

Computational Study of Radiant Cooling Effects on Turbulent Natural Convection in Rectangular Enclosed Rooms

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Abstract

The radiant cooling effects on turbulent natural convection in rectangular enclosed rooms were investigated using a computational fluid dynamics (CFD) technique. The standard $k-\varepsilon$ model and the discrete ordinates method (DOM) were employed for solving the problem. The numerical results were validated, step by step, with available experimental and numerical published works. Good agreement with those works was achieved. For the main problem, the temperature boundary conditions at the radiant cooling panels were set to be isothermal, while at other walls, constant heat fluxes were prescribed. The Rayleigh numbers were in the range of $2.8 \times 10^{10} \leq Ra \leq 1.38 \times 10^{11}$ and the temperature of the cooling panel was varied from 4°C to 16°C. The aspect ratio of the room was varied from 0.6 to 1.33. The temperature and velocity distributions in the room were studied. In terms of human thermal comfort, the appropriate cooling panel temperatures for different room types were suggested.

Keywords: Radiant cooling, Turbulent natural convection, CFD, Thermal comfort

1. Introduction

In Thailand, more than seventy percent of energy consumption in buildings comes from air conditioning systems [1]. Radiant cooling system is an alternative way for efficient air conditioning. It offers various advantages over the conventional system especially in terms of more energy saving and better comfort environment [2]. Over the past decade, many researchers have paid attention to the study of this system by both experimentation and computation, for example, Causone et al. [3], Catalina et al. [4] and Velusamy et al. [5], among others.

Causone et al. [3] has recently done an experiment on the radiant floor cooling combined with displacement ventilation and found that floor cooling did not increase draught risk at ankle level but it did increase the vertical air temperature differences. Catalina et al. [4] reported experimental and computational study of a radiant cooling ceiling installed in a test room. They found that the air velocity at ankle/feet zone which was greater than 0.2 m/s, caused local discomfort to the resident. A radiosity method for calculating the mean radiant temperatures for different positions was proposed in this work. Velusamy et al. [5] investigated the interaction effects between surface radiation and turbulent natural convection in rectangular enclosures. It could be seen that surface radiation enhanced the

velocity and turbulent levels in the boundary layers along the enclosure walls, thus resulted in higher convective heat transfer.

Although there have been extensive researches on radiant cooling as already mentioned, few studies were found on the overall effects of radiant cooling on the flow characteristics in a room. This paper numerically studies the air velocity distribution, temperature distribution and thermal comfort of a rectangular enclosed room with radiant cooling systems in tropical climate at the cooling surface temperature between 4°C to 16°C. A well-established commercial code, FLUENT [6], together with the standard $k-\varepsilon$ model [7] and discrete ordinates method is employed for the calculation and the results have been validated with available case studies.

2. Mathematical model and governing equations

The geometry in Fig. 1 is a rectangular enclosed room with constant temperature at cooling panel and constant heat flux of 25 W/m² at other walls. The physical properties are assumed to be constant. The air flow is assumed to be Newtonian, incompressible, turbulent and two dimensional at steady state. The participating

medium is assumed to be gray surface and the surface emissivity is assumed to be constant.

The time averaged Reynolds equations with Boussinesq approximation and standard $k-\varepsilon$ model is applied to predict the turbulent flow in the enclosed room. The conservation equations of mass, momentum and energy can be described as follows:

Conservation of mass

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \quad (1)$$

Conservation of momentum

$$\begin{aligned} \frac{\partial}{\partial x}(\rho uu) + \frac{\partial}{\partial y}(\rho vu) = & -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left[2\mu_{eff} \frac{\partial u}{\partial x} \right] \\ & + \frac{\partial}{\partial y} \left[\mu_{eff} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(\rho vv) = & -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left[\mu_{eff} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \right] \\ & + \frac{\partial}{\partial y} \left[2\mu_{eff} \frac{\partial v}{\partial y} \right] + \rho \beta g (T - T_0) \end{aligned} \quad (3)$$

Conservation of energy

$$\begin{aligned} \frac{\partial}{\partial x}(\rho u C_p T) + \frac{\partial}{\partial y}(\rho v C_p T) = & \frac{\partial}{\partial x} \left[K_{eff} \frac{\partial T}{\partial x} \right] \\ & + \frac{\partial}{\partial y} \left[K_{eff} \frac{\partial T}{\partial y} \right] - \nabla \cdot q_r \end{aligned} \quad (4)$$

