AMM-1031

Improvement of a Mathematical Model for Estimation of Sliding Loss in a Spur Gear Pair

Natcha Ponchai[1] , Surin Phongsupasamit[1] and Chanat Ratanasumawong[1]*

[1] Department of Mechanical Engineering. Faculty of Engineering Chulalongkorn University.
Phayathai Road, Pathumwan, Bangkok 10330.
*E-mail chanat.r@eng.chula.ac.th

Abstract

This paper presents the improvement of the mathematical model used to estimate the sliding loss of a spur gear pair. In the former mathematical model, the contact of a spur gear pair is considered as full tooth contact. The contact at any meshing position is a line contact which has contact length equal to the face width of the gear, and the load acting on the gear tooth surface is considered to distribute uniformly along the contact line. However, in the actual operation partial tooth contacts are also normally occur. With this reason the improvement in this paper is done by using contact stress theory to create more realistic model for calculating the contact of gear and the distribution of load .This makes the mathematical model more practical.

In this paper, the gear used in the experiment is assumed to have crowning modification about 5 microns along tooth trace direction. In the presented model, the calculated contact length is shorter than the face width, and the contact force is distributed elliptically along the contact line. These changing make the friction coefficient increase, therefore the power loss estimated from the presented model is slightly higher than the former one. Comparing with the experimental results, the power loss estimated by the improved model is close to the experimental results and better than the former model in many cases. The patterns of load distribution on the gear tooth surface calculated from the model are also agreeable with the contact pattern obtained from the experiment.

***Keywords**:* – Power loss, mathematical model, contact stress, contact line, load distribution

1. Introduction

Gear is an important component that is widely used in power transmission. Although the efficiency of a gear pair is quite high, normal transmission system consists of several pairs of gear that cause the increasing in total power loss considerably. In the energy crisis situation nowadays, the power loss in the gear pair becomes more important and should be considered carefully as well as the strength of gear in the design of the transmission system.

To understand the mechanism of gear power loss, there are many researches focused on the mathematical model for determining gear power loss [1-4]. The authors' group [1,2] has also created mathematical model for estimating sliding loss of a spur and a helical gear pair. The model can be used to estimate the sliding loss and used to study the effect of geometrical parameters on the sliding loss. Although the model is simple and probable to estimate the effect of gear parameters in many cases, in some cases such as the effect of helix angle or face width, the model cannot estimate the results precisely. One of the reasons is probably that the contact of gear teeth is assumed to be full teeth contact. The contact at any meshing position is a line contact which has contact length equal to the face width of the gear, and the load acting on the gear tooth surface is considered to distribute uniformly along the contact line. However, in the actual operation partial tooth contacts are also normally occur. This can be shown by the preliminary tooth contact experiment [5]. In this paper, the improvement of mathematical model is done by adding the effect of contact area into the former model to make the model more practical.

2. Concept of mathematical model

2.1 Sliding power loss

Power loss attributed to gears can be categorized into four categories, which are sliding loss, rolling loss, windage loss and oil churning loss. For small gears operated at low and medium speed such as gears used in passenger vehicles, rolling loss and windage loss are very small comparing with the sliding loss. Moreover for the gear pair using jet lubrication as the test rig used in this research, oil churning loss can also be neglected. With these reasons, the sliding loss is focused in this paper. The sliding mechanism of the gear pair that is the cause of sliding loss is described further. The tooth profile of spur gears is designed to be involute curve to keep a constant angular velocity ratio and proper transmission power.

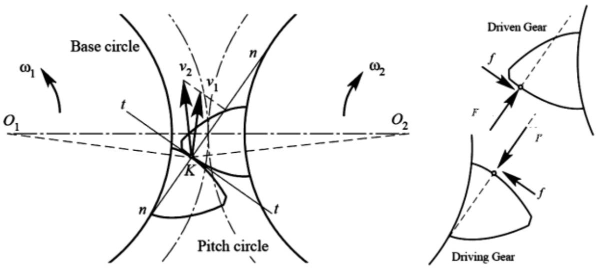


Fig. 1 Relative velocity and friction [1]

Figure 1 shows the sliding of gear tooth and the forces acting on gear tooth surface during meshing. From Fig.1 because the velocities at the meshing point of driving and driven gear are different, the relative velocity is occurred. Sliding loss of the gear pair ($P_{sliding}$) can be calculated from the product of friction force (f) and relative velocity ($V_{relative}$) and can be represented by

$$P_{sliding} = f \cdot V_{relative} \ . \qquad (1)$$

The friction force can be calculated from the relation of transmission force (F_{t-t}) and friction coefficient (μ) as shown by

$$f = \mu \cdot F_{t-t} \ . \qquad (2)$$

The calculation of sliding loss of a spur gear pair in this paper is based on the method presented in reference [1]. The sliding loss ratio (φ) that is the ratio between the sliding loss and the input power can be calculated from

$$\varphi = \frac{H_3}{H_1} = \frac{H_1 - H_2}{H_1} \ , \qquad (3)$$

Where H_1 is power input, H_2 is power output, and H_3 is power loss.

Sliding loss ratio can be written in the term of gear parameters and meshing position and shown in equation

$$\varphi = \frac{-n \cdot \tan\alpha \cdot \mu \cdot (1 + \mathrm{m_w})}{1 - (n+1) \cdot \tan\alpha \cdot \mu} \ , \qquad (4)$$

Where n is the position ratio relation to meshing position, α is the pressure angle, $\mathrm{m_w}$ is the speed reduction ratio.

Relation between sliding loss ratio calculated by Eq.(4) and meshing position is plotted in Fig.2.

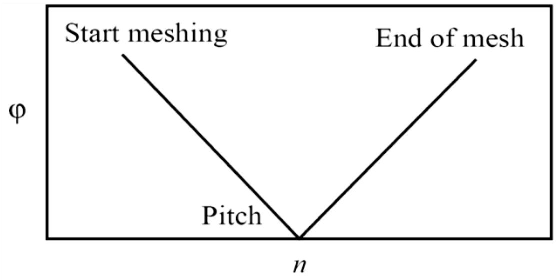


Fig.2 Sliding loss ratio [1]

2.2. Contact and load distribution on spur gear

In the former mathematical model, the contact of a spur gear pair is considered as full tooth contact. The contact at any meshing position is a line contact which has contact length equal to the face width of the gear, and the load acting on the gear tooth surface is considered to distribute uniformly along the contact line. However, in the actual operation partial tooth contacts are also normally occur. Figure 3 shows a sample of tooth contact pattern obtained from a preliminary experiment. The gear teeth were painted before doing the experiment. After the experiment the photos of teeth were collected. The area that the painted is peeled off represents the actual teeth contact area. This evidence confirms that the assumption of full tooth contact used in the former model is not suitable for all case. In this paper the contact stress theory [6] is applied to construct more realistic model including the contact of gear teeth surface and load distribution along the line of contact into consideration.

Here the gear tooth surface of spur gears is assumed to have small amount of crowning modification along the tooth trace direction as shown in Fig. 4, therefore the contact of gear -

Fig.3 contact pattern experiment [5]

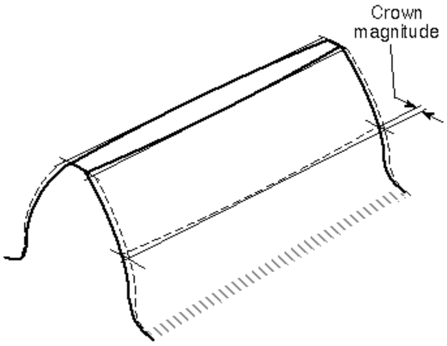
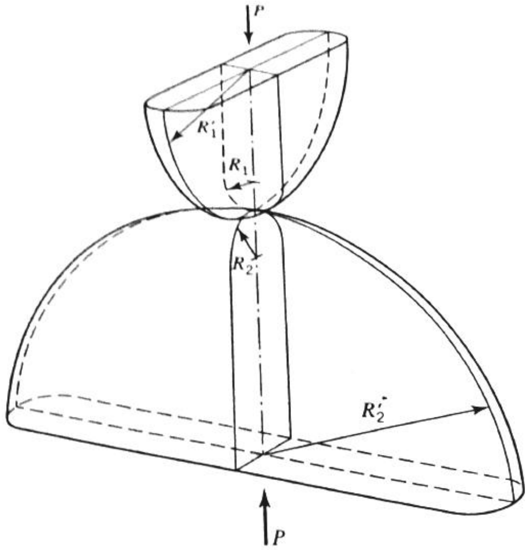


Fig.4 Crowned gear and non crowned gear

(A)



(B)

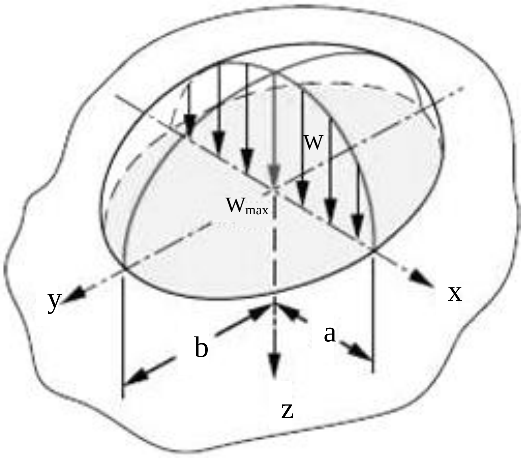


Fig.5 The contact area [6]

tooth surface of a spur gear pair can be considered as same as the contact of two curve surface as shown in Fig.5 (A). The radii of curvature of the curve surface correspond to the radius of curvature of gear profile and the radius of curvature of crowning modification. The shape of contact area of two curve surface is an elliptic shape as shown in Fig.5 (B) . By using contact stress theory the length of the major axis (a) and for the minor axis (b) can be calculated from

$$a = \frac{b}{k} , \tag{5}$$

$$b = c_b \sqrt[3]{P\Delta} , \tag{6}$$

where P = Force,

$\Delta = \frac{2(1-\nu^2)}{(A+B)E}$,

E = Young's modulus,

ν = Poisson ratio.

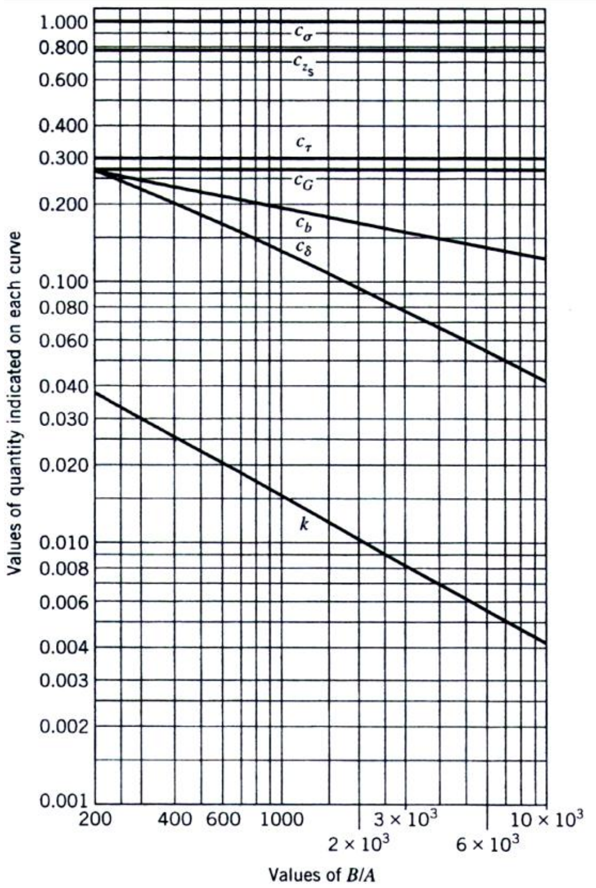


Fig.6 c_b and k constant [6]

Note that the values of cb and k in Eqs.(5) and (6) can be known by using the graph in Fig.6. These values depend on the values of A and B that are the parameters corresponding to the geometrical parameters and can be calculated by

$$A = \frac{1}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_1^{'}}+\frac{1}{R_2^{'}}\right)$$
$$-\frac{1}{4}\sqrt{\left[\left(\frac{1}{R_1}-\frac{1}{R_1^{'}}\right)+\left(\frac{1}{R_2}-\frac{1}{R_2^{'}}\right)\right]^2-4\left(\frac{1}{R_1}-\frac{1}{R_1^{'}}\right)\left(\frac{1}{R_2}-\frac{1}{R_2^{'}}\right)\sin^2\alpha} ,$$

$$B = \frac{1}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_1^{'}}+\frac{1}{R_2^{'}}\right)$$
$$+\frac{1}{4}\sqrt{\left[\left(\frac{1}{R_1}-\frac{1}{R_1^{'}}\right)+\left(\frac{1}{R_2}-\frac{1}{R_2^{'}}\right)\right]^2-4\left(\frac{1}{R_1}-\frac{1}{R_1^{'}}\right)\left(\frac{1}{R_2}-\frac{1}{R_2^{'}}\right)\sin^2\alpha} ,$$

Where

R_1 = distance from center of driving gear to meshing position,

R_2 = distance from center of driven gear to meshing position,

R'_1 = radius of curvature on driving gear face,

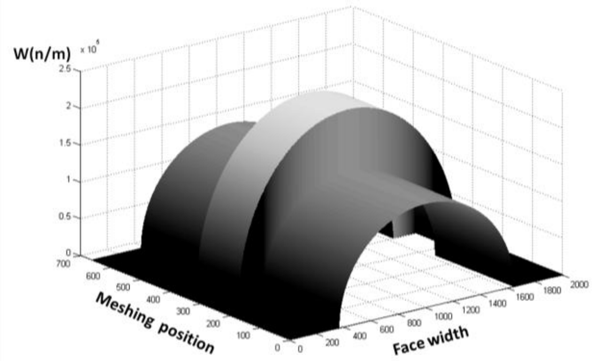
R'_2 = radius of curvature on driven gear face,

α = angle between curve planes having radius of curvature R_1 and R_2.

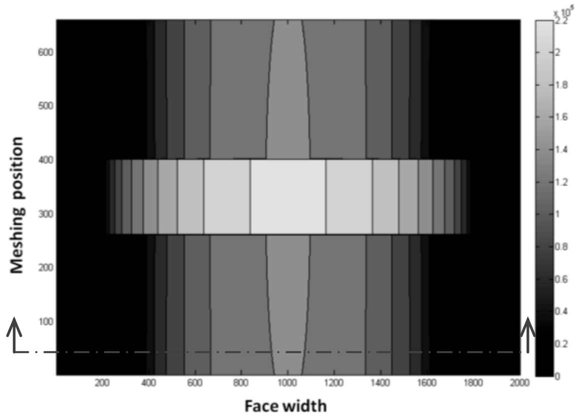
Normally amount of crowing modification is about 5-10 micrometer. This makes the radius of curvature in the tooth trace direction is very large, and then the length of minor axis (b) in Fig. 5 is very small, hence the shape of contact area can be simplified from elliptic shape to be a line that its length equal to 2×(the length of major axis, a). The load distribution along the line of contact (W_0) is the elliptical shape. The Maximum load distribution (W_{0max}) can be determined from

$$W_{0,\max} = \frac{4P}{\pi a} . \tag{7}$$

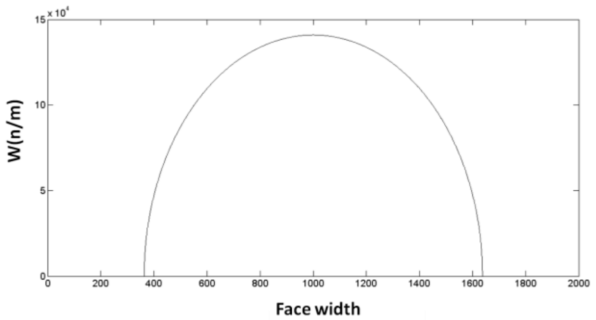
Figure 7 shows the load distribution along tooth trace direction at various meshing position calculated from the method described above. Figure 7(A) and (B) are the 3D-plot and the contour plot of load distribution, respectively. Figure 7(C) shows the load distribution at one meshing position that shown by the dash line in Fig.7 (B). The suddenly increasing of load distribution in Fig.7 is caused by the changing from double teeth meshing to single tooth meshing.



(A)



(B)



(C)

Fig.7 Load distribution

2.3. Friction coefficient

Friction coefficient (μ) is a very important parameter affecting to the power loss calculation. There are many researches proposed many empirical formulas for calculation of friction coefficient. In this calculation, the formula proposed by ISO TC60 [7] is used since it provides the good results in the former study [2]. Here the friction coefficient is calculated by using the equation

$$\mu = 0.12[WS/(RV_r \nu)]^{0.25}, \qquad (8)$$

where W is load per length, S is surface roughness, R is combined radius of curvature, and Vr is sum of the rolling velocities.

The friction coefficient calculated from Eq.(8) are shown in Fig.8. In the figure, the changes of friction coefficient along face width at various meshing are shown. Figures 8(A) and (B) are the 3D-plot and the contour plot of the friction coefficient respectively, whereas fig.8(C) represents the variation of friction coefficient along face width at the meshing position that shown by the dash line in Fig.8(B). In calculation here, the friction coefficient used to calculate the sliding loss is determined by averaging the friction coefficient along face width direction at each meshing position. Figure.8(C) shows the averaged friction coefficient used in calculation. The suddenly change of friction coefficient in the figure is caused by the changing from double teeth to single tooth meshing that results in suddenly change of load.

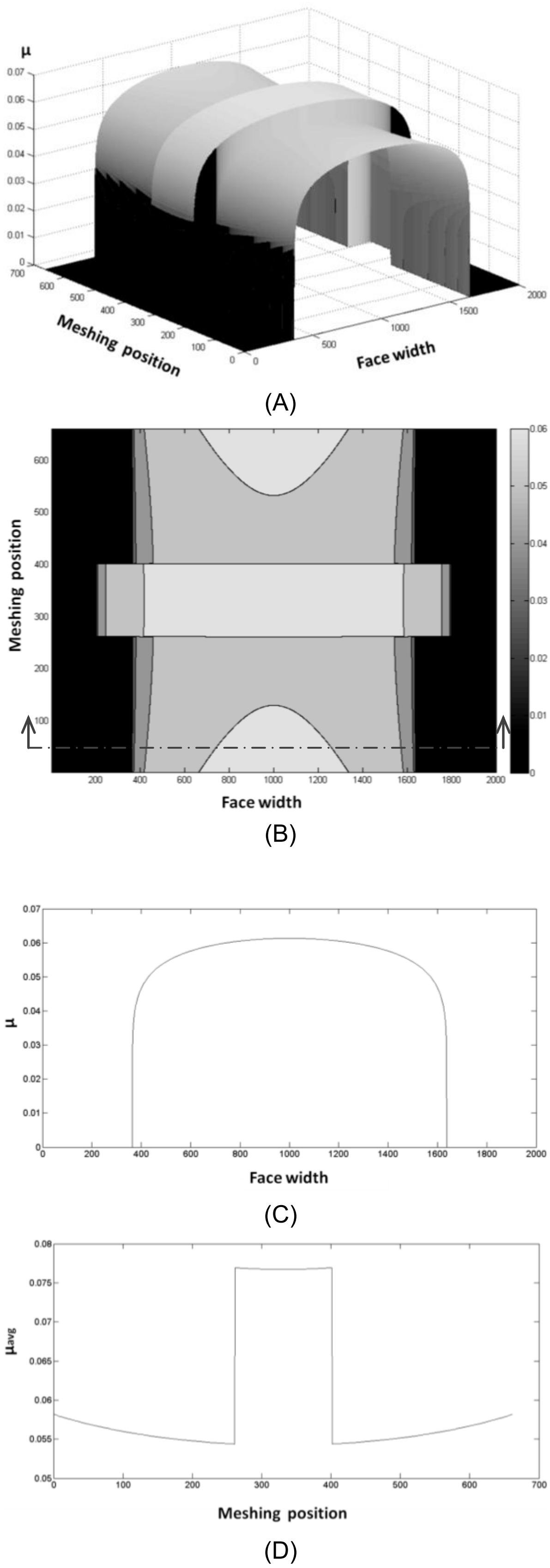


Fig.8 Friction coefficient distribution

2.4. Sliding loss calculation procedure

The flow chart of sliding loss calculation is shown in Fig.9. The calculation is start from input the values of gear parameters, surface roughness, radii of curvature of gear tooth surface, viscosity of lubricant oil, and working condition into calculation program. After that, the gear parameters are used to calculate the contact length (length 2a in Fig.5) in each meshing position. This length of contact line is used along with gear parameters and working condition to determine load distribution (W_0). Then the variation of friction coefficient along face width direction of each meshing position is calculated by using empirical formula proposed by ISO TC60. The input gear parameters, surface roughness, viscosity of lubricant oil and working condition are used to calculate friction coefficient in this step. Next, the calculated friction coefficients are averaged along face width direction. The averaged friction coefficient is used to calculate the sliding loss ratio and sliding loss of a gear pair further.

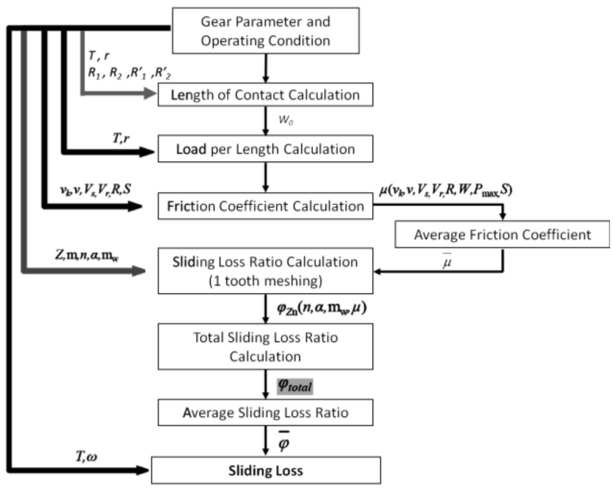


Fig.9 Flow chart of power loss calculation

3. Gear parameter

To verify the applicability of the model, the estimated results are compared with the experimental results done by a back-to-back gear test rig [5]. The parameters of two gear sets that their results are used for comparison are shown in Table 1. The operating conditions are rotating speed 2000 rpm and applied load 50-250 Nm. Surface roughness of run in gear ranges from 0.508 μm to 1.143 μm. The value of surface roughness used in this calculation is 0.508 μm. About the crowning modification, normally the amount of crowing is about 5-10 μm. In this calculation, the crowning magnitude is set to be 5 μm.

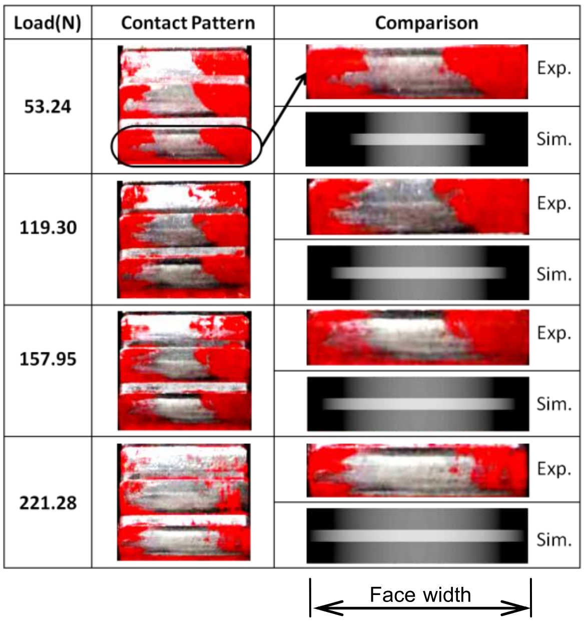
Table 1 Gear parameter

Parameter	Gear Set	
	A	B
Number of Teeth	30	30
Module (mm)	3	3
Pressure Angle (degree)	14.5	20
Face Width (mm)	20	20
Pitch Diameter (mm)	90	90
Gear Material	SCM415	SCM415
Surface roughness (µm)	0.508	0.508
Crowning (µm)	5	5

4. Contact pattern simulation

The load distribution described in section 2.2 can be used to estimate the tooth contact pattern. Table 2 shows the comparisons between contact patterns obtained from the tooth contact experiments and the results obtained from load distribution calculation of gear set B. In the photos obtained from tooth contact experiments, the area that the paint is peeled off (the highlight-area) represents the tooth contact area. In the calculated results, the black color means the area that the tooth surfaces do not contact. The gray areas represent the contact area. The lighter color means the larger load acted at that area. From the results, it is obvious that the estimated results agree well with the experimental results. At light load the contact is occurred at the center of tooth surface, and the contact area is extended to the rims of tooth surfaces when the load is increased. This confirms that the contact stress theory can be used to explain the contact of gear tooth surface, and is suitable to apply to the mathematical model for estimation of sliding loss.

Table 2 Contact pattern simulation



5. Results and discussion

The sliding losses estimated from the improved model are compared with the estimated results from the former model and from the experiments. Figures 10 and 11 show the results of gear set A and gear set B respectively. It is found that estimated results are close to the results obtained from the former model, and also close to the experimental results. The calculated results from the new model are slightly higher than that obtained from the former model. This is because the use of contact stress theory makes the length of contact shorter than the former model, hence the load distribution and the friction coefficient will be increased, and consequently the power loss is increased.

.

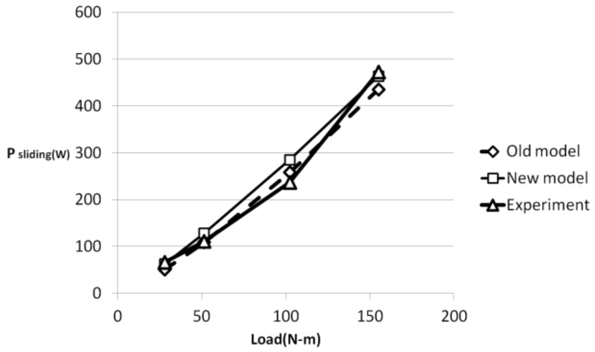


Fig.10 slidind power loss of gear set A

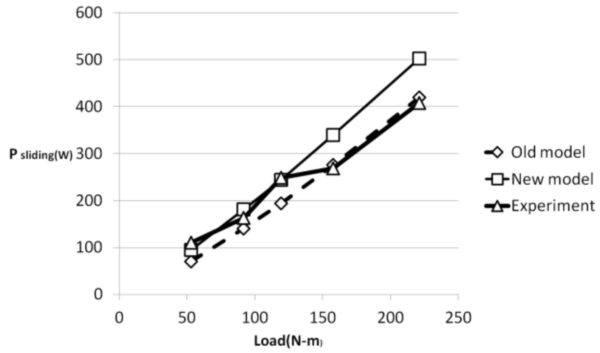


Fig.11 slidind power loss of gear set B

6. Conclusion

The mathematical model used to estimate the sliding loss of a spur gear pair had been improved in this paper. The improvement was done by including the effect of gear tooth contact into the model by using the contact stress theory. The contact patterns obtained from the experiments agree well with the load distribution results. The estimated sliding losses are close to experimental results and are slightly higher than the results estimated from the former model.

7. Acknowledgement

This research is part of a research project on "The study on Mechanism and Parameters Affecting to Power Loss in Parallel Axis Gear Pair" (MRG5480164) and was financially supported by Office of the Higher Education Commission, The Thailand Research Fund, and Chulalongkorn University.

8. References

[1] Yenti, C., Phongsupasamit, S. and Ratanasumawong, C. (2013). Analytical and experimental investigation of parameters affecting sliding loss in a spur gear pair, *Engineering Journal*, vol.17 Issue1, January 2013, pp. 79 - 93.

[2] Ratanasumawong, C., Asawapichayachot, P., Phongsupasamit, S., Houjoh, H. and Matsumura, S., (2012). Estimation of Sliding Loss in a Parallel-Axis Gear Pair, Journal of Advanced Mechanical Design, Systems, and Manufacturing, Vol.6, No.1, pp.88-103.

[3] Teraucha, Y., Nagamura, K., and Ikejo, K. (1991). Study on Friction Loss of Internal Gear Drives, JSME International Journal, Series III, Vol.34, No.1,pp.106-113.

[4] Michlin, Y. and Myunster, V.(2002). Determination of Power Loss in Gear Transmission with Rolling and Sliding Friction Incorporated, Machanism and Machine Theory, 37, pp. 167-174.

[5] Bhumrattanarak, S. Chaikittisilp, S. Denariyakul, S. (2012). The Study on Parameters Effecting to Sliding Loss in Spur Gear and Helical Gear , *Bachelor of engineering project report*, (in thai)

[6] Arthur P. Boresi, and Richard J. Schmidt, ADVANCED MECHANICS OF MATERIALS, 6th edition, JOHN WILEY & SONS, New York.

[7] ISO TC 60, DTR 13989