Modeling of inflow freestream turbulence for large-eddy simulation of bypass transition on a flat plate boundary layer using data driven approaches

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Abstract

Free-stream turbulence plays an important role in a receptivity mechanism of bypass transition and performance prediction of compressors and turbines in modern gas turbines. Free-stream turbulence may be fully characterized in laboratory condition but not in real engine operation conditions. Ad hoc assumptions are needed in order to fully characterize this inflow free-stream turbulence in computational models. There is no guarantee that the modeled free-stream turbulence will reflect realistic free-stream turbulence in those conditions or the results obtained will match experimental measurements. This work proposes a method to model inflow free-stream turbulence for large-eddy simulation (LES) using data-driven approaches. Synthesized inflow free-stream turbulence for the simulation is characterized by turbulent intensity and length scale. T3A bypass transition in a flat-plate boundary layer is used as a test-bed. Optimization based on response surface models drives those two parameters in synthesized free-stream turbulence to achieve the minimization of the total squared error of local wall shear stress coefficients compared to experimental data. The proposed method removes trial-and-error process and shows a promising result.

Keywords: Inflow Modeling, Synthesized Free-stream Turbulence, Data-Driven Approaches, Response Surface Models, Bypass transition, Design of Experiments.

1. Introduction

Bypass Transition is the process of transition formation during free-stream and turbulence perturbation combination. The process can be found among engineering applications such as flows in turbomachines. The study of bypass transition started on the flat plate in order to understand the development of perturbation under the zero-pressure gradient condition. A simulation is built to predict the period of transition occurrence including flow characteristics during the transition process.

Rai and Moin[1] used a prescribed value of the root-mean-square intensity, length scale, and range of frequencies for inflow generation. The simulation of the study is direct numerical solution (DNS). Despite coarse grid resolution, the result is well qualitative.

The later research of Voke and Yang[2] studied the similar topic. Their study conducted the comparison with T3b case of ERCOFTAC experiment in addition to the previous research of Rai and Moin. Moreover, it used different method which is large eddy simulation (LES) with the sub-grid scale model (SGS) selection of Smagorinsky and Lilly.

In this study, the precursor simulation which has pseudorandom disturbance at the inflow was used to generate more realistic free-stream turbulence for input into non-leading edge successor simulation with a smooth cutoff of free-stream simulation. The simulation resulted in a qualitative outcome in spite of coarse grid resolution.

Jacobs and Durbin[3] researched further from the two previous researches. However, the study selected to implement DNS for the simulation with simulation by using the eigenfunctions of the Orr–Sommerfeld to generate inflow of fine grid which provided the quite successful results close to both T3a and T3b experiment.

Xu et al.[4] applied LES simulation in their study. However, the difference are the sub-grid scale model and the grid quality which changed to be Kinetic energy and finer grid. The simulation comparison of T3a was made in the study which generated result improvement from Voke and Yang.

It can be inferred from the literatures regarding bypass transition simulation that there is a problem occurred in all researches in which turbulence intensity of numerical simulation decays faster than natural free-stream turbulence.

According to the problem, there are no systematic way to predict turbulence intensity and length scale generation.

This research aim to model bypass transition systematically by using design of experiment (DOE) and response surface method.

2. Methods

2.1 Finite-volume LES

The model set up is divided into 2 scales; large and small scale. The large scale applied three-dimensional filter Navier-Stokes equation in Eqs. (1) - (2) the calculation as follows:

$$\frac{\partial \bar{u}}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(\frac{\bar{P}}{\rho}\delta_{ij} + \tau_{ij}\right) + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \tag{2}$$

where $\bar{u}_i, \bar{p}, \rho, \delta_{ij}$ and ν are the filtered velocity field, the filtered static pressure, the density, the Kronecker delta and the kinematic viscosity, respectively. The small scale calculation used kinetic energy model in SGS without wall function.

2.2 Resolution

A method to solve this problem is create a domain which can generate more natural stream movement (Rai and Moin 1992).

The creation of domain to support T3A experiment based on ERCOFTAC (Roach and Brierly 1990) with a free-stream turbulence level of 3% which based on the research of Xu et al. presented in Fig. 1.

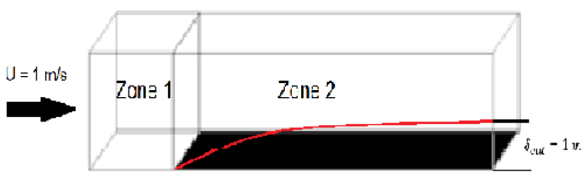


Fig. 1 Geometry of domain. Left, zone 1; right, zone 2.

Zone 1 and zone 2 of the domain have streamwise, wall-normal and spanwise length of $(L_{x_1}, L_{y_1}, L_{z_1}) = (3.53\delta_{out}, 13.4\delta_{out}, 4\delta_{out})$ and $(L_2, L_2, L_2) = (134\delta_{out}, 13.4\delta_{out}, 4\delta_{out})$ where x, y and z are streamwise, wall-normal and spanwise direction, respectively. While δ_{out} is outlet Blasius boundary layer thickness at Rex = 448900 which used to normalize the size of domain. The total grid points are $(N_x \times N_y \times N_z) = (780 \times 80 \times 64)$. A resolution provided by these dimensions are $\Delta x^+ = 28.94, \Delta z^+ = 10.26$ and Δy^+ varying from 0.93232 at the wall to 142.51 in wall units which is based on the friction velocity just after transition is completed.

$$\Delta t^+ = \frac{\Delta t u_\tau^2}{\nu} \tag{3}$$

Where u_τ = 0.049 m/s and ν =0.00029851 m^2/s. From Eq.3, the time steps depend on turbulent intensity of each case but it is limited and maintained below 0.5 by the courant numbers in all simulations.

This study used finite-volume central difference as a spatial discretization and second order implicit is used as a temporal discretization. Fractional-time steps is selected as a pressure velocity coupling scheme.

2.3 Boundary condition

The Inlet boundary of zone 1, this research used U = 1 m/s, V = 0 m/s and W =0 m/s as upstream boundary with the free-stream disturbances by using vortex method limited to turbulent intensity (Tu) and length scale (L) as in Table. 1. The lower boundary of this zone applied a symmetry condition.

Table. 1 Turbulent intensity and length scale of each case.

Case	Tu (%)	L (δ_{out})
Case1	3.00	1.00
Case2	3.00	2.00
Case3	4.00	1.00
Case4	4.00	2.00

The no-slip condition applied together with adiabatic-wall condition as lower boundary of zone 2. Periodic and symmetry boundary conditions are used in spanwise direction of the domain and on upper surface for both zones.

3. Result

Zone 1 of the domain was built to make the behavior of free-stream turbulence close to the isotropic turbulence where $u' = v' = w'$ before moving to Zone 2 in order to be close to natural free-stream turbulence.

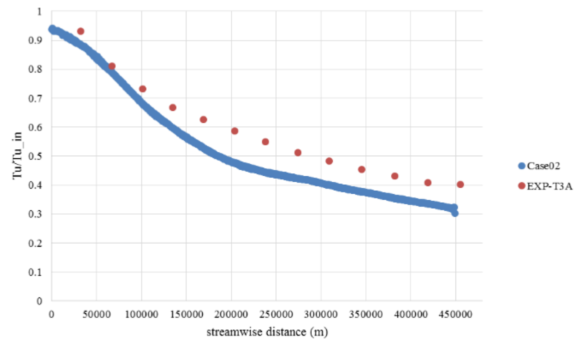


Fig. 2 Free-stream decay

Decay rate of the model can be illustrated as Fig.2 which included the result of T3a. It is obvious that the decay speed of the used model is faster than experiment due to flow simulation.

3.1 Initial Cases

Skin friction coefficient (Cf) versus Reynolds number is the parameter which provides a good overview of onset point and transition length. The onset point has the lowest Cf while transition length is the distance from the lowest to the highest Cf. Fig. 3 exhibits case 1 as the only one case that turbulence completed beyond the domain.

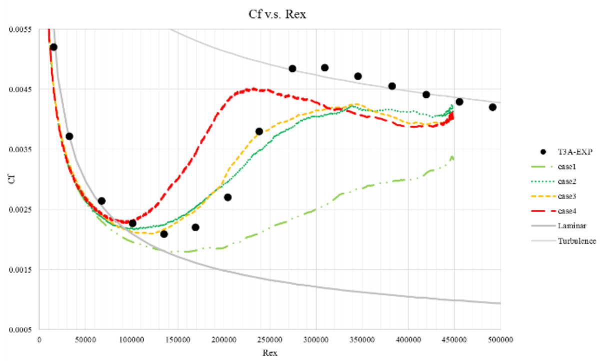


Fig. 3 Averaged skin friction coefficient

The characteristic of the case is the lowest Tu and L. Considering Fig. 4, it explains case 1 that flow of the case near the wall area is stimulated but it cannot become turbulence which is later taken away from the domain by free-stream. Therefore, third order Polynomial equation is used for extrapolation to find the transition length.