The turbulent kinetic energy (TKE) and the dissipation of TKE equations are shown below:

$$\begin{aligned} \frac{\partial}{\partial x}(\rho uk) + \frac{\partial}{\partial y}(\rho vk) = & \frac{\partial}{\partial x} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x} \right] \\ & + \frac{\partial}{\partial y} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] + P_k + G_k - \varepsilon \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial}{\partial x}(\rho u \varepsilon) + \frac{\partial}{\partial y}(\rho v \varepsilon) = & \frac{\partial}{\partial x} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x} \right] \\ & + \frac{\partial}{\partial y} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial y} \right] + [C_{\varepsilon 1} (P_k + C_{\varepsilon 3} G_k) - C_{\varepsilon 2} \varepsilon] \frac{\varepsilon}{k} \end{aligned} \quad (6)$$

where

$$\begin{aligned} P_k = & \mu_t \left[2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right], \\ G_k = & -\frac{\mu_t}{\sigma_T} g \beta \frac{\partial T}{\partial y}; \mu_{eff} = \mu + \mu_t; \mu_t = C_\mu \frac{\rho k^2}{\varepsilon}, \end{aligned}$$

$$\text{and } K_{eff} = K + \frac{\mu_t C_p}{\sigma_T}.$$

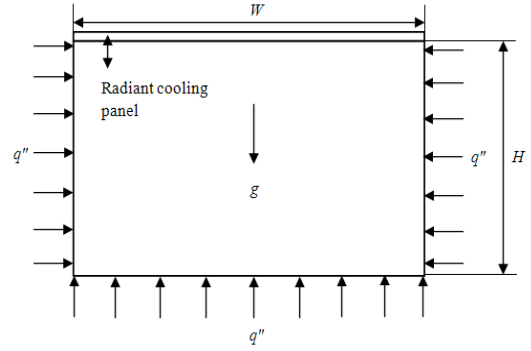


Fig. 1 The geometry of the enclosed room and boundary conditions

The turbulence model constants are $C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $C_{\varepsilon 3} = \tanh(v/u)$, $\sigma_T = 1.0$, $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.3$ [7]. The boundary conditions of Eqs. (1) – (6) are $u = 0$, $v = 0$, $k = 0$ and $\varepsilon = \infty$ on all walls [5].

From the energy equation, Eq. (4), the local divergence of radiation flux ($\nabla \cdot q_r$) is related to the local intensities by:

$$\nabla \cdot q_r = \kappa \left[4\pi I_b(r) - \int_{4\pi} I(r, \Omega) d\Omega \right] \quad (7)$$

The radiative transfer equation (RTE) is solved to achieve the radiation intensity field and the local divergence of radiation flux [8]:

$$\begin{aligned} (\Omega \cdot \nabla) I(r, \Omega) = & -\beta I(r, \Omega) + \kappa I_b(r) \\ & + \frac{\sigma_s}{4\pi} \int_{4\pi} I(r, \Omega') \Phi(\Omega, \Omega') d\Omega' \end{aligned} \quad (8)$$

Thus the radiative heat flux on boundary surfaces is:

$$q_r = \varepsilon \left[\pi I_b(r_w) - \int_{n \cdot \Omega' < 0} I(r_w, \Omega') |n \cdot \Omega'| d\Omega' \right] \quad (9)$$

The radiative boundary condition for diffusely reflecting surface in Eq. (8) is:

$$\begin{aligned} I(r_w, \Omega) = & \varepsilon I_b(r_w) \\ & + \frac{(1-\varepsilon)}{\pi} \int_{n \cdot \Omega' < 0} I(r_w, \Omega') |n \cdot \Omega'| d\Omega' \end{aligned} \quad (10)$$

The radiative transfer equation (RTE) in Eq. (8) is solved by using the discrete ordinates method (DOM). The RTE is replaced by set of M discrete equations for a finite number of directions Ω_m while each integral is replaced by the quadratured series of the form [9, 10]:

$$\begin{aligned} (\Omega_m \cdot \nabla) I(r, \Omega_m) = & -\beta I(r, \Omega_m) + \kappa I_b(r) \\ & + \frac{\sigma_s}{4\pi} \sum_{k=1}^M w_k I(r, \Omega_k) \Phi(\Omega_m, \Omega_k) \end{aligned} \quad (11)$$

and the boundary condition is:

$$I(r_w, \Omega_m) = \varepsilon I_b(r_w) + \frac{(1-\varepsilon)}{\pi} \sum_{n \cdot \Omega_k} w_k I(r_w, \Omega_k) |n \cdot \Omega_k| \quad (12)$$

where w_k is the ordinate weight.

In Cartesian coordinates, the original equation is transformed by angular approximation into a set of coupled differential equations:

$$\xi_m \frac{\partial I_m}{\partial x} + \eta_m \frac{\partial I_m}{\partial y} + \mu_m \frac{\partial I_m}{\partial z} + \beta I_m = \beta S_m \quad (13)$$

$$S_m = (1-\omega) I_b(r) + \frac{\omega}{4\pi} \sum_{k=1}^M w_k I(r, \Omega_k) \phi(\Omega_m, \Omega_k) \quad (14)$$

where ξ_m , η_m and μ_m are the directional cosines of Ω_m , S_m is the source term and $\omega = (\sigma_s/\beta)$ is scattering albedo.

3. Numerical method

FLUENT is a commercial CFD code package for simulation of airflow and temperature distributions in the enclosed room of interest. In the present work, the standard $k-\varepsilon$ model is applied to solve the turbulent natural convection and DOM to solve the radiation at the surface. The buoyancy effect of Boussinesq approximation is considered in this model. The convection terms are discretized by the second-order upwind scheme and the pressure by the method of pressure staggering option (PRESTO!). The velocities and pressure are coupled by the SIMPLE algorithm [11].

4. Validation

Accuracy of the present calculation is validated with simple case studies, i.e., the problems of natural convection with and without surface radiation in enclosed room. The natural convection in enclosed room problem is divided into laminar natural convection and turbulent natural convection. The laminar natural convection case is verified with experimental data of Krane and Jessee [12] and numerical results of Barakos et al. [13] while the turbulent case is verified with experimental data of Cheesewright et al. [14] and numerical results of Choi et al. [15]. For the case of natural convection with surface radiation in enclosed room, there is a limitation on experimental study therefore the results are validated with other numerical simulation only. Similar to the previous case, the problem is classified into laminar and turbulent natural convection with surface radiation which

are validated with computational results of Lari et al. [10] and Velusamy et al. [5], respectively.

4.1 Laminar natural convection

The laminar natural convection in a square enclosure is considered. The upper and lower walls are adiabatic. The temperature difference between the left and right isothermal walls is 20 °C. The Rayleigh number is 1.89×10^5 . Fig. 2 shows the comparison of the present solution with the experimental results [12] and numerical simulations [13].

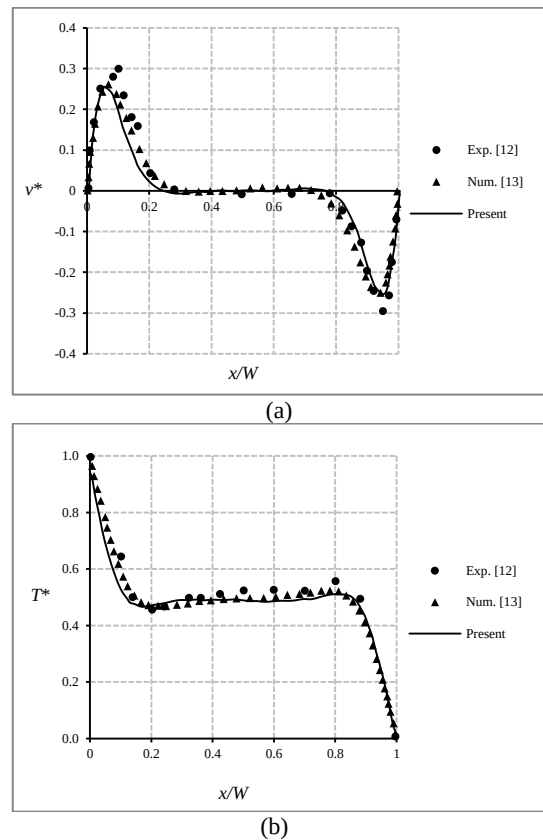


Fig. 2 Comparison of present solution with experimental data and other numerical solution for $Ra = 1.89 \times 10^5$, $AR = 1$; (a) mid-height vertical velocity and (b) mid-height temperature

4.2 Turbulent natural convection

The turbulent natural convection case is evaluated in a room with aspect ratio of 5. The upper and lower walls are insulated as in the laminar case. The temperature difference between the left and the right wall is 45.8 °C and the Rayleigh number is 4.5×10^{10} . Fig. 3 shows the validation of present solution with the experimental data [14] and numerical results [15].

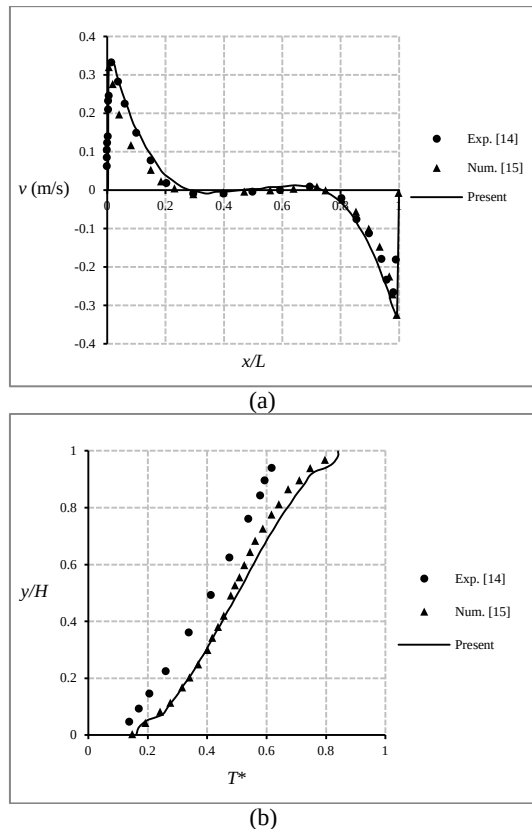


Fig. 3 Comparison of present solutions with experimental data and other numerical solution for $Ra = 4.5 \times 10^{10}$, $AR = 5$; (a) mid-height vertical velocity and (b) mid-width temperature

4.3 Laminar natural convection with surface radiation

The square enclosure is insulated at the upper and lower walls. The left and right walls are isothermal and the temperatures are 310 K and 290 K, respectively. The emissivity is set to unity at all surfaces and the Rayleigh number is 10^6 . Fig. 4 shows the comparison of present solution with the numerical simulation [10].

4.4 Turbulent natural convection with surface radiation

The geometry of this case is similar to the case in 4.3. The temperature difference between the left and right walls is 50 K and the Rayleigh number is 10^{11} . The radiation comes from top and bottom walls with the surface emissivity of 0.9. Fig. 5 displays the validation of present solution with other numerical simulation [5].

4.5 Discussion of validation results

For the cases of laminar natural convection with and without surface radiation and turbulent natural convection with surface radiation, the results agree well with the published experimental data and other numerical works.

However, for the case of pure turbulent natural convection, the calculated temperatures deviate from the experimental data, as shown in Fig. 3b. The deviation trend is similar to the results of Choi et al. [15]. This is due to heat loss from insufficient insulation at upper and side walls in the experiment [14].

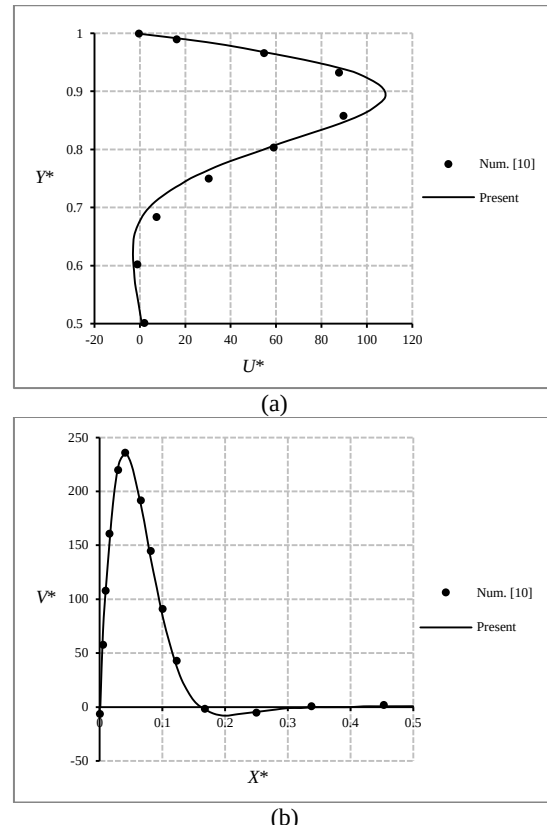


Fig. 4 Comparison of present solutions with other numerical simulation for $Ra = 10^6$, $AR = 1$; (a) mid-width horizontal velocity and (b) mid-height vertical velocity

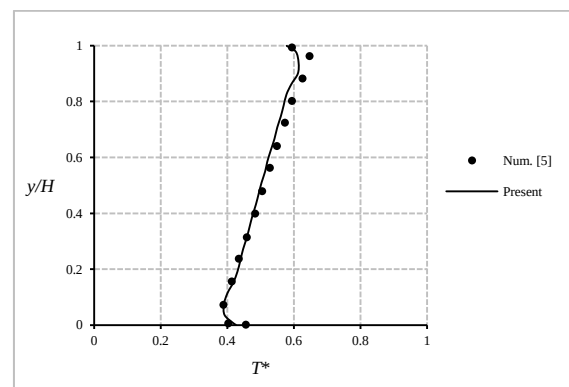


Fig. 5 Comparison mid-width temperature of present solutions with other numerical simulation for $Ra = 10^{11}$ and $AR = 1$

5. Results and discussion of the main problem

The present work focuses on the air velocity distribution, temperature distribution and thermal comfort of the room with radiant cooling system. Three room sizes are investigated, i.e., a square room (3m×3m, AR = 1), a wide room (5m×3m, AR = 0.6) and a tall room (3m×4m, AR = 1.33). The temperatures of radiant floor and ceiling cooling panels are varied from 4°C to 16°C and the Rayleigh number is in the range of $2.8 \times 10^{10} \leq Ra \leq 1.38 \times 10^{11}$.

5.1 Air velocity distribution

The air speed is an important indicator for thermal comfort of human [16]. If the air speed is higher than 0.2 m/s in the feet and ankle zone, problems will arise for the occupants [4]. Such kind of problem can be seen in the room with radiant ceiling cooling of AR = 1.33 ($T_{RCC} = 4^\circ\text{C}$, 6°C and 8°C). The example of 8°C T_{RCC} case is illustrated in Fig. 7a.

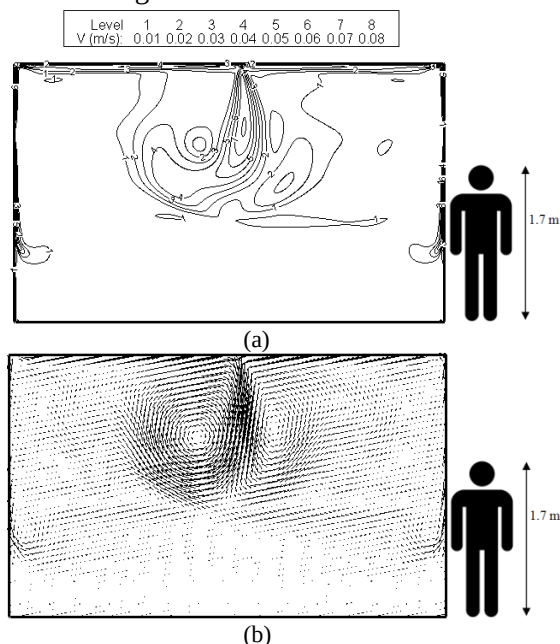


Fig. 6 Velocity fields for $T_{RFC} = 14^\circ\text{C}$, AR = 0.6 and $Ra = 3.3 \times 10^{10}$; (a) Velocity contour and (b) Velocity vector

The maximum value of the mean velocity within the occupied zone of the room with radiant ceiling cooling is around 0.1 m/s for the room with AR of 1.33, while for the room with radiant floor cooling it is around 0.02 m/s for the room with AR of 0.6 (Fig. 6a).

It can be seen from Figs. 6b and 7b that air velocity distribution of the room with radiant ceiling cooling is more diffuse than the case of room with radiant floor cooling since the former has more circulation zones.

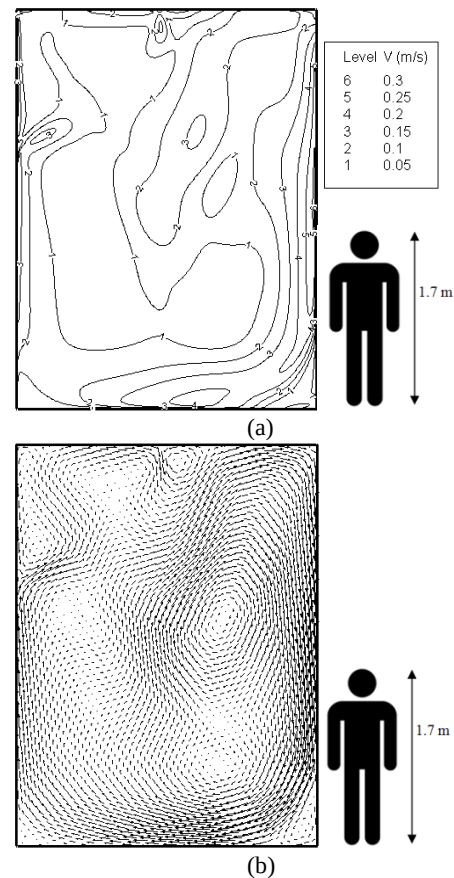


Fig. 7 Velocity fields for $T_{RