k)\end{bmatrix} \end{matrix}\right\} \tag{36}$$

The future control input is denoted as

$$\Delta u(k_i),\Delta u(k_i+1),\cdots,\Delta u(k_i+N_C+1) \tag{37}$$

Where N_C is the control horizon, which is the number of parameters used to capture the future control trajectory, and the future state variables are predicted for N_P step. Where N_P is called the prediction horizon. This work denotes the future state variables as

$$x(k_i+1|k_i),\ x(k_i+2|k_i),\ x(k_i+m|k_i),\cdots,x(k_i+N_P|k_i) \quad (38)$$

Where $x(k_i+m|k_i)$ is the prediction state variables at k_i+m time step with given current plant information k_i. The control horizon N_C is chosen to be less than or equal to the prediction horizon N_P.

Based on the augmented state-space model system matrix A, control input matrix B and the output matrix C, the future state variables are calculated sequentially using the set of future control parameters as

$$\begin{aligned}
x(k_i+1|k_i) &= Ax(k_i)+B\Delta u(k_i)\\
x(k_i+2|k_i) &= Ax(k_i+1|k_i)+B\Delta u(k_i+1)\\
&= A^2x(k_i)+AB\Delta u(k_i)\\
&\quad +B\Delta u(k_i+1)\\
&\ \vdots\\
x(k_i+N_P|k_i) &= A^{N_P}x(k_i)+A^{N_P-1}B\Delta u(k_i)\\
&\quad +A^{N_P-2}B\Delta u(k_i+1)+\cdots+\\
&\quad +A^{N_P-N_c}B\Delta u(k_i+N_c-1)
\end{aligned} \quad (39)$$

The set of prediction output variables is calculated sequentially as

$$\begin{aligned}
y(k_i+1|k_i) &= CAx(k_i)+CB\Delta u(k_i)\\
y(k_i+2|k_i) &= CA^2x(k_i)+CAB\Delta u(k_i)\\
&\quad +CB\Delta u(k_i+1)\\
&\ \vdots\\
y(k_i+N_P|k_i) &= CA^{N_P}x(k_i)+CA^{N_P-1}B\Delta u(k_i)\\
&\quad +CA^{N_P-2}B\Delta u(k_i+1)\\
&\quad +\cdots+\\
&\quad +CA^{N_P-N_c}B\Delta u(k_i+N_c-1)
\end{aligned} \quad (40)$$

Define the prediction states as

$$X=\left[y(k_i+1|k_i)\ y(k_i+2|k_i),\cdots,y(k_i+N_P|k_i)\right]^T \quad (41)$$

Define the future control input as

$$U=\left[\Delta u(k_i)\ \Delta u(k_i+1),\cdots,\Delta u(k_i+N_C\text{-}1)\right]^T \quad (42)$$

The dimension of X is N_P and the dimension of U is N_C. The sequent of output variables can be rewritten into a compact matrix form as

$$X(\bar{k})=FX(\bar{k})+\Phi U(\bar{k}) \quad (43)$$

The matrix F can be written as

$$F=\begin{bmatrix} CA\\ CA^2\\ \vdots\\ CA^{N_P}\end{bmatrix} \quad (44)$$

The matrix Φ can be written as

$$\Phi=\begin{bmatrix} CB & 0 & 0 & \cdots & 0\\ CAB & CB & 0 & \cdots & 0\\ CA^2B & CAB & CB & \cdots & 0\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ CA^{N_P\text{-}1}B & CA^{N_P\text{-}2}B & CA^{N_P\text{-}3}B & \cdots & CA^{N_P\text{-}N_c}B \end{bmatrix} \quad (45)$$

This paper define the cost function $\bar{J}(X(\bar{k}),U(\bar{k}))$ that reflects the control objective as

$$\bar{J}(X(\bar{k}),U(\bar{k}))=X^T(\bar{k})QX(\bar{k})+U^T(\bar{k})RU(\bar{k}) \quad (46)$$

Where the first term is represented to the objective of minimizing the prediction states while the second term reflects the consideration given to the size of the future control input when the cost function is made to be as small as possible. The weight matrix Q and R are diagonal matrix for tuning the closed-loop performance. Substitution the compact form of output variables into the cost function and uses the first derivative as

$$\frac{D\bar{J}(X(\bar{k}),U(\bar{k}))}{DU}=2F^TQFX(\bar{k})+2(F^TQF+R)U(\bar{k}) \quad (47)$$

The optimal solution of the control signal can be determine by setting the first derivative to zero, so the optimal control signal can be shown as

$$U\left(\bar{k}\right)=\left(\Phi^{T}Q\Phi+R\right)^{-1}\Phi^{T}QFX\left(\bar{k}\right) \qquad (48)$$

The state feedback gain in the form of model predictive control can be written as

$$K_{mpc}=\overbrace{\begin{bmatrix}1 & 0 & \cdots & 0\end{bmatrix}}^{N_C}\left(\Phi^{T}Q\Phi+R\right)^{-1}\Phi^{T}QF \qquad (49)$$

The state feedback gain in the form of model predictive control is used to implement for controlling of heat conduction in rod.