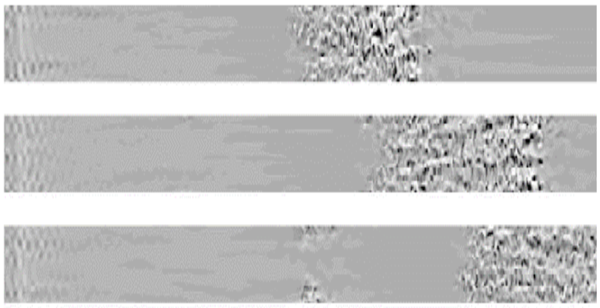
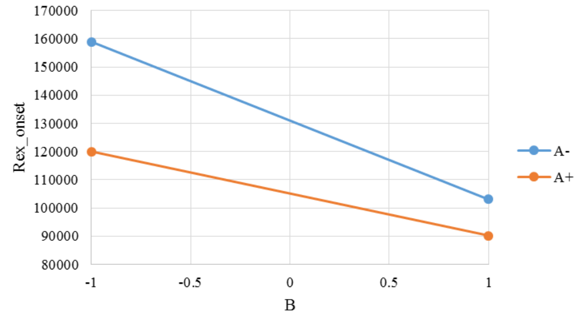


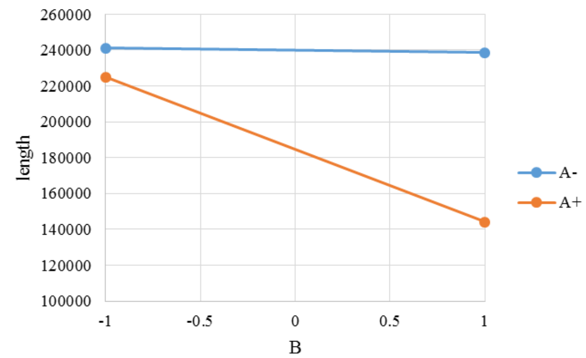
Fig. 4 Streamwise Velocity of Case 1

3.2 Optimization

The data appears in Fig. 5 is normalized and showed its relationship of Turbulence intensity (Tu) and Length scale (L).



(a)



(b)

Fig.5 (a) Relationship of Tu and L affecting onset (b) Relationship of Tu and L affecting transition length.

The value A represents the coded turbulent intensity while B represents the coded length scale. Thus, AB is the interaction between A and B. The selection of full factorial equation required consideration of the interaction between Tu and L which are considered by using two factors:

3.2.1 The result of parameter affecting Re_onset

According to part (a) of Fig. 5, it can be illustrated that at the same level of L, the increase in Tu moves the onset closer to upstream. While at the same turbulence level, the increase in L can also move the onset to the same direction. However, the results of L appears lesser when Tu increased.

3.2.2 The result of parameter affecting Re_length

Part (b) of Fig. 5 shows that when Tu is low, L is hardly affect the length of transition. On the other hand, when Tu is high, length scale has an effect on transition length in the way that the larger length scale is, the faster flow become turbulence.

The interaction between Tu and L illustrated both in onset point and transition length which adjusted full factorial equation to be:

$$\hat{Y} = \bar{Y} + \frac{Y_1}{2}A + \frac{Y_2}{2}B + \frac{Y_3}{2}AB \qquad (3)$$

Where $\hat{Y}$ is the modeling Re_onset, $\bar{Y}$ is an average value of onsets, Y_1 is the difference between the average minimum value of A and the average maximum value of A. Y_2 is the difference between the average minimum value of B and the average maximum value of B. Y_3 is the difference between the average minimum value of AB and the average maximum value of AB. In this case, the parameters of Re_onset (Y_o) and Re_length (Y_1) shown as Eq. 4.

$$Y = \begin{bmatrix} Y_o \\ Y_1 \end{bmatrix} = \begin{bmatrix} Re_{onset} \\ Re_{length} \end{bmatrix} = \begin{bmatrix} 135000 \\ 174000 \end{bmatrix} \qquad (4)$$

Table.2 Design of experiment of the research

Case	Tu	L	A	B	AB	Y_0	Y_1
1	3	1	-	-	+	158886	241114
2	3	2	-	+	-	103069	238624
3	4	1	+	-	-	119903	225039
4	4	2	+	+	+	90075	144119

The response surface equation used in modeling can be conducted from the data from Table.2 which are presented in Eq. (5)

$$\begin{bmatrix} 135000 \\ 174000 \end{bmatrix} = \begin{bmatrix} 117983 - 12994.25A - 21411.15B + 6497.185AB \\ 212224 - 27648.03A - 20852.51B - 38224.05AB \end{bmatrix} \qquad (5)$$

The study found that Tu in modeling case is 3.916% and L in the similar case is 1.537 m.

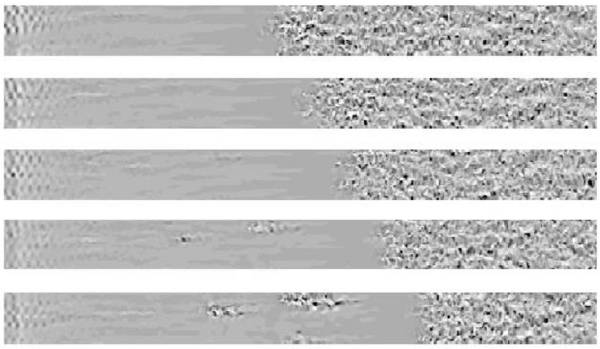
3.2.3 Modeling case analysis

According to the error comparison in Table.3, case DOE has the closest transition length to the experiment. However, if the point where transition occurred is considered, the case with the closest value to the experiment is case 3.

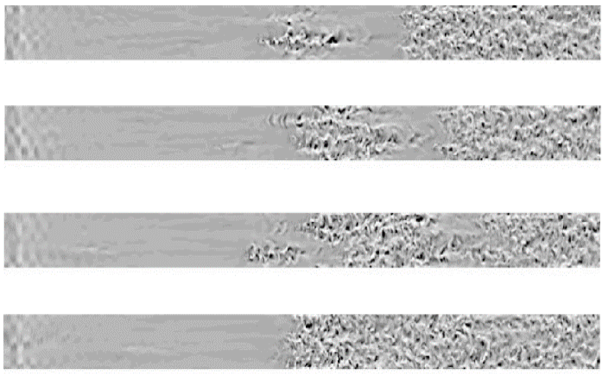
Table.3 Comparison of Errors to the experiment

Case	Re_{onset} Error (%)	Re_{length} Error (%)
Case 1	17.6930542	38.57148
Case 2	-23.6526474	37.1401
Case 3	-11.1833086	29.33261
Case 4	-33.278099	-17.1728
Case DOE	-30.8717407	-10.7232
Experiment	0	0

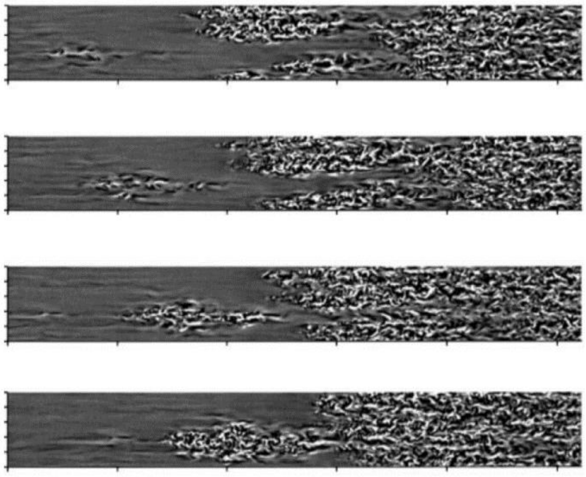
It appeared in Fig. 6 part (a) that streak occurred at upstream moving toward downstream together with turbulent region. This phenomena can be seen from part (c) as well.



(a)



(b)



(c)

Fig. 6 Instantaneous contour of fluctuating wall-normal velocity. (a) case 3; (b) case DOE. (c) Jacobs and Durbin

It can be seen from Fig. 6 that case 3 has the period in transition occurrence longer than Case DOE which corresponds to the flow physics that when the length scale increases, the transition process becomes shorter [5]. Fig. 7 presents the clear transition length.

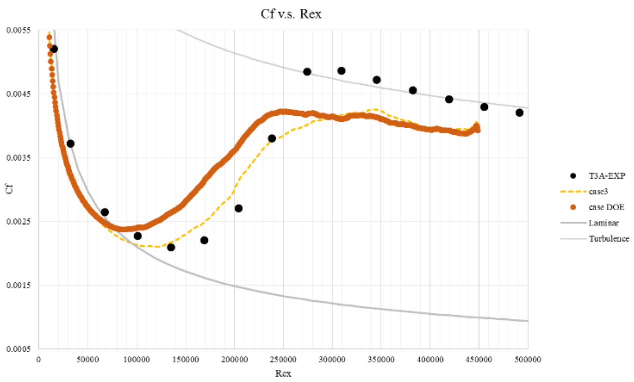


Fig. 7 C_f comparison of case DOE, case 3 and Jacobs and Durbin.

4. Conclusion

The problem regarding decay speeds of turbulence intensity can be solved by the systematic process of DOE and response surface method which can help in turbulent intensity and length scale prediction.

The value of Tu and L received from modeling is still within the scope of the initial parameter that if the values Tu and L used to predict the onset and transition lengths are within the limits set by the research.

The use of full factorial allows the possibility to model the transition length well. Moreover, conducting DOE inform that there is an interaction between Tu and L which has an impact on Transition length.

The case result from DOE modeling has the basic characteristics of transition and turbulent flow which can be seen from the comparison of wall-normal fluctuation velocity and mean skin friction coefficient.

5. Acknowledgement

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