Tracking Control of UGV using the Fundamental Equation

Tachakorn Hansawangkit and Thanapat Wanichanon[1,*]

[1] Department of Mechanical Engineering, Faculty of Engineering, Mahidol University, Thailand
*Corresponding author: 999 Phuttamonthon 4 Road, Salaya 73170
E-mail: thanapat.wan@mahidol.edu

Abstract

Nowadays, they are many methods provided to solve for the equations of motion of the constrained mechanical system. In this paper, we use the fundamental equation of constrained motion method because it is a closed-form equation that can be used to solve linear/nonlinear motions even for the system subjected to redundant constraints or conditions. We consider the trajectory tracking control of a mobile robot. Velocity profiles of robot wheels are represented by kinematical information that is obtained from the closed-form equation of the fundamental equations. To show the efficiency of the control, the tracking path is chosen as a circle path. And then the real non-holonomic robot is developed to use with the velocity control signal to verify the effectiveness of the control in real life application.

Keywords: The Fundamental Equation, Tracking, Non-holonomic Robot, Constraint Equation

1. Introduction

In the history of robot revolution, one of well-known first robot [1] is called Greek God Nepheastus. It is a simple mechanical robot that is used for opening and closing water valves. In these days, robots now can substitute human for doing many works such as applications from Industrial robots, Service robots, Educational robots, Modular robots, Collaborative robots, and Mobile robots.

One of those mobile robots is called an Unmanned Ground Vehicle (UGV), which is mechanized equipment that moves across the ground [2]. The UGV needs a path planning to work properly. Thus, many engineers and scientists have studied and discovered for the best equations for path planning. Some of them are as following.

First path planning was founded by Dubin in 1920s [3]. He used the simple geometry, circle and straight line, to find the shortest curve that connects two points in two dimensional planes. This is the based model of path planning at the present time. Next, Journ'es [4] improves Dubin path for non-holonomic robot (NHR), the robot that has degree of freedom less than controlled degree of freedom [5]. He calls this developed path as Suboptimal Continuous-Curvature Paths (SCCP). SCCP is 10 times more accurate than Dubin Path.

Currently, Dubin path is still be widely used. For example, Xian-Zhong, et al. improve the Dubin path to Bounded Curvature Path (BCP) by applying Dubin path for dynamic equation. BCP can plan a pathway through points but it has not got a perfect trajectory tracking yet.

After that there are many researchers who aimed to propose for the perfect trajectory tracking, such as N.E. Pears [7] who used steering control as shown in Fig. 1, Gregor klancar [8] who used First Order Kinematic Model (speed) for path planning of NHR as seen on Fig. 2, Mohammed A.H. Ali, et al. [9] who used Hybrid Method, which compounds of Resolve Acceleration Control (RAC), Active Force Control (AFC), and RAC-AFC as illustrated on Fig. 3.

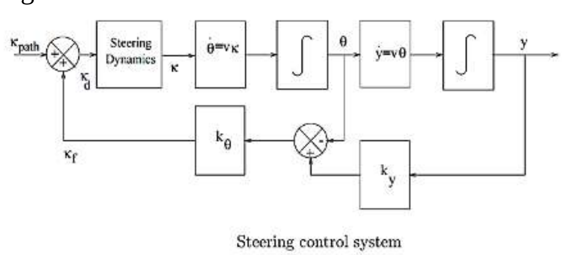


Fig. 1 N.E. Pears scheme

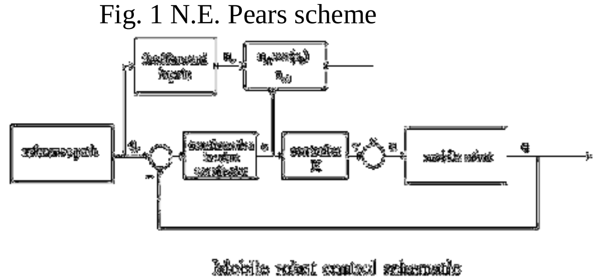
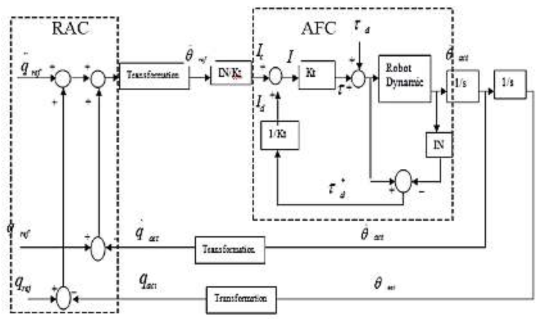


Fig. 2 Gregor klancar scheme



Illustrate the RAC-AFC control Scheme.

Fig. 3 Mohammed A.H. Ali, et.al. scheme

All mentioned methodologies are highly accurate for trajectory tracking. However, they have complex controlling schemes so that their controllers are high cost and are rarely found in the market.

Pioneer 3-AT is a good robot for NHR. It embedded controller within itself. Xiaoming Lang [10] planned a path of Pioneer 3-AT by controlling the wheels from kinematic model. The robot has a perfect trajectory tracking while carrying goods but its cost is still pretty high (about 30,000 baths).

This work is about making an accurate trajectory tracking with low price NHR. For path planning, we choose the fundamental equation of constrained motion because it represents in a closed-form and can be used with redundant constraints and conditions. The fundamental equation was previously used for NHR by Hao Sun, et.al. [12], which they called their application as Udwadia-Kalaba Approach. They used a torque controller for tracking control of mobile robots. And since a torque controller is not easy to find and is difficult to make when comparing with a velocity controller, in this work we improve their approach by using a velocity profile of NHR wheels instead of the torque profile. Then Arduino controller is used to execute the velocity command signal. This makes the NHR's price lower to about 2,000 baths.

2. Dynamics of the UGV system

We utilize the conceptualization of the fundamental equation [13] in obtaining our UGV system. First the unconstrained equation is described in which its coordinates are all assumed independent of each other. The equation of motion of this system is given, using Lagrange's equation, by

$$M(r,\theta)\ddot{q} = Q(r,\theta,\dot{r},\dot{\theta}) \qquad (2.1)$$

with the initial conditions

$$r_0 = q_r\,, \theta_0 = \ q_\theta, \dot{r}_0 = \dot{q}_r\,, \dot{\theta}_0 = \ \dot{q}_\theta \qquad (2.2)$$

where $q = [r,\theta]^T$ is the generalized coordinate 2-vector; M >0 is the 2 by 2 mass matrix, and Q is a given force vector, which is a known function of $r, \dot{r}, \theta$ and $\dot{\theta}$.

From Eq. (2.1), we find the acceleration ($\ddot{q}$) of the uncontrolled system given by

$$\ddot{q} = M^{-1}(r,\theta)Q\big(r,\theta,\dot{r},\dot{\theta}\big) := a. \qquad (2.3)$$

Second, we impose a set of control requirements as constraints on this uncontrolled system. We suppose that the uncontrolled system is now subject to the p sufficiently smooth control requirements given by

$$\varphi_i\big(r,\theta,\dot{r},\dot{\theta}\big) = 0\,; i = 1,2,3,\dots,p \qquad (2.4)$$

where $s \leq p$ equations in the equation set of Eq. (2.4) are functionally independent. The control constraints described by Eq. (2.4) include all the usual varieties of holonomic and/or non-holonomic constraints and they do not permit all the components of the initial conditions $r_0, \dot{r}_0, \theta_0$ and $\dot{\theta}_0$ to be independently assigned. We shall assume that the initial conditions in Eq. (2.2) satisfy the p control requirements. (If not, the control constraints can be expressed in an alternative form so that they are asymptotically satisfied [14].

Differentiating the control requirements in Eq. (2.4), we obtain the relation

$$A\big(r,\theta,\dot{r},\dot{\theta}\big)\ddot{q} = b(r,\theta,\dot{r},\dot{\theta}), \qquad (2.5)$$

where A is an p by n matrix whose rank is s and b is an p-vector. We note that each row of A arises by appropriately differentiating one of the p control requirements in the set given in Eq. (2.4).

In the third step, the equation of motion of the constrained (controlled) system is given by

$$M(r,\theta)\ddot{q} = Q\big(r,\theta,\dot{r},\dot{\theta}\big) + Q^c\big(r,\theta,\dot{r},\dot{\theta}\big) \qquad (2.6)$$

where Q^c is the control force n-vector that arises to ensure that the control requirements in Eq. (2.5) are satisfied. Using the fundamental equation, the explicit equation of the constrained UGV system is given by

$$M(r,\theta)\ddot{q} = Q + \ A^T(AM^{-1}A^T)^+(b - Aa), \quad (2.7)$$

where in the various quantities have been defined in the previous two steps and the superscript "+" denotes the Moore-Penrose inverse of a matrix [15]. In the above equation and in what follows, we shall suppress the arguments of the various quantities unless required for clarity.

We note that Eq. (2.7) is valid (i) whether or not the control requirements are holonomic or non-holonomic, (ii) whether or not they are nonlinear functions of their argument, and (iii) whether or not they are functionally dependent. The control force that the uncontrolled system is subjected to, because of the presence of the control requirements in Eq. (2.4), can be explicitly expressed as

$$Q^c(r,\theta,\dot{r},\dot{\theta}) = A^T(AM^{-1}A^T)^{+}(b - Aa). \quad (2.8)$$

The control force given in Eq. (2.8) is optimal in the sense that it minimizes the control cost at each instant of time.

Pre-multiplying both sides of Eq. (2.7) with M^{-1}, the acceleration of the constrained system that satisfies the constraint in Eq. (2.4) can be expressed as

$$\ddot{q} = M^{-1}Q + M^{-1}A^T(AM^{-1}A^T)^{+}(b - Aa), \quad (2.9)$$

a relation which we shall require later on.

3. Path planning for a circle path

For illustrative purpose, this paper is going to show the method of path planning for a circle path. Using the conceptualization from the fundamental equation [13] as mentioned in Section 2, we obtain:

3.1 The unconstrained system

We first use the center of mass of the wheel of the NHR to define the coordinate for the NHR in the XY plane and assume that it is unconstrained (see Fig. 4).

Then, obtaining the equation of the unconstrained motion by using Lagrange's equation, we define the position of the NHR as

$$x_i = r_i cos\theta_i \quad (3.1)$$

$$y_i = r_i sin\theta_i \quad (3.2)$$

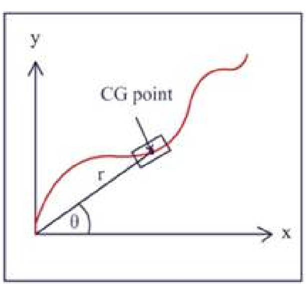


Fig. 4 The center of mass of the NHR

where

x_i is a position of the ith wheel of the NHR in the x-plane

y_i is a position of the ith wheel of the NHR in the y-plane

r_i is a position of the ith wheel in polar coordinate

θ_i is an orientation angle of the NHR in polar coordinate

From kinetic energy equation, we get

$$T = \frac{1}{2}\sum_{i=1}^{2} m_i(\dot{x_i}^2 + \dot{y_i}^2) + \frac{1}{2}I_i(\dot{\theta_i}^2)$$
$$= \frac{1}{2}m_1\left(r_1^2\dot{\theta}_1^2 + I_1\dot{\theta}_1^2 + r_1^2\right)$$
$$+\frac{1}{2}m_2(r_2^2\dot{\theta}_2^2 + I_2\dot{\theta}_2^2 + r_2^2) \quad (3.3)$$

where we mark:

m = mass putting on each wheel of the NHR

l = displacement between both wheels of the NHR

$\dot{\theta}$ = Angular velocity

And since the NHR moves in the XY-plane, the potential energy

$$V = 0 \quad (3.4)$$

Thus using (3.3) and (3.4) in the Euler – Lagrange's equation,

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}}T\right) - \frac{\partial}{\partial q}T + \frac{\partial}{\partial q}V = Q(r,\theta,\dot{r},\dot{\theta}), \quad (3.5)$$

we thus obtain

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_1r_1^2 + I_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_2r_2^2 + I_2 \end{bmatrix}\begin{bmatrix} \ddot{r}_1 \\ \ddot{\theta}_1 \\ \ddot{r}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} m_1r_1\dot{\theta}_1^2 \\ -2m_1r_1\dot{r}_1\dot{\theta}_1 \\ m_2r_2\dot{\theta}_2^2 \\ -2m_2r_2\dot{r}_2\dot{\theta}_2 \end{bmatrix}. \quad (3.6)$$

From (2.3) and (3.6), we get the unconstrained acceleration

$$a = \begin{bmatrix} \ddot{r}_1 \\ \ddot{\theta}_1 \\ \ddot{r}_2 \\ \ddot{\theta}_2 \end{bmatrix} =$$

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_1r_1^2 + I_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_2r_2^2 + I_2 \end{bmatrix}^{-1}\begin{bmatrix} m_1r_1\dot{\theta}_1^2 \\ -2m_1r_1\dot{r}_1\dot{\theta}_1 \\ m_2r_2\dot{\theta}_2^2 \\ -2m_2r_2\dot{r}_2\dot{\theta}_2 \end{bmatrix}. \quad (3.7)$$

3.2 The constraint equation

The NHR is forced to maneuver a circle path in this step. First, we assume that the NHR is moving freely in the XY-plane.

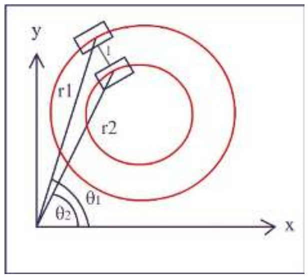


Fig. 5 Dynamics and Variables of NHR

After that, we consider a circle path as a constraint (see Fig. 5).

The first constraint is for keeping one wheel in a circular form with radius R,

$$(r_1 cos\theta_1)^2 + (r_1 sin\theta_1)^2 = R^2, \quad (3.8)$$

and the second one is for maintaining the distance between the two wheels as a constant l,

$$(r_2cos\theta_2 - r_1cos\theta_1)^2 + (r_2sin\theta_2 - r_1sin\theta_1)^2 = l^2 \quad (3.9)$$

Then, we appropriately differentiate Eqs. (3.8) and (3.9) to the form of Eq. (2.5) as

$$A\ddot{q} = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \Delta_{24} \end{bmatrix} \begin{bmatrix} \ddot{r}_1 \\ \ddot{\theta}_1 \\ \ddot{r}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -\dot{r}_1{}^2 \\ \alpha_{21} \end{bmatrix} = b, \quad (3.10)$$

where

$\Delta_{21} = r_1 - r_2\cos(\theta_1 - \theta_2)$,
$\Delta_{22} = r_1r_2sin(\theta_1 - \theta_2)$,
$\Delta_{23} = r_2 - r_1cos(\theta_1 - \theta_2)$,
$\Delta_{24} = r_1r_2sin(\theta_1 - \theta_2)$,

and

$$\begin{aligned}\alpha_{21} = -\dot{r}_1{}^2 - \dot{r}_2{}^2 + 2\dot{r}_1\dot{r}_2\cos(\theta_1 - \theta_2) \\ - 2\dot{r}_1r_2(\dot{\theta}_1 - \dot{\theta}_2)\sin(\theta_1 - \theta_2) \\ -2r_1\dot{r}_2(\dot{\theta}_1 - \dot{\theta}_2)sin(\theta_1 - \theta_2) \\ -r_1r_2(\dot{\theta}_1 - \dot{\theta}_2)^2cos(\theta_1 - \theta_2).\end{aligned}$$

3.3 The constrained system

Using Eqs. (3.6) and (3.10) in Eq. (2.9), we obtain the constrained acceleration $\ddot{q}$. We can further achieve other kinematical information (velocity, $\dot{q}$ and displacement, q) by appropriately integrating the acceleration. All these information are of the UGV when it travels in the circular format.

4. Numerical simulation for NHR

In order to verify the approach, we use a computer simulation by first identifying the physical properties of the NHR. We define the inertia of the NHR [16] and choose the radian (R) for the circle path as the following.

Moment of inertia of NHR w.r.t the left wheel: I_1 = 0.3125 kg-m^2
Moment of inertia of NHR w.r.t the right wheel: I_2 = 0.3125 kg-m^2
Mass of NHR w.r.t the left wheel: m_1 = 1.25 kg
Mass of NHR w.r.t the right wheel: m_2 = 1.25 kg
Distance between the left and right wheels: l = 0.2 m
Radius of circle path of the right wheel: R = 0.5 m
Time = 26 Seconds

Next, we begin the numerical simulation using all information from Section 3.3 with the initial displacement

$$(r_1, \theta_1, r_2, \theta_2) = (50,0.25,30,0.25).$$

and initial velocity

$$(\dot{r}_1, \dot{\theta}_1, \dot{r}_2, \dot{\theta}_2) = (0,0.25,0,0.25).$$

We then get the velocity profile of the wheels of the NHR and the tracking path of the NHR as seen on Fig. 6 and Fig. 7 respectively. We note that the red lines refer to as the response of the left wheel of NHR, whereas the one of the right wheel are shown in blue.

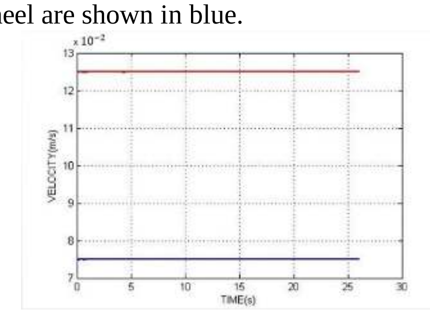


Fig. 6 Velocity of the wheels of the NHR

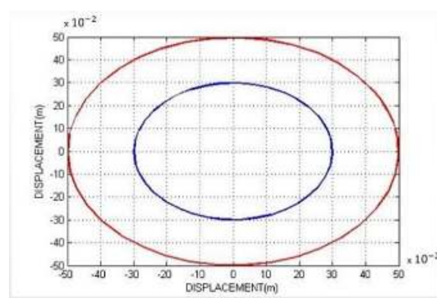


Fig. 7 Displacement of the wheels of the NHR

The errors in satisfying the constraints are shown in Fig. 8 where the red line is the error in displacement of the first constraint Eq. (3.8). The blue line is the error in velocity of the first constraint Eq. (3.8). The yellow line is the error in displacement of the second constraint Eq. (3.9). And the green line is the error in velocity of the second constraint Eq. (3.9). It can be seen that all these errors are very small. This means that our approach satisfies the trajectory tracking of the circle path.

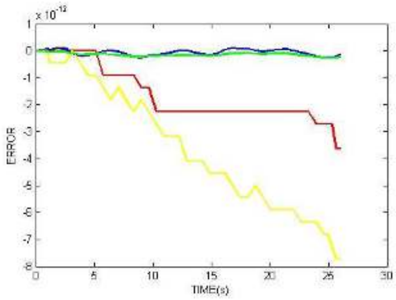


Fig. 8 Errors in satisfying the constraints

In the end, The NHR, seen on Fig. 9, is tested by using the velocity controlled 12.5×10^{-2} m/s for the first wheel and 7.5×10^{-2} m/s for the second wheel. We choose the Arduino, pulse width modulation [17], to be the velocity controller. It uses the tracking time of one round of circle path for 27.83 second. The physical tracking path confirms with the simulation result shown above.

Fig. 9 The NHR for testing

5. Conclusion

While each day mobile robots keep developing, this research proposes for another way in advance the mobile robot technology. The fundamental equation of constrained motion approach can be used to regulate the NHR by controlling velocity of its wheels.

The velocity control is a great idea for controlling the NHR. Because some engines cannot accurately feedback torque profiles, for example, all combustion engines, gas turbine, and steam turbine. This result could be the inspiration for the combustion engine robot.

In practical, we have tried the velocity control profile with our own making NHR, driven by DC motor. We use the mentioned circle path for this test. The result shows that the NHR tracks nearly to the circle path while taking roughly the time as shown in the computer simulation. This proves that our approach has the reliability of the real life application. However, it might be better to add an uncertainty controller to compensate for any modeling errors and disturbance for more accurate tracking.

Future work is progressing on applying this methodology to the more complex dynamic model of mobile robots and more realistic performance with the great uncertainty controller.

6. Acknowledgement

We express our deep gratitude to Dr. Thanapat Wanichanon, Aut Chatongyot, Sitti Egkamol and Kultol Sinsakul for advising to pass this project. We hope this project will inspire other projects of our senior EGME or the good thing is we can change the world with just an idea.

7. Reference

[1] Wikipedia, *Robot*, URL: en.wikipedia.org/wiki/Robot, accessed on 9/12/2015

[2] Douglas W. Gage, *A Brief History of Unmanned Ground Vehicle (UGV) Development Efforts*, Unmanned Systems Magazine, 1995, pp. 1-7.

[3] Wikipedia, *Dubins Path*, URL :en.wikipedia.org/wiki/Dubins path, accessed on 3/08/2015

[4] Journées des Jeunes, *SubOptimal Continuous-Curvature Path Planning for Non-Holonomic Robots*, Inria Rhône-Alpes &Gravir Lab, 1999, pp. 1-6.

[5] Wikipedia, *Nonholonomic system*, URL:en.wikipedia.org/wiki/Nonholonomic_system, accessed on 11/04/2015

[6] Xian-ZhongGao, Zhong-Xi Hou, Xiong-FengZhu, Jun-Tao Zhang, Xiao-Qian Chen, *The Shortest Path Planning for Manoeuvres of UAV*, Acta Polytechnica Hungarica, 2013, *pp.* 221-239.

[7] N. E. Pears, *Mobile Robot Tracking of Pre-Planned Paths*, Department of Computer Science, 2010:1-19.

[8] GregorKlancar, DragoMatko, SasoBlazic, *Mobile Robot Control on a Reference Path*, Mediterranean Conference on Control and Automation, 2005, pp. 1343-1356.

[9] Mohammed a.h. ali, Musa, Wan azhar b. m. yussof, Zamzuri bin hamedon, Zulkili

b.m. yussof, *Wheeled Mobile Robot Path Control in a Complex Trajectory using Hybrid Methods*, Recent Advances in Electrical *Engineering*, 2015, pp. 38-43.

[10] Xiaoming Lang, *A Trajectory Tracking Controller Design for a Nonholonomic Mobile Robot*, Agricultural and Biological Engineering Department *UIUC*, 2013, pp. 1-10.

[11] Firdaus E, Udwadia, ThanapatWanichanon, *Control of Uncertain Nonlinear Multibody Mechanical Systems*, Journal of Applied Mechanics, 2014, pp. 81-92.

[12] Hao Sun, Han Zhao, Shengchao Zhen, Kang Huang, Funmin Zhao, *Application of the Udwadia-Kalaba Approach to Tracking Control of Mobile Robots*, Aries Systens Corporation, 2015, pp. 1-18

[13] Thanapat Wanichanon, *Control of Uncertain Nonlinear Multibody Mechanical Systems*, Journal of Applied Mechanics, 2014, pp. 1-10

[14] Udwadia, F. E., *2005, "Equations of Motion for Constrained Multi-Body Systems and Their Control,"* J. Optim. Theory Appl., 127(3), pp. 627–638.

[15] J.C.A. Barata, M.S. Hussein, *The Moore-Penrose Pseudpinverse. A Turorail Review of the Theory*, Instituto de Fisica, 2011, pp. 1-23.

[16] The Physics Astronomy department, Michigan State University, United State of America (2013). PHY251 Undergraduate Physics Laboratory I, *Lab 7 moment of inertia*, pp.77. URL: www.pa.msu.edu/courses/2013summer/phy251/labfiles/Lab%207%20moment%20of%20inertia.pdf

[*17*] Tutorial, Arduino LLC, Italy (2016). PWM. URL: https://www.arduino.cc/en/Tutorial/PWM