An Inverse Kinematic Method for Performance Determination of Flight Simulation using Stewart Platform

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Abstract

In this paper, a theoretical method for determining the performance limitation of the Stewart platform operation with constraints on actuator speed and force is described. The limitation of platform in the generation of oscillating surge, sway, and heave motions as well as roll and pitch angular motions is investigated using the inverse kinematic equation of Stewart platform and a numerical method. The proposed method also accounts for both inertial mass and rotational mass of actuators in the determination of operating limitation. The implementation of the proposed method for an example platform is presented and the performance determination results are discussed.

Keywords: Stewart Platform; Rigid Body Dynamic Simulation.

1. Introduction

Flight simulators are being widely used in the modern world. Not only for training purposes, they are also used for evaluation and research purposes. The Stewart platform [1] is a classic mechanism widely used for a motion control device. Not only used in flight simulation, the platform was also utilized for other applications such as high precision positioning devices [2] and machining centers [3]. The advantages of this mechanism are on the wide range of motion, high rigidity, and accurate positioning capability.

This work focuses on the use of Stewart platform mechanism to simulate basic flight maneuvers for hardware-in-the-loop (HITL) simulation purpose for autonomous aerial systems and flight instruments. The load (device/vehicle) is attached to the moving platform to simulate response due to flight maneuvers. In order to achieve accurate flight response, it is important to know the operation boundary of the platform hardware. A simple formula for computing actuating forces and speed for a Stewart platform presented in [4] assumed negligible actuator inertia in which sufficient accuracy could be obtained for the case of high load inertia. However, accounting of actuator inertia in dynamics model is required for control performance [5].

In this work, we focuses on finding the operation limitation for Stewart platform with given actuator characteristics (actuating speed and force). The mathematical model and a brief derivation of the dynamic equations are presented in Section 2. Section 3 describes details of the proposed method for Stewart platform limitation determination. Section 4 presents the implementation of the proposed method in an example platform and result discussion. Finally, the conclusion is presented in Section 5.

2. Mathematical Model

The Stewart platform considered in this study is a 6-DOF parallel mechanism. A moving rigid platform is connected to a fixed base through six independent, identical linear actuators (Fig. 1). Spherical joints are applied to the connection at both end of each actuator. The change in the length of the linear actuators is the command variable used to control the motion of the moving platform.

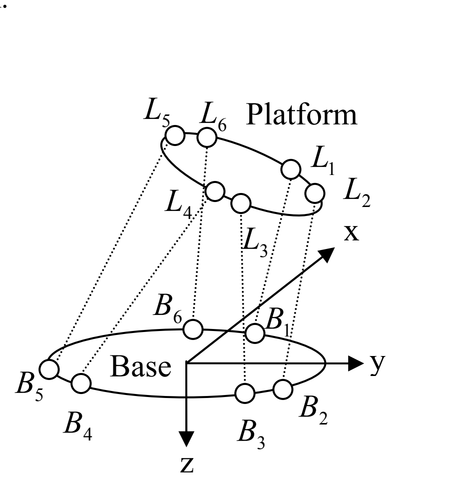

Fig. 1 schematic diagram of Stewart platform

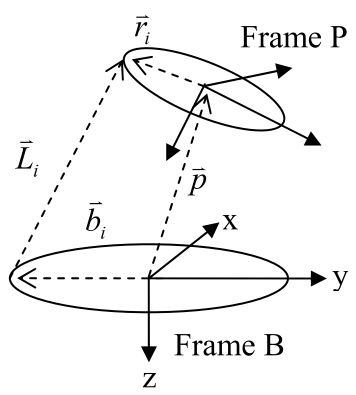

Fig. 2 frame and variable definitions

As shown in Fig. 2, the actuator element can be considered as position vector between the connecting point at the platform and that of the base. The desired position and orientation of the moving platform can be used to determine the required length of each actuator. The actuator element, $\vec{L}_i$, can be obtained by

$$\vec{L}_i = L\hat{s}_i = R(\phi,\theta,\psi)\vec{r}_i^{\,P} + \vec{p} - \vec{b}_i \tag{1}$$

where L is the actuator length, $\hat{s}$ is a unit vector representing the actuator direction, $\vec{p}$ is the position of the platform frame, $\vec{b}_i$ is the position of the actuator-connecting point at the base, $\vec{r}_i^{\,P}$ is the position of the actuator-connecting point of the platform in frame *P*, and $R(\phi,\theta,\psi)$ is the rotation matrix representing the orientation of the platform in the form of the Euler angles (roll, pitch, and yaw).