4. Fuzzy weighting method

The multi-model predictive controller share the effect of local controller from low value and high value of thermal conductivity into global controller depend on the average temperature in aluminum rod as shown in Fig. 4.

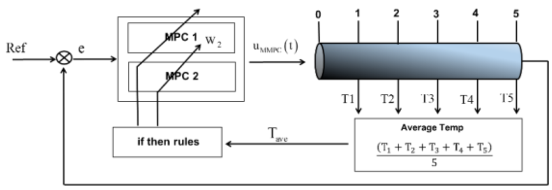

Fig. 4 The diagram of multi-model predictive controller in aluminum rod.

From Fig. 4, the average temperature is calculated from temperature value in each part of aluminum rod. The average temperature is used to adjust the effect of each local controller according to the membership function as shown in Fig. 5.

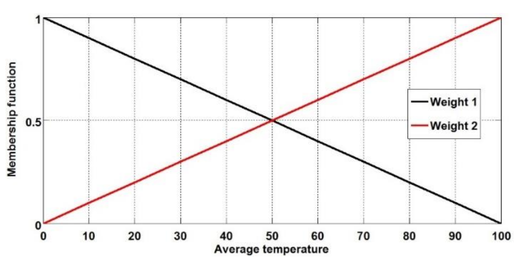

Fig. 5 Membership function of multi-model predictive controller.

From Fig. 4, the controller MPC1 is designed for controlling a low value of thermal conductivity. The system matrix from Eq. (25) and control input matrix from Eq. (26) used to calculate the state feedback gain of local model 1 by using Eq. (49). The state feedback gain of MPC2 is calculated from Eq. (49) by using the system matrix and control input matrix from Eq. (27) and (28), respectively.

From Figure 5, the value of the weight in each thermal conductivity can be calculated according to the average value of temperature from each part of an aluminum rod. The weight1 is a weight in the case of low value of thermal conductivity and the weight2 is the weight of high value of thermal conductivity. The first weight condition can be written as Eq. (50) according to the membership function in Fig. 5, when an average temperature below 0 $^{\circ}C$

$$w_1 = 1,\ w_2 = 0 \qquad (50)$$

The second weight condition, when the range of an average temperature operates between 0 $^{\circ}C$ and 100 $^{\circ}C$. The weight of each local controller is share as

$$w_1 = \frac{T_{ave}-100}{0-100},\ w_2 = 1-w_1 \qquad (51)$$

Where T_{ave} is an average temperature of aluminum rod. The control effort of multi-model predictive control $\left(u_{MMPC}\right)$ can be calculated from the effect of the local controller as shown in Eq. (52).

$$u_{MMPC}=\left(w_1\times MPC1\right)+\left(w_2\times MPC2\right) \qquad (52)$$

5. Simulation result

To demonstrate the performance of the proposed control system, this work uses multi-model of heat conduction in aluminum rod. The state equation of multi-model can be written as

$$x\left(k+1\right)=\left(\sum_{i=1}^{2}w_iA_{kci}\right)x\left(k\right)+\left(\sum_{i=1}^{2}w_iB_{kci}\right)u\left(k\right) \qquad (53)$$

The tip temperature of aluminum rod (T_5) as shown in Fig. 4 is a control parameter. In the simulation study, this work uses two reference signal of a tip temperature at 60 $^{\circ}C$ for the first 7000 second and the reference signal is shifted to 80 $^{\circ}C$ for the second part of the simulation as shown in Fig. 6. Fig. 7 is show the average temperature of aluminum rod 7(a) and weight of multi-model controller 7(b).

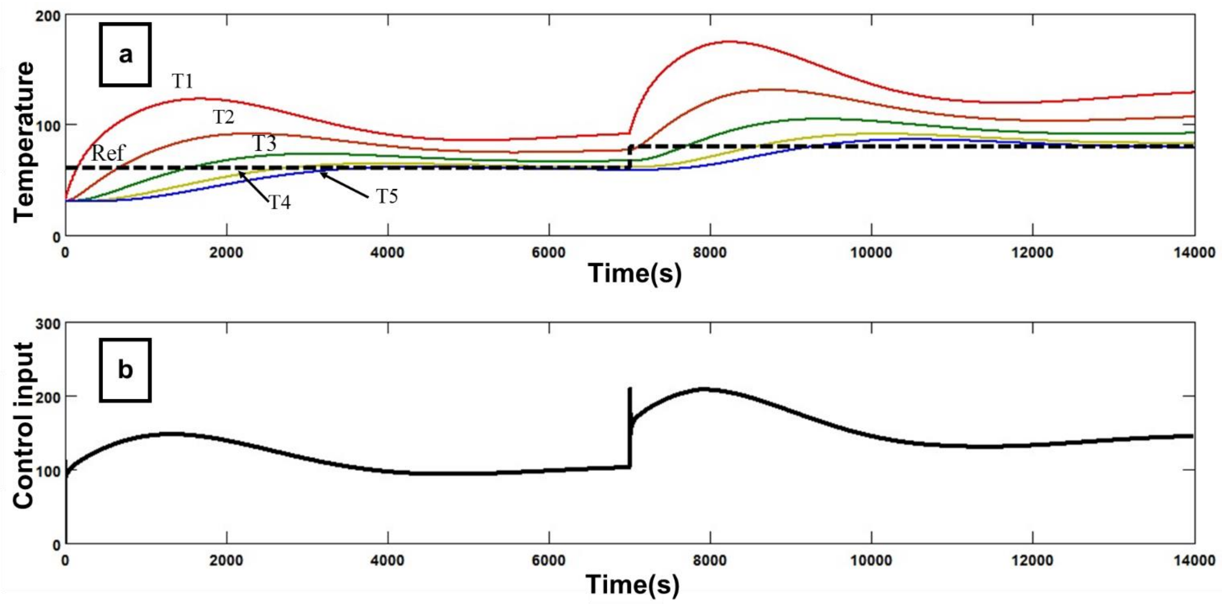

Fig. 6. (a): Temperature profile in each part of aluminum rod when using multi-model predictive control as a controller, (b): Control input signal.

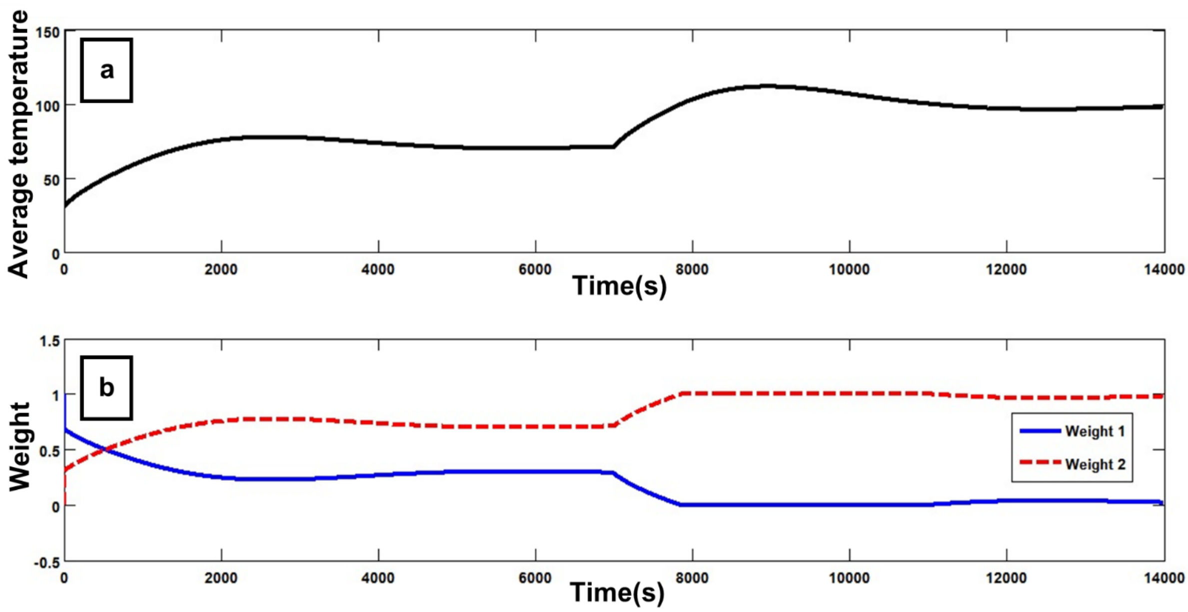

Fig. 7. (a): Average temperature of aluminum rod, (b): The dynamic of the weight value.

From Fig. 6(a), the tip of aluminum rod $\left(\mathrm{T}_5\right)$ can track the two-state reference signal. The temperature of a first part of aluminum rod $\left(\mathrm{T}_1\right)$ is reach to the maximum temperature at 160 $^\circ\mathrm{C}$, when the reference signal is changed to 80 $^\circ\mathrm{C}$. From Fig. 7(b), the system became local model 1 $\left(\mathrm{A}_{\mathrm{kc1}},\mathrm{B}_{\mathrm{kc1}}\right)$ at the beginning of the simulation. Then, the percentage of local model 1 $\left(\mathrm{A}_{\mathrm{kc1}},\mathrm{B}_{\mathrm{kc1}}\right)$ is decreasing and the percentage of local model 2 $\left(\mathrm{A}_{\mathrm{kc2}},\mathrm{B}_{\mathrm{kc2}}\right)$ is increasing depend on the average temperature of aluminum rod. When the average temperature of aluminum rod is reached the temperature at 50 $^\circ\mathrm{C}$. The weight of local model 1 $\left(\mathrm{A}_{\mathrm{kc1}},\mathrm{B}_{\mathrm{kc1}}\right)$ and local model 2 $\left(\mathrm{A}_{\mathrm{kc2}},\mathrm{B}_{\mathrm{kc2}}\right)$ are share the dynamic together at 50 % or $\mathrm{w}_1=\mathrm{w}_2=0.5$. The weight of local model 2 become prominent and the weight of local model 2 become inferior according to weighting condition as shown in Eq. (51), where the average temperature of an aluminum rod is high. The control effort according to Eq. (52) for controlling the tip temperature of aluminum rod is shown in Fig. 6(b).

Fig. 8(a) is shown a comparison between the tips of aluminum rod temperature, when using difference controller algorithm. The temperature response in difference controller is same, when system operates at the first reference temperature as shown in Fig. 8(a). When the temperature reference is changed to 80 $^\circ\mathrm{C}$, the response of temperature at tip location by using multi-model predictive control as a controller is better than the response of conventional model predictive control in term of overshoot and steady-state error as shown in Fig. 8(a). The magnitude of control input signal, when using multi-model predictive control as a controller is less than conventional model predictive control as shown in Fig. 8(b).

5. Conclusion

From simulation result, the multi-model predictive controller based on the model predictive controller in each operating region can reduce the disadvantage behavior of the system such as, overshoot, rise time and steady-state error according to wide range operating condition.

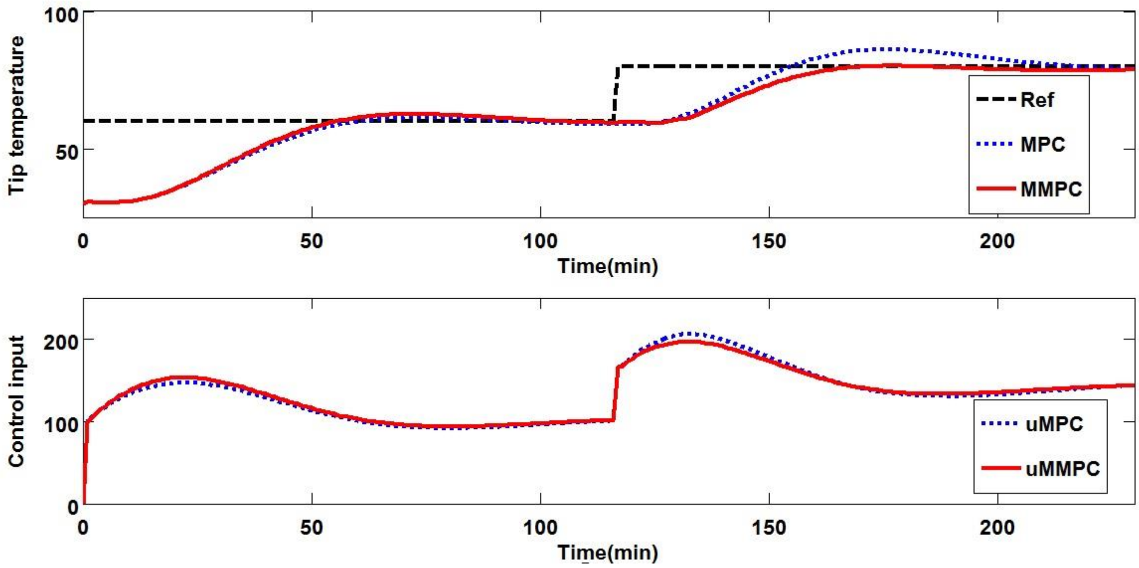

Fig. 8. (a): The temperature response of tip location of aluminum rod when using multi-model predictive controller (solid-line) and comparison with conventional model predictive control (dot-line), (b): Control input signal of multi-model predictive controller (solid-line) and comparison with conventional model predictive control (dot-line).

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