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Study on controller design of a flexible link manipulator using wave based control method

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Abstract. Wave based control method is one of the vibration control method for flexible structure. The method can suppress the vibration by reducing the development of standing wave, and is generally applied to a lumped system, but it is difficult to apply them to continuous systems. Therefore, there is room for further research into the application of them to continuous systems and consequently this paper proposes a method to apply wave control method to continuous system, and a flexible link manipulator is introduced as a controlled object. In order to derive the mathematical model for controller design, i.e. the model to which wave based control method is applied, Absolute Nodal Coordinate Formulation (ANCF) which is one of the nonlinear finite element method is employed. In order to derive the controller by wave based control method, a lumped system is derived from some manipulation and coordinate transformation of ANCF model which is derived for flexible link manipulator and the wave based method is applied to the derived lumped model. The validity of the proposed method is demonstrated by numerical simulation and applicability of the proposed method is discussed.

1. Introduction

In recent years, space structure has been made of flexible element. The behavior of the flexible object is very complex and oscillatory. Therefore, study on vibration problem of flexible body is important. The control method based on mode analysis is one of the general approaches to vibration problems. However, it is not appropriate to apply such a control method to a flexible object which has nonlinearity. Conventionally, wave control method[1][2] has been studied as methods to solve these problems. In the wave control method, the traveling wave in the structure is canceled by the controller in order to avoid the generation of standing wave and consequently vibration of the structure is damped. Since the traveling wave is controlled, the wave control method can suppress wave from an earlier stage before standing wave arise. Therefore, it can be expected to damp more rapidly than the other conventional control method based on mode analysis. However, there are few cases where the wave control method has been applied to general distributed parameter systems such as flexible objects.

Analysis including large deformation and large rotation is very important for vibration control of flexible object. Absolute Nodal Coordinate Formulation (ANCF) proposed by Shabana[3] is one of the analysis methods of flexible object which is subject to large deformation and large rotation and is a kind of nonlinear finite element method. The nodal coordinates are represented by absolute coordinate in ANCF and such a representation of coordinate leads to constant mass matrix and nonlinear stiffness matrix, which is the typical feature of ANCF. A lot of study on ANCF have focused on the improvement of analytical ability, however there is few study on the controller design by ANCF model.

The purpose of this paper is to propose a controller design method for vibration control of flexible structure by applying wave control method to ANCF beam model. Furthermore, the proposed method is validated by numerical analysis.

This paper is composed as follows. Section 2 introduces a controlled object and the mathematical model for the controlled object is derived. In Section 3, conventional wave control method is introduced briefly. Section 4 proposes a method to control the flexible structure by wave control method. In Section 5, numerical analysis shows the validity of our proposed method and conclusion is given in Section 6.

2. Model of flexible object

2.1. Controlled object

A flexible beam is introduced as the controlled object simulating a planar single flexible link robot as Figure 1 indicates. The link is uniform, one end of the link is fixed by pined support and control torque is applied to that end and the other end is free. In addition, internal damping of the link, air drag, influence of gravity and friction force on the joint are neglected.

2.2. Formulation of the controlled object by ANCF

The derivation of equation of motion by ANCF beam is shown below briefly. Flexible link is divided into N element and motion of equation of each element is formulated by ANCF. Figure 2 shows the i-th elements. The nodal coordinate of i-th element is given as:

$$\mathbf{e}_i = \left\{ r_{ix}\big|_{x=0} \quad \frac{\partial r_{ix}\big|_{x=0}}{\partial x} \quad r_{iy}\big|_{x=0} \quad \frac{\partial r_{iy}\big|_{x=0}}{\partial x} \quad r_{ix}\big|_{x=l} \quad \frac{\partial r_{ix}\big|_{x=l}}{\partial x} \quad r_{iy}\big|_{x=l} \quad \frac{\partial r_{iy}\big|_{x=l}}{\partial x} \right\} \tag{1}$$

where 1st and 3rd elements are global position of one end in the direction of X and Y respectively, 5th and 7th elements are global position of the other end in the direction of X and Y respectively. Furthermore, 2nd and 4th elements are global slopes of one end in the direction of X and Y respectively, and 6th and 8th elements are global slopes of the other end in the direction of X and Y respectively. Then, position vector of arbitrary point on the beam $\mathbf{r}_i$ is as follows.

$$\mathbf{r}_i = \begin{Bmatrix} r_{ix} \\ r_{iy} \end{Bmatrix} = \mathbf{S}_i \mathbf{e}_i \tag{2}$$

Where, $\mathbf{S}_i$ is the shape function for the i-th element. Using definition of position of the point on the beam given by equation (2), kinetic energy, potential energy and virtual work are derived. Furthermore, applying Lagrange's equation to those obtained energy and virtual work, the equation of motion for i-th element are given by

$$\mathbf{M}_i \ddot{\mathbf{e}}_i + \mathbf{K}_{ti}(\mathbf{e}_i) + \mathbf{K}_{li}(\mathbf{e}_i) = \mathbf{Q}_i \tag{3}$$

where $\mathbf{M}_i$ is mass matrix for i-th element, $\mathbf{K}_{ti}$ is bending stiffness matrix for i-th element and $\mathbf{K}_{li}$ is axial stiffness matrix for i-th element. Deriving constraint equation which describes the relationships between each element and deriving the Jacobian and the acceleration equation, a differential equation is obtained as

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_q{}^T \\ \mathbf{C}_q & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_E \\ \boldsymbol{\gamma} \end{bmatrix} \tag{4}$$

where $\mathbf{M}$ is block mass matrix which consists of $\mathbf{M}_i$, $\mathbf{q}$ is generalized coordinate vector which consists of nodal coordinates of each element, $\mathbf{C}_q$ is the Jacobian obtained by partially differentiating the constraint equation with a generalized coordinate vector, $\boldsymbol{\lambda}$ is Lagrange multiplier, $\mathbf{Q}_E$ is generalized external force. In addition, Newton Euler equation as shown in equation (5) is obtained in order to derive the control design model

$$\mathbf{M}_{in}\ddot{\mathbf{q}}_{in} + \mathbf{K}_{tin}(\mathbf{q}_{in}) + \mathbf{K}_{lin}(\mathbf{q}_{in}) = \mathbf{Q}_{in} \tag{5}$$

where the subscript "in" indicates that it is related to independent coordinates.

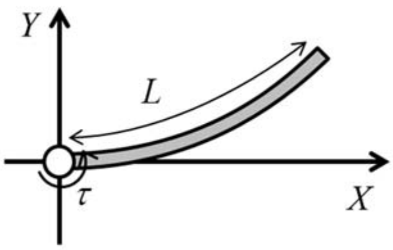

Figure 1. Controlled object.

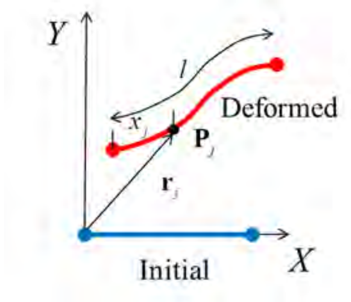

Figure 2. *i*-th element of ANCF.

2. Wave control method

2.1. Model of wave control

In our study, we employ the wave control method proposed by O'Connor[2]. In this model, the control object is expressed lumped system composed of springs and lumped masses. The Figure 3 indicates the wave control model. m_i, k_i, x_i are mass, spring constant and displacement of *i*-th, respectively. Note that x_0 and k_0 are virtual displacement and spring constant for the sake of expression of control input f_0. Then, control input f_0 can be given as

$$f_0 = (x_0 - x_1)k_0 \tag{6}$$

The wave control model assumes that the displacement of each mass x_i is separated into two traveling wave components. One is component a_i which travels rightward, the other is b_i which travels leftward and $x_i = a_i + b_i$ is satisfied. As Figure 4 shows, the wave propagation is given by traveling wave components A_i and B_i which consists of X_i, and transfer functions G_i, H_i and F. Note that X_i, A_i, B_i are x_i, a_i, b_i in the Laplace domain. Then, each traveling wave has following relationships:

$$A_i = G_i A_{i-1} \tag{7}$$

$$B_i = H_i B_{i+1} \tag{8}$$

$$B_n = FA_n \tag{9}$$

Note that Equation (7), (8) and (9) corresponds to the relation of the rightward propagation, the leftward propagation, and the reflection at the boundary, respectively. O'Connor has mentioned in the reference [2] that the transfer functions G_i and H_i can be approximated to second-order system and we also employ second-order system in this study as

$$G_i = \frac{\omega_{Gi}{}^2}{s^2 + \omega_{Gi}s + \omega_{Gi}{}^2}, \quad \omega_{Gi} \equiv \sqrt{\frac{2k_i}{m_{i+1}}} \tag{10}$$

$$G_i = \frac{\omega_{Hi}{}^2}{s^2 + \omega_{Hi}s + \omega_{Hi}{}^2}, \quad \omega_{Hi} \equiv \sqrt{\frac{2k_i}{m_i}} \tag{11}$$

Furthermore, considering the boundary condition at the right end in Figure 3, i.e. free end, the transfer function F is given as unity.

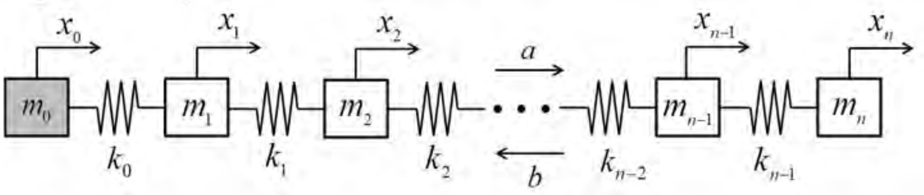

Figure 3. Model of the wave control method.

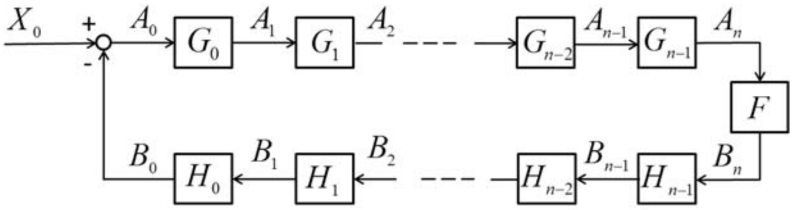

Figure 4. Block diagram of the wave control method.

2.2. Method of wave control

In the wave control method, vibration control of structure is performed by applying control input so as to cancel the traveling waves. If reflection of traveling waves can be avoided at the boundary, the standing waves are not generated, that is, if the reflected wave A_0 at the left end in Figure 3 can be cancelled, vibration does not continue, in other words, vibration is suppressed. In order to achieve cancellation of the reflected wave at left end, the incident wave B_0 to the boundary at left end should be measured so as to derive the control input given as

$$X_0 = B_0 + X_{ref}/2 \tag{12}$$

where X_{ref} is reference displacement and incident wave B_0 to the boundary is given by

$$B_0 = \frac{H_0(1-G_0)}{1-H_0G_0}X_1 - \frac{H_0G_0}{k_0(1-H_0G_0)}F_0 \tag{13}$$

3. Application of wave control method to flexible structure

The model of controlled object is different from the model used in wave control method, therefore the wave control method cannot be directly applied to model of control object. In this section, we propose the process of applying the wave control method to the model of our controlled object. In our proposed method, we utilize the coordinate transformation which transforms the coordinates of wave control model to the coordinates of ANCF model. On the other hand, Joseph has also proposed the method which utilizes the diagonalized model[4] and systematic process is developed in that study. But in the context of controller design freedom, we employ numerical method as shown later.

3.1. Controller design model

In order to design controller, controller design model is derived from Equation (5). In general, longitudinal deformation is quite small compared to the transverse deformation, then the influence of longitudinal deformation on the behaviour of the system can be neglected. Furthermore, as is clear from structure of the motion of equation given by Equation (5), expression of motion becomes redundant if longitudinal deformation is neglected. That is, expression become redundant. Therefore, we employ the motion of equation for Y directions for controller design as

$$\mathbf{M}_y\ddot{\mathbf{q}}_y + \mathbf{K}_{ty}\mathbf{q}_y = \mathbf{Q}_y \,, \tag{14}$$

which process is also used in another study[5]. The number of nodal coordinate of Y direction about ANCF beam consisting of N elements is $2N+1$. The number of mass (number of DOF) of the lumped system for wave controller design need to be equal to the number of DOF of ANCF beam model. Therefore, the lumped system transformed from the ANCF beam model has to consist of $2N+1$ masses and $2N$ springs.

3.2. Determination of unknown parameters

Because parameters of the lumped system is generally unknown, it is difficult to directly apply coordinate transformation to ANCF model in order to derive the corresponding lumped system. Therefore, applying diagonalization to the equation of motion of each model, both diagonalized model is used as intermediate model for coordinate transformation.

Equation (15) is the equation of motion of the lumped system to which the wave control method is applied.

$$\mathbf{M}_w\ddot{\mathbf{x}} + \mathbf{K}_w\mathbf{x} = \mathbf{Q}_w \tag{15}$$

where $\mathbf{M}_w$ is the mass matrix, $\mathbf{K}_w$ is the stiffness matrix, $\mathbf{Q}_w$ is the external force matrix. The equation of motion of lumped system can be diagonalized by solving the generalized eigenvalue problem of Equation (15), and the diagonalized equation of motion is given by

$$\begin{bmatrix} \overline{m}_{w1} & & & 0 \\ & \overline{m}_{w2} & & \\ & & \ddots & \\ 0 & & & \overline{m}_{w(2N+1)} \end{bmatrix} \ddot{\overline{\mathbf{x}}} + \begin{bmatrix} \overline{k}_{w1} & & & 0 \\ & \overline{k}_{w2} & & \\ & & \ddots & \\ 0 & & & \overline{k}_{w(2N+1)} \end{bmatrix} \overline{\mathbf{x}} = \mathbf{T}_w{}^T\mathbf{Q}_w \tag{16}$$

where $\mathbf{T}_w$ is coordinate transformation matrix in which eigenvectors of the system given by Equation (15) are aligned, $\mathbf{x} = \mathbf{T}_w\overline{\mathbf{x}}$. Similarly, the equation of motion of ANCF model can be diagonalized by solving the generalized eigenvalue problem of Equation (14).

$$\begin{bmatrix} \overline{m}_{c1} & & & 0 \\ & \overline{m}_{c2} & & \\ & & \ddots & \\ 0 & & & \overline{m}_{c(2N+1)} \end{bmatrix} \ddot{\overline{\mathbf{q}}}_y + \begin{bmatrix} \overline{k}_{c1} & & & 0 \\ & \overline{k}_{c2} & & \\ & & \ddots & \\ 0 & & & \overline{k}_{c(2N+1)} \end{bmatrix} \overline{\mathbf{q}}_y = \mathbf{T}_c{}^T\mathbf{Q}_y \tag{17}$$

where $\mathbf{T}_c$ is coordinate transformation matrix in which eigenvectors of the system given by Equation (14) are aligned, $\mathbf{q}_y = \mathbf{T}_c\overline{\mathbf{q}}_y$. Unknown parameters of lumped system are determined by numerical search so that the diagonal elements of equations (16) and (17) coincide with each other. Applying coordinate transformation by obtained two matrices $\mathbf{T}_w$ and $\mathbf{T}_c$, mass and stiffness matrix for lumped system is derived as

$$\mathbf{M}_w = \mathbf{T}_w{}^{-T}\mathbf{T}_c{}^T\mathbf{M}_y\mathbf{T}_c\mathbf{T}_w{}^{-1} \tag{18}$$

$$\mathbf{K}_w = \mathbf{T}_w{}^{T^{-1}}\mathbf{T}_c{}^T\mathbf{K}_{ty}\mathbf{T}_c\mathbf{T}_w{}^{-1} \tag{19}$$

Note that the number of unknown parameters is $4N+1$, which is larger than the conditions that the unknown parameters have to satisfy, and therefore additional conditions can be added to the parameter determination. In other words, there is degree of freedom in parameter determination. This degree of freedom is utilized in considering the external force term as described next section.