$$R(\phi,\theta,\psi) = R(\psi)R(\theta)R(\phi) = \begin{bmatrix} c\psi c\theta & R_{12} & R_{13} \\ s\psi c\theta & R_{22} & R_{23} \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{2}$$

where $s(\cdot)$ and $c(\cdot)$ indicate the sine and cosine functions, respectively, and

$$\begin{aligned} R_{12} &= c\psi s\theta s\phi - s\psi c\phi \\ R_{13} &= c\psi s\theta c\phi + s\psi s\phi \\ R_{22} &= s\psi s\theta s\phi + c\psi c\phi \\ R_{23} &= s\psi s\theta c\phi - c\psi s\phi \end{aligned} \tag{3}$$

The motion of the actuator can be derived as

$$\dot{L}\hat{s}_i + \omega_i \times L\hat{s}_i = \dot{R}(\phi,\theta,\psi)\vec{r}_i^{\,P} + \vec{v}_P \tag{4}$$

where ω_i is the angular motion of the actuator element, $\vec{v}_P$ is the velocity of the platform and $\dot{R}(\phi,\theta,\psi)\vec{r}_i^{\,P}$ can be written as a function of the angular rate of Euler angle;

$$\dot{R}(\phi,\theta,\psi)\vec{r}_i^{\,P} = G(\phi,\theta,\psi)\cdot\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{5}$$

Eq. (4) can also be written as a function of the angular motion in the body coordinate system of the platform:

$$\dot{L}\hat{s}_i + \omega_i \times L\hat{s}_i = R(\phi,\theta,\psi)(\omega_P^P \times \vec{r}_i^{\,P}) + \vec{v}_P \tag{6}$$

where

$$\omega_P^P = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{7}$$

The relationship between the angular rate in the body axis and the rate of change of the roll, pitch, and yaw angles can be obtained from

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi c\theta \\ 0 & -s\phi & c\phi c\theta \end{bmatrix}\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{8}$$

and

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s\phi s\theta / c\theta & c\phi s\theta / c\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi / c\theta & c\phi / c\theta \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{9}$$

From Eq. (6), the acceleration at the connection between actuator and the platform can be derived as

$$\begin{aligned} &\ddot{L}\hat{s}_i + \alpha_i \times L\hat{s}_i + \omega_i \times (\omega_i \times L\hat{s}_i) \\ &+ 2\omega_i \times \dot{L}\hat{s}_i = \vec{a}_P + R(\phi,\theta,\psi) \\ &(\alpha_P^P \times \vec{r}_i^{\,P} + \omega_P^P \times (\omega_P^P \times \vec{r}_i^{\,P})) \end{aligned} \tag{10}$$

To obtain the actuating force, an actuator model is considered. The force and moment balance must be applied to the platform and actuators. Note that frictions in the actuators and their joints are neglected in this work.

For the actuator model, the free body diagram shown in Fig. 3 is considered, the force balance of the actuating part (the upper part) is

$$F_{a,i}\cdot\hat{s}_i - (m_1 g\cdot\hat{s}_i) - F_i\cdot\hat{s}_i = m_1\ddot{L} \tag{11}$$

and the moment balance about the joint at the base is

$$(m_1 l_1 + m_2 l_2)\hat{s}_i \times g - (I_1 + I_2)\alpha_i - \omega_i \times (I_1 + I_2)\omega_i - L_i \hat{s}_i \times F_{T,i} = 0 \quad (12)$$

where $F_{a,i}$ is the force generated from the i-th actuator, F_i is the actuating force on the platform, $F_{T,i}$ is the inertia force due to the actuator angular motion, I_1 and I_2 are the moment of inertia of the actuator, l_1 and l_2 are the distance from the base joint to the center of the gravity of m_1 and m_2, respectively.

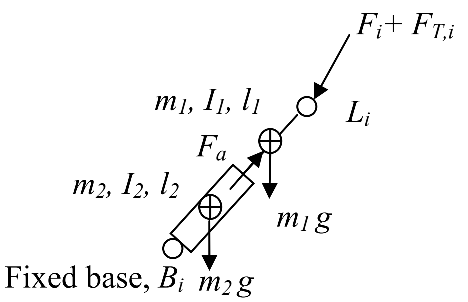

Fig. 3 free body diagram of actuator

For the platform, the force and moment balance can be described by:

$$\sum F_i + \sum F_{T,i} + m_P g = m_P \vec{a}_P \quad (13)$$

$$\sum r_i \times F_i + \sum r_i \times F_{T,i} = I_P \alpha_P + \omega_P \times I_P \omega_{Pi} \quad (14)$$

where m_P and I_P are the total mass and moment of inertia including those of the platform and the carried load.

3. Determining the Limitation

The equations of motion in Eqs. (1) – (14) represent the motion of the platform and its actuators. The required actuating motions (speeds and accelerations) can be determined for desired platform motions. Then Eqs. (13) – (14) can be combined to obtain the following matrix form in which the actuating forces can be obtained using inverse matrix:

$$\begin{bmatrix} [s]_{3\times 6} \\ [r\times s]_{3\times 6} \end{bmatrix} \begin{bmatrix} |F_{a,1}| \\ \vdots \\ |F_{a,6}| \end{bmatrix} = \begin{bmatrix} [H_1]_{3\times 1} \\ [H_2]_{3\times 1} \end{bmatrix} + H_3 \quad (15)$$

where

$$H_1 = m_P \vec{a}_P - m_P g - \sum F_{T,i}$$

$$H_2 = I_P \alpha_P + \omega_P \times I_P \omega_{Pi} - \sum r_i \times F_{T,i}$$

$$H_3 = \begin{bmatrix} [s]_{3\times 6} \\ [r\times s]_{3\times 6} \end{bmatrix} \begin{bmatrix} (m_1 g \cdot \hat{s}_1) + m_1 \ddot{L}_1 \\ \vdots \\ (m_1 g \cdot \hat{s}_6) + m_1 \ddot{L}_6 \end{bmatrix} \quad (16)$$

Given the desired motion of the platform, the motion of actuators can be determined and used to obtain the inertia force, $F_{T,i}$, acting on the platform due to the inertia of actuator from angular motion. Then, the force required on the platform for the desired motion can be computed. Finally, the actuating force can be determined. The computation process is shown in Fig. 4.

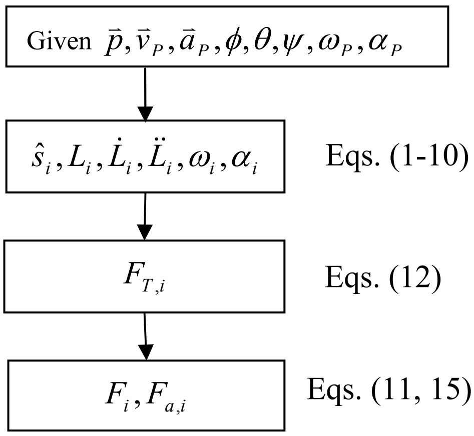

Fig. 4 Computation diagram

After the forces and speeds of actuators due to desired platform motion are determined, the maximum speed and force and their corresponding actuators can be identified. For the similar motion with increased speed and load, it is likely that these identified actuators will also require the highest performance among the platform actuators. Therefore, the platform performance is limited by the performance of these actuators.

For the speed limitation, the platform movement can be easily found using the maximum limit of the actuator in Eq. (6). Unfortunately, for the force limitation, Eq. (15) must be modified to have unknowns in the vector on the left-hand side and the solution is then obtained using the inverse matrix method. For example, the heaviest load that results specified force on the actuating force can be obtained from solving:

$$\begin{bmatrix} g-\vec{a}_P & s_2 \cdots s_6 \\ \underline{0} & [r \times s]_{3\times5} \end{bmatrix} \begin{bmatrix} m_P \\ |F_{a,2}| \\ \vdots \\ |F_{a,6}| \end{bmatrix} \tag{17}$$
$$= \begin{bmatrix} F_{a,i,\max} s_1 - \sum F_{T,i} \\ [H_2]_{3\times1} \end{bmatrix} + H_3$$

4. Implementation and Results

In this section, the developed method is implemented for the example of Stewart platform mechanism. The properties of the Stewart platform in the example are presented in Table 1. The position of the connecting joints at the base and platform is defined by the angles ψ_B and ψ_P, respectively (as show in Fig. 5).

Table. 1 Properties of the Stewart platform in the example

Description	Limitation
Platform+load	
Mass [kg]	4
Radius [m]	0.2
ψ_P [deg]	5
Inertia [kg m^2]	[0.09, 0.09, 0.18]
CG	[0,0,-0.05]
Base	
Radius [m]	0.25
ψ_B [deg]	55
Actuator	
Length [m]	0.3 (0.6 fully extended)
Speed [m/s]	0.02
Force [N]	90
Mass 1 [kg]	0.25
Mass 2 [kg]	0.25
Inertia 1 [kg m^2]	[0.01,0.01,0] (at L = 0.3)
Inertia 2 [kg m^2]	[0.01,0.01,0]

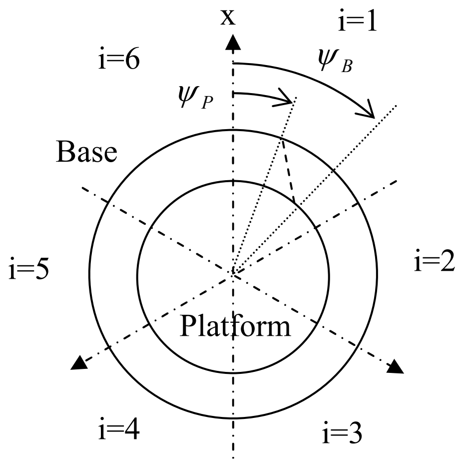

Fig. 5 position of actuator joints at the base and platform (top view)

In this paper, the desired platform motions are represented using the following sinusoidal functions. For angular motion, the center of rotation is located at the center of gravity of the load (as shown in Fig. 6).

$$\begin{aligned} A &= A_0 + A_1 \cos(\omega_A t) \\ \dot{A} &= -\omega_A A_1 \sin(\omega_A t) \\ \ddot{A} &= -\omega_A{}^2 A_1 \cos(\omega_A t) \end{aligned} \tag{18}$$

where A represents the considered variable (i.e., surge, sway, heave, roll, and pitch motions), A_0 is the initial value of the variable, A_1 is the magnitude of oscillation and ω_A is the oscillation frequency of the variable.

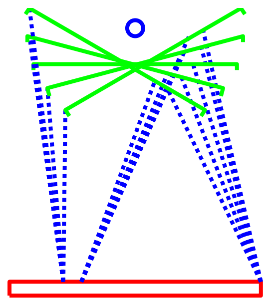
Fig. 6 angular motion of the platform

Figs. (7) – (16) present the maximum force and speed of the actuators in order to achieve the desired motion specified by Eq. (18) for surge, sway, heave, roll, and pitch motions in the example Stewart platform.

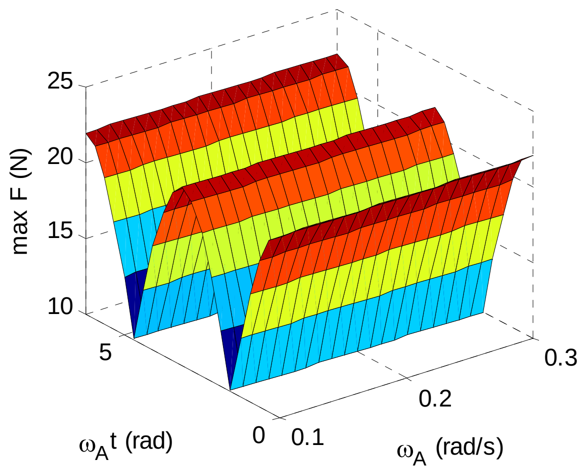

Fig. 7 maximum actuating force from surge motion (A_1= 0.15m)

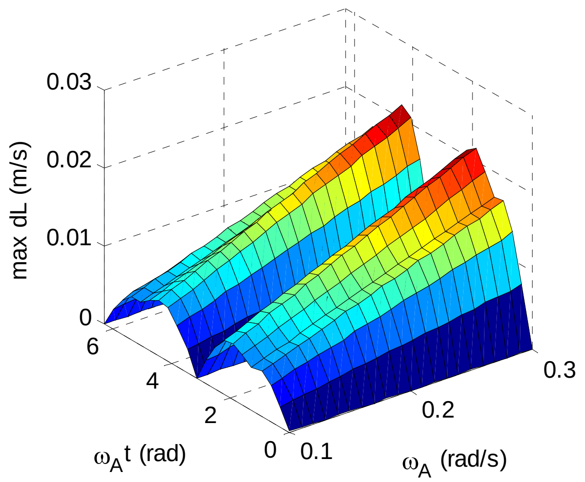

Fig. 8 maximum actuating velocity from surge motion (A_1= 0.15m)

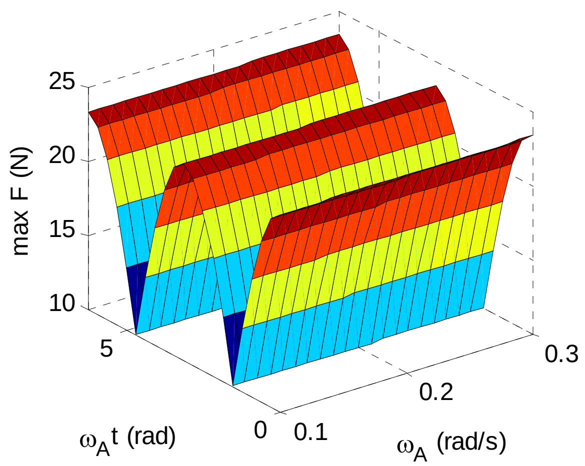

Fig. 9 maximum actuating force from sway motion (A_1= 0.15m)

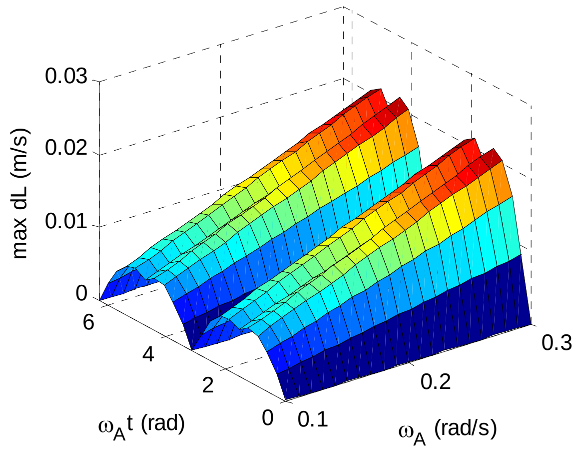

Fig. 10 maximum actuating velocity from sway motion (A_1= 0.15m)

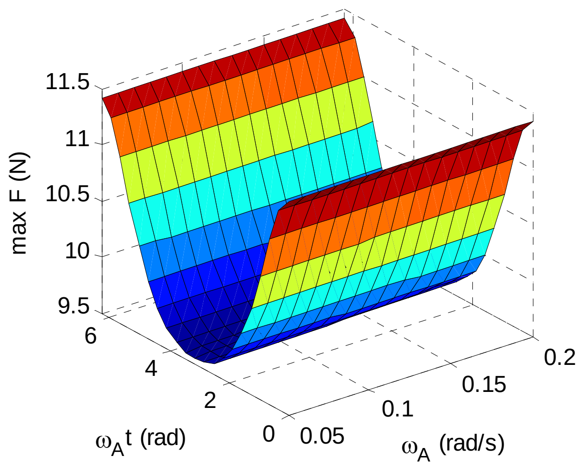

Fig. 11 maximum actuating force from heave motion (A_0= 0.45m, A_1= 0.15m)

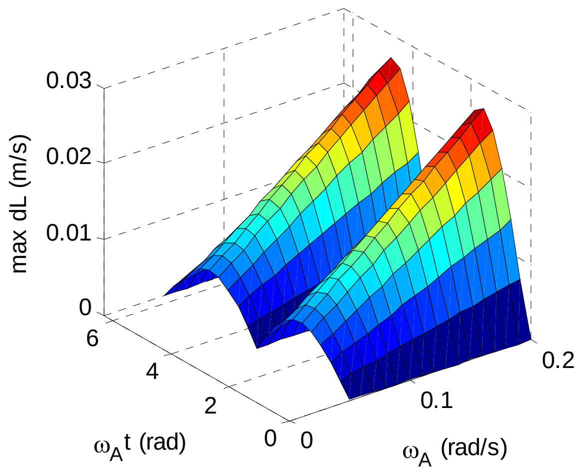

Fig. 12 maximum actuating velocity from heave motion (A_0= 0.45m, A_1= 0.15m)

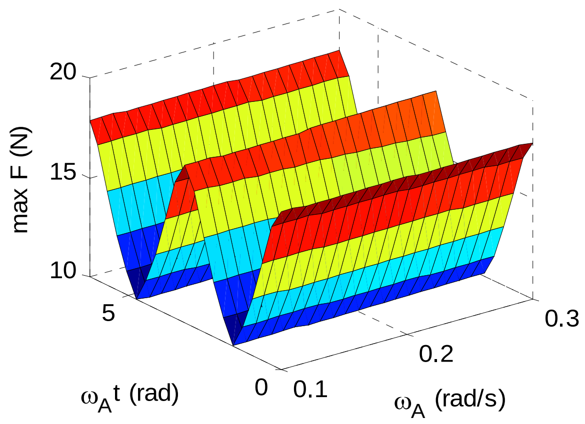

Fig. 13 maximum actuating force from roll motion ($A_1 = 45$deg)

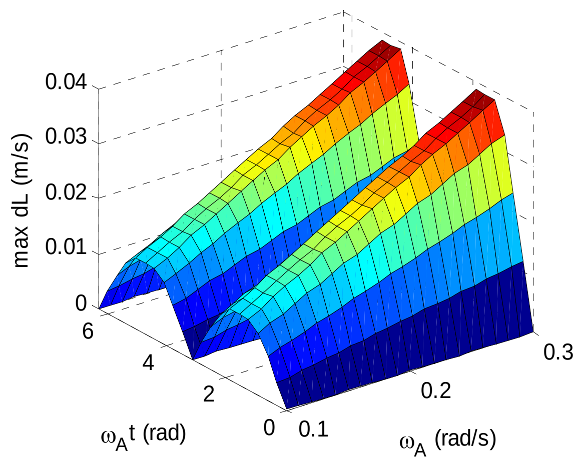

Fig. 14 maximum actuating velocity from roll motion ($A_1 = 45$deg)

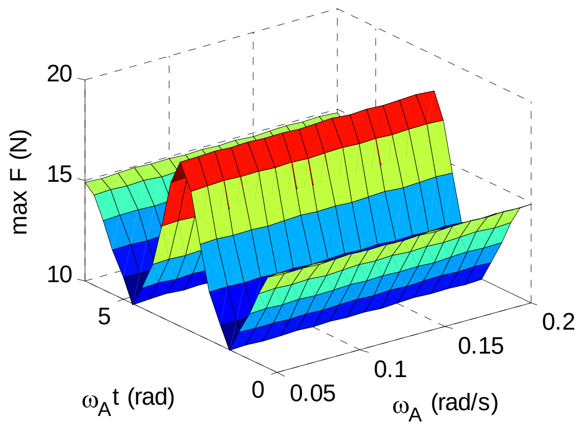

Fig. 15 maximum actuating force from pitch motion ($A_1 = 45$deg)

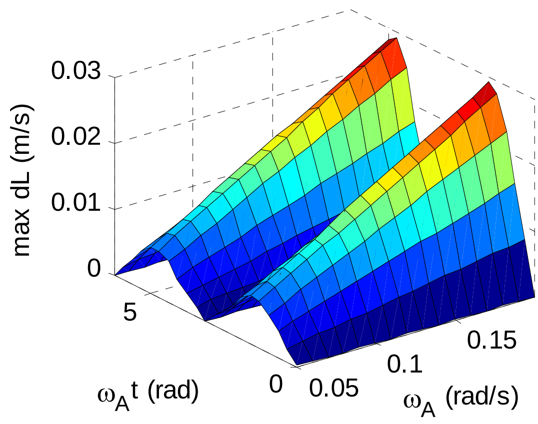

Fig. 16 maximum actuating velocity from pitch motion ($A_1 = 45$deg)

From the results, the capability of the example Stewart platform mechanism for the specified motions is limited by the speed of the actuator. This is common for electric linear actuators which are known for high actuating force with slow movement speed. For the example platform, the maximum actuating speed for the desired motion is at $\omega_A t = \pi/2$ and $3\pi/2$. For the maximum actuating force, it is happened at $\omega_A t = 0, \pi, 2\pi$ for all motions; expect the heave and pitch motions.

In these test cases, the effect of rotational mass of the actuators is insignificant. The results show negligible effect of actuator moment of inertia. This is because the motion of the platform is very slow so that the actuators require small angular acceleration as well as excessive force due to their moment of inertia.

5. Conclusion

This paper presents a method for the determination of required actuator force and speed for the desired oscillating motion of the Stewart platform. The limitation of the platform can be determined using the performance of the actuator. The method utilizes theoretical equations of motion of Stewart platform, assumes no friction in the actuators and their joints, and accounts for the inertia of the actuators.

6. Acknowledgement

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7. References

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