Blood Flow and Stresses in the Left Ventricle

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Abstract

The heart is an important organ for the maintenance of life because it is the main part of the blood circulatory system that drives blood flow to all parts of the body. In order for blood flow to occur, the heart must perform several complex activities to result in a successful circulatory system. At the present time, we know a lot about heart physiology and functions; yet many of heart's processes are still in a state of controversy and remain undiscovered and unexplained. Understanding this not only raising inquiring questions about heart failure problems, but is also leading to the improvement of strategies to treat the cardiac patients.

Calculations of the flow patterns of the blood in the left ventricle used a modification of the moving immersed boundaries numerical method. The difference was that the velocity of the heart wall endocardium was used as the boundary condition. The position of the endocardium as a function of time was measured from the echocardiograms. The shear stress and normal stress were estimated from the blood flow calculation.

The heart wall boundary conditions from the echocardiograms of left ventricle are just two-dimensional representation of a three dimensional process that includes twist of the heart. Despite that, but the flow pattern results from the simulation of blood flow in the left ventricle looks quite reasonable. The shear stress from successive heart beats had almost same shape and same time of peaking. The systolic-diastolic volume changes correlated well with increasing heart rate, the systolic-diastolic volume change increases until a maximum point, and after that the systolic-diastolic volume change decreases.

***Keywords**: Heart wall stress, Blood flow in heart, Blood flow modeling, Echocardiogram

1. Introduction

An echocardiogram, often referred to in the medical community as a cardiac echo or simply an echo, is a sonogram of the heart. It uses standard ultrasound techniques to create moving images of two-dimensional slices of the heart. The echocardiogram allows doctors to see the heart beating, and to see many of the structures of the heart. There are three types of echocardiography involved with heart research transthoracic echocardiography, stress echocardiography, and transesophageal echocardiography. Ordinarily, the echocardiogram test and electrocardiogram test, a test that records the electrical activity of the heart, are recorded at the same time. The movement of the left ventricle recorded as part of the echocardiogram test was used in blood flow model.

2. Prepare Model's Input Data

To generate the data for the immersed boundaries, two-dimensional slices of the left side of the heart are taken from the echocardiogram. For the simulation model there are six steps shown in figure 1, in each heart beat: Starting with time T_0 when the heart mitral valve and aortic valve both are closed and the ventricle chambers are relaxing. The next step is time T_1 where the mitral valve is at the fully open position and the aortic valve is still closed. Blood flows from the left atrium to fill the left ventricle. The left atrium contracts to force the blood into the ventricle. Therefore at end of this step the atrium chamber has fully contracted and the ventricle chamber fully expanded. At step T_2 the mitral valve starts to close while the aortic valve is still closed, and the left ventricle chamber expands to the maximum volume. Step T_3, the mitral valve is completely closed and the aortic valve starts to open. The ventricle chamber starts to contract and blood flows out. At time step T_4 blood flows through aortic valve and ventricle chamber wall boundary contracts. Step T_5 the aortic valve is at the maximum open position. The process then is repeated for the next heart beat. The ventricle chamber wall boundary is now fully contracted. The steps T_0-T_3 when the atrium is contracted is called diastole and steps T_3-T_6 when the ventricle is contracted is called systole.

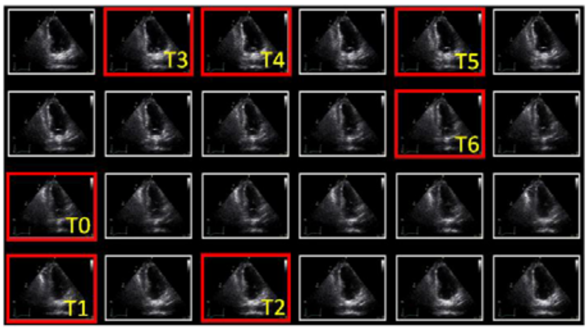

Fig. 1 Show selected individual steps of echocardiography

An echocardiogram image was selected at each time step as shown in figure 2. The appropriate atrium model was attached as shown in figure 3. The edge was traced and used as the boundary value for the calculation. This was repeated at each time step.