3.3. Conditions on external force term

Following equation indicates the coordinate transformation of the external force term.

$$\mathbf{Q}_y = \mathbf{T}_c{}^{T^{-1}} \mathbf{T}_w{}^T \mathbf{Q}_w \tag{20}$$

The components of the equation (20) are given by the equations (21) and (22).

$$\mathbf{Q}_y = \begin{bmatrix} \tau \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{21}$$

$$\mathbf{T}_c{}^{-T} \mathbf{T}_w{}^T \mathbf{Q}_w = \begin{bmatrix} T_{11} f_0 \\ T_{21} f_0 \\ \vdots \\ T_{n1} f_0 \end{bmatrix}, \quad \mathbf{T}_c{}^{-T} \mathbf{T}_w{}^T \equiv \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1n} \\ T_{21} & \ddots & & \\ \vdots & & \ddots & \\ T_{n1} & & & T_{nn} \end{bmatrix} \tag{22}$$

The elements of external force term of controlled object is zero except for first element as Equation (21) shows, however external force term of the transformed system from lamped system may has non-zero value in the elements except for first element as Equation (22) shows. In order to satisfy the equation (20), it is possible to reduce the influence of the control force which does not exist in the original system by using the degree of freedom as described above and introducing the following evaluation value

$$\alpha = \sum_{i=2}^{n} \left(\frac{T_{i1}}{T_{11}} \right)^2 \tag{23}$$

Equation (23) indicates that the smaller value α corresponds to smaller value of the elements in the external force except for first element. Therefore, minimization of Equation (23) and numerical search for determination of $\mathbf{T}_w$ and $\mathbf{T}_c$ are carried out simultaneously.

4. Simulation

The proposed method described in Chapter 4 was validated by numerical analysis. In the numerical analysis, link is moved by control torque from the initial angle (0 degree) to the target angle (10 degree). Parameters of the link is shown in Table. 1. The control input for the system is derived by using Equation (12) and (20). In numerical simulation, the value of the element of external force except for first element are approximated by 0.

Table 1. Parameters for numerical analysis

Young’s modulus of the flexible beam [GPa]	70
Density of the flexible beam [kg/m^3]	2700
Length of the flexible beam [m]	1
Thickness of the flexible beam [m]	0.005
Width of the flexible beam [m]	0.01
Element number of ANCF model	1

In order to show the validity of the proposed method based on wave control method, control result by the proposed method was compared with the control result by a linear quadratic regulator (LQR). Figure 5 shows time history of tip displacement in the *Y* direction and two results are shown for comparison between the proposed method and LQR controller. As the results show, tip position converges to the target position by the proposed controller. Furthermore, the results by the proposed controller shows faster convergence to the target position than that by LQR.

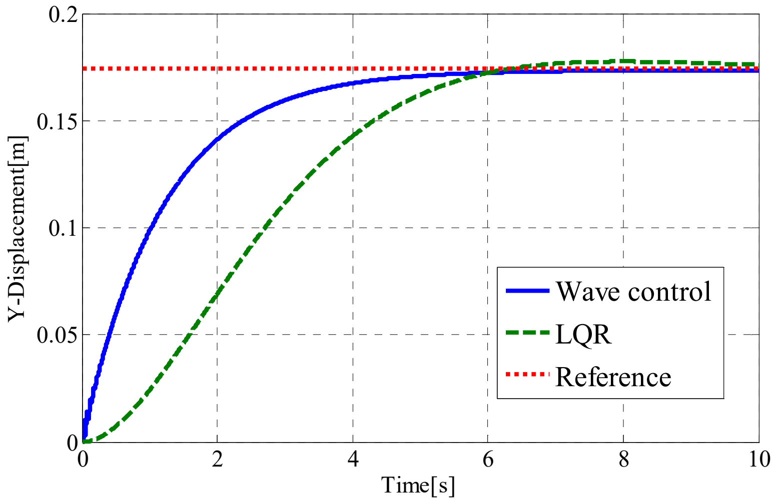

Figure 5. Comparison by control method.

5. Conclusion

In this paper, we proposed a method of applying wave control method to the flexible structure by using coordinate transformation, moreover we demonstrated the validity of the proposed method by numerical analysis. As the result of numerical analysis, the proposed method shows faster convergence to the target position, and validity of the proposed method is confirmed.

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