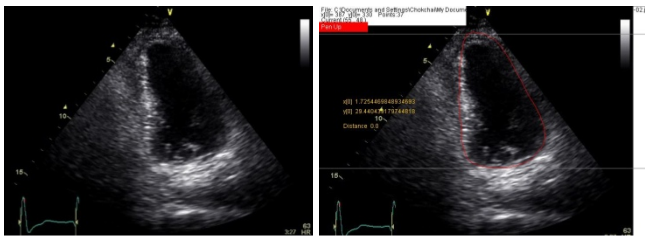

Fig. 2 Show the left ventricle edge detection form echocardiogram

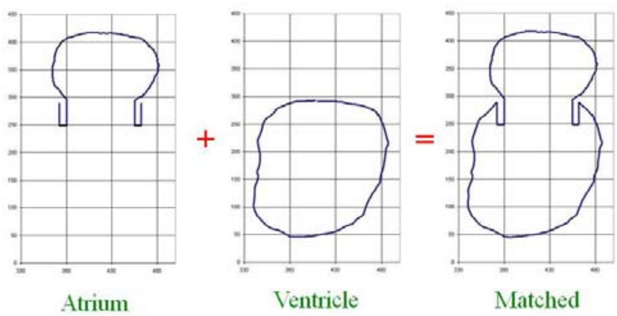

Fig. 3 Matched between left ventricle and left atrium

To make sure that at each step the heart wall edges are correctly positioned , the position of the mitral valve was fixed at the same x-y coordinate as shown in figure 4 In the lower left corner of the electrocardiogram the heart rate is shown as beats per minute.

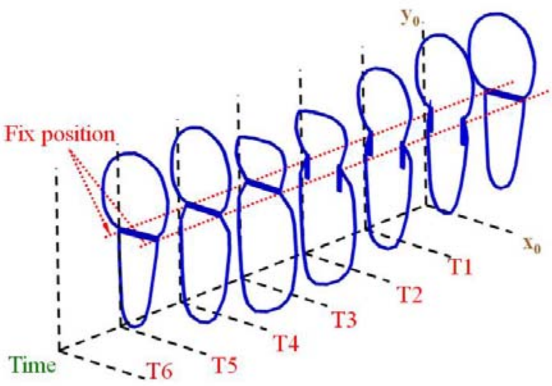

Fig. 4 Fixed position of individual steps of echocardiography

3. Modeling and Algorithm

Because the heart wall is an elastic tissue, the calculation of the flow in the left side of heart has to have updated boundaries in correspondence with the moving of the heart wall. The mathematical formulation for immersed boundary method will follow in topic 3.1.

3.1 Mathematical Modeling and Algorithm

The equation of motion in a closed elastic curve immersed in a two-dimensional incompressible fluid can be explained by the Navier-Stokes equation on the x-y Cartesian [2-6] co-ordinate system as

$$\rho\left(\frac{\partial \hat{u}}{\partial t}+\hat{u}\bullet\nabla\hat{u}\right)+\nabla p=\mu\nabla^2\hat{u}+\hat{F} \qquad (1)$$

$$\nabla\bullet\hat{u}=0 \qquad (2)$$

The equation of motion in this case is between the fluid and non-fluid regions. Therefore the force density $F(\mathrm{x},\mathrm{t})$ should be applied at the surface of the non-fluid regions. The calculation of the boundary force density of the system may be written as:

$$F(\mathrm{x},\mathrm{t})=\int_0^L f(\mathrm{s},\mathrm{t})\delta(x-X(\mathrm{s},\mathrm{t}))\,d\mathrm{s} \qquad (3)$$

Where as

$$\frac{\partial X(\mathrm{s},\mathrm{t})}{\partial t}=\int u(\mathrm{x},\mathrm{t})\delta(x-X(\mathrm{s},\mathrm{t}))\,d\mathrm{x} \qquad (4)$$

Here $u(\mathrm{x},\mathrm{t})$ is the fluid velocity, $p(\mathrm{x},\mathrm{t})$ is the fluid pressure, ρ is the constant blood density and μ is the constant blood viscosity. The function $f(\mathrm{x},\mathrm{t})$ is the force on the boundary element at point s. The value of calculating $f(\mathrm{s},\mathrm{t})$ on [4] was to assume an equation of state for the heart material. In the present case the function $f(\mathrm{s},\mathrm{t})$ can be obtained in finite difference form from $F_r=-8v^{-1}G(v)v_r$ on [4]:

$$x_{\mathrm{k}}^{*}=[x_{\mathrm{k}}^{\mathrm{n}}+\Delta \mathrm{t}u_{\mathrm{k}}^{\mathrm{n}}]+\lambda f_{\mathrm{k}}(x_1^{*}Lx_{\mathrm{n}}^{*}) \qquad (5)$$

The superscript * indicates next time step.

where x is defined on the Cartesian system and X is the point on the Lagrangian system. The solution is obtained using a discrete time step n so that $u^{\mathrm{n}}(x)=u(x,\mathrm{n}\Delta\mathrm{t})$.

1) Find the boundary force f^{n} for the boundary configuration X^{n}:

2) Apply the force f^{n} to the grid of fluid computation:

$$F^{\mathrm{n}}(\mathrm{x})=\sum_{\mathrm{s}} f^{\mathrm{n}}(\mathrm{s})\delta_{\mathrm{h}}(x-X^{\mathrm{n}}(\mathrm{s}))\Delta\mathrm{s} \qquad (6)$$

where as $x=(\mathrm{x},\mathrm{y})$

$$\delta_h(x) = \delta_h(\mathrm{x})\delta_h(\mathrm{y})$$

and

$$\delta_h(\mathrm{x}) = \begin{cases} \frac{1}{4h}\left(1+\cos\frac{\pi \mathrm{x}}{2h}\right) & |\mathrm{x}| \le 2h \\ 0 & |\mathrm{x}| \ge 2h \end{cases} \quad (7)$$

3) Update the fluid velocity under the influence of the force density F^n . Solve the following systems successively for $u^{n+1,0}, u^{n+1,1}, u^{n+1,2}, u^{n+1,3}, \ldots \ (u^{n+1}, p^{n+1})$:

$$\rho\frac{u^{n+1,0} - u^{n}}{\Delta t} = F^{n} \quad (8)$$

$$\rho\left(\frac{u^{n+1,1} - u^{n+1,0}}{\Delta t} + u_x^n D_x^0 u^{n+1,1}\right) = \mu D_x^+ D_x^- u^{n+1,1} \quad (9)$$

$$\rho\left(\frac{u^{n+1,2} - u^{n+1,1}}{\Delta t} + u_y^n D_y^0 u^{n+1,2}\right) = \mu D_y^+ D_y^- u^{n+1,2} \quad (10)$$

$$\rho\left(\frac{u^{n+1} - u^{n+1,2}}{\Delta t}\right) + D_x^0 D_y^0 p^{n+1} = 0 \quad (11)$$

$$D_x^0 D_y^0 u^{n+1} = 0 \quad (12)$$

4) Interpolate the new velocity to the old boundary positions and move the boundary points:

$$X^{n+1}(\mathrm{s}) = X^{n}(\mathrm{s}) + \Delta t \sum_{\mathrm{s}} u^{n+1}(\mathrm{x})\delta_{\mathrm{h}}(x - X^{n}(\mathrm{s}))h^2 \quad (13)$$

Here D^+, D^-, D^0 the forward, backward, and centered divided difference operator.

As the system is closed it is necessary to supply the blood from a source and the outflow is simulated by sinks as shown in figure 5 let $Q(\mathrm{t})$ be the volume flow rate. From the continuity equation $\nabla.u = 0$ the source can be written as:

$$\nabla.u = \psi(x,\mathrm{t}) = Q(\mathrm{t})\psi_0(x) \quad (19)$$

Because blood flow in left side of heart is the periodic domain, the integral of $\nabla.u$ is identically zero, $\nabla.u = \int \psi_0(x)\,dv = 0$. Therefore, the sinks must match the source.

$$\psi_0(x) = w_a(x - X_a) - w_e(x) \quad (20)$$

where X_a is a point the middle of the left atrium, w_a and w_e the spatial distribution of the source and sink. The continuity equation, the mass input is equal to the mass output; is satisfied.

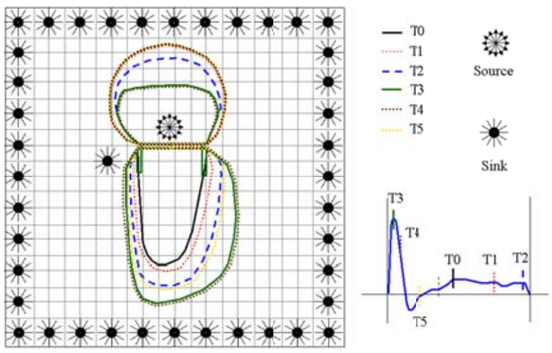

Fig. 5 Show the blood flow simulation model of heart's left side

4. Result

The numerical models of blood flow in the left ventricle provide the systolic-diastolic volume change, blood velocity pattern, blood pressure, and computed normal stress also wall shear stress.

4.1 Systolic-Diastolic Volume Change

The systolic-diastolic volume change was determined from the area difference between the maximum expansion contour and minimum contraction contour of the left ventricle wall edge detection. This is the maximum volume change of the one heart beat.

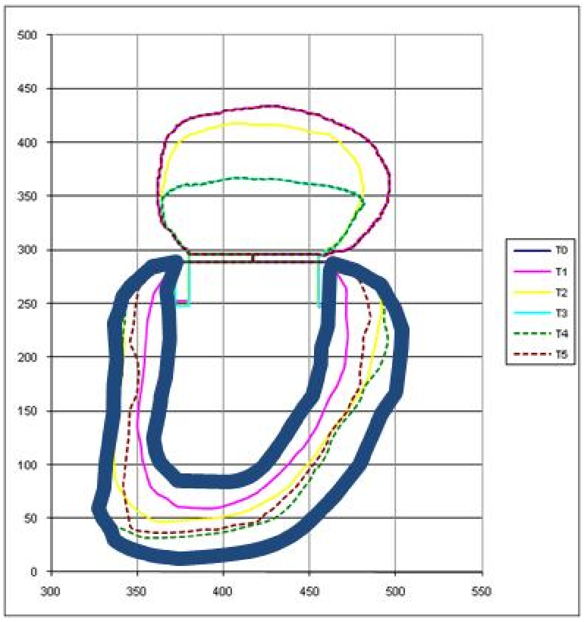

Fig. 6 Left ventricle volume (area unit^2) change

When the dosage of dobutamine was increased the systolic-diastolic volume also increased until a maximum value was achieved. As shown in figure 7.

Table. 1 Left ventricle volume (area) change of dobutamine dosage

mics	Patient1	Patient2	Patient3	Patient4	Patient5
0	13119.5000	9162.5000	15982.5000	16190.0000	10150.3750
10	22278.0000		22334.0000	16762.5000	14527.2500
20	21262.0000	8288.9161	18779.2500	16819.1535	12161.5000
30		7390.6655		15341.0000	10482.8750
35	14589.7500				
40		9348.7500	12339.5000	11563.5000	

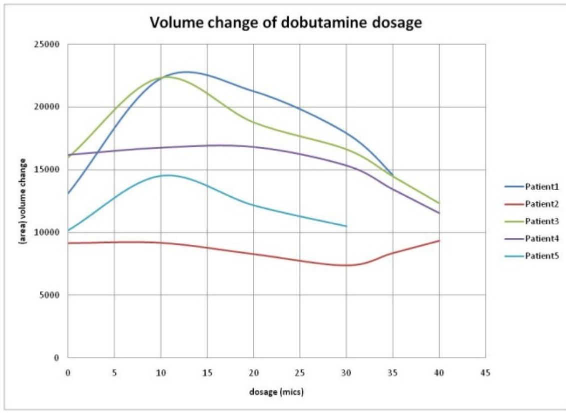

Fig. 7 Left ventricle volume (area) change of dobutamine dosage

4.2 Heart Wall Stress

At each step time the velocity profile in the left ventricle was plotted and used for the calculation of the heart wall stress. The shear stress was calculated in the fluid as close to the wall as the data allowed. It was assumed that the stress was continuous at the wall boundary. Velocity profiles in the left ventricle were plotted with the boundary change to confirm that the left ventricle shape change according to the heart beat cycle and also to confirm that the flow direction correlated with boundary changed.

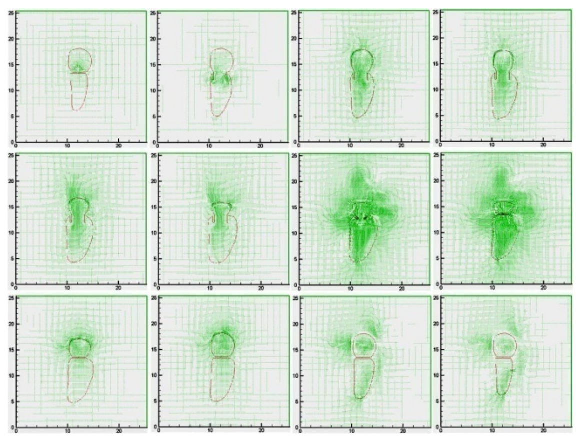

Fig. 8 Velocity profile in the left ventricle patient 1 HR92 20 mics

There are two heart wall stresses determined in this simulation, normal stress and shear stress. Each kind of stress was calculated for five different locations of left ventricle: apex, endocardium, mitral valve, septum, and whole left ventricle for every dosages of dobutamine. The heart wall stresses will be used in the correlation models.

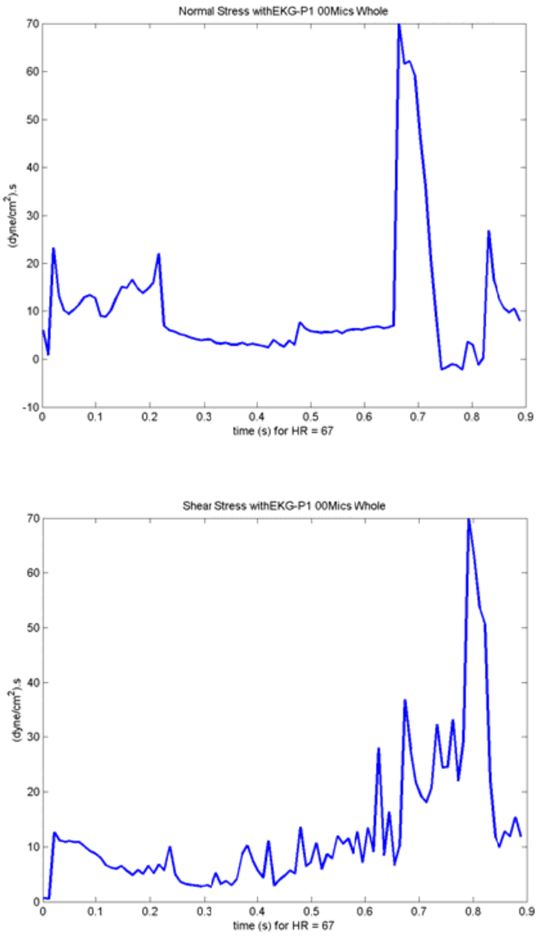

Fig. 9 Patient 1 ventricle wall stress at 0 mics

The average of normal stress and shear stress was plotted against the change of heart rate for each patient. Increasing the patient heart rate has a tendency to increase the average of normal stress and shear stress. The values for the average wall normal stresses for all patients are between 1-25 (N/m^2), whereas the average wall shear stresses for all patients are between 0.1-2 (N/m^2). However the shear stress is important as the ventricular muscles are parallel to the walls and resist the shearing force.

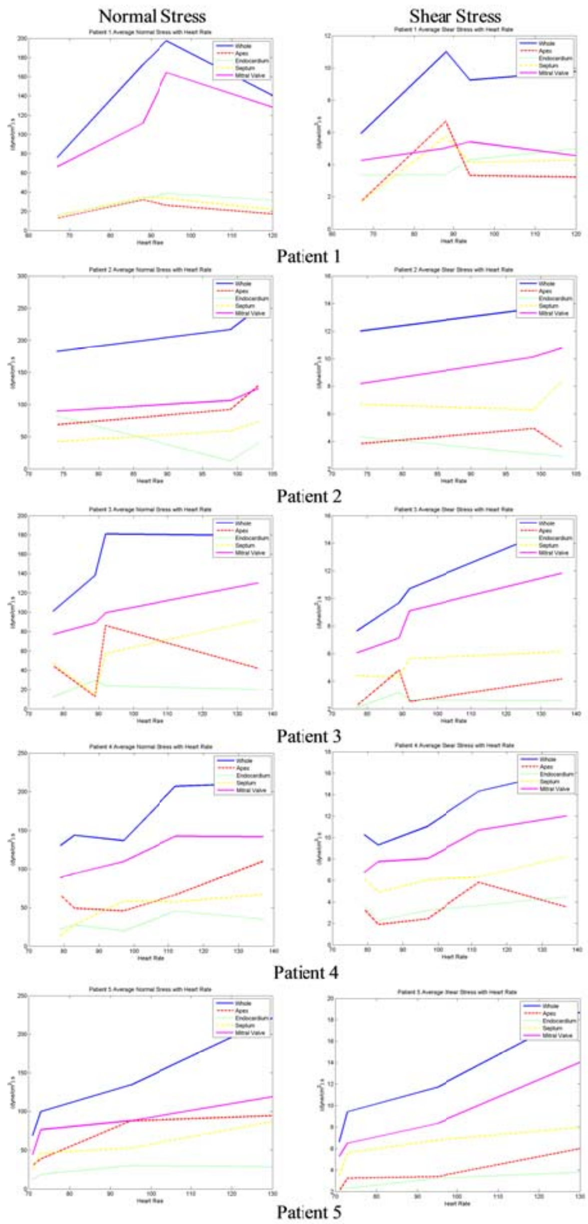

Fig. 10 Wall stress against heart rate

5 Conclusion

In this current paper use was made of the modified moving immersed boundary numerical method to solve the flow pattern of the blood in the left ventricle. Model sensitivity of the model to errors in the mapping of the shape of the ventricle was checked and found to be satisfactory. The blood flow model in the left ventricle enabled the calculation of the systolic-diastolic volume change, blood flow pattern, blood velocity, normal heart wall stress, and shear heart wall stress. This output will be used for simulations later in the dissertation.

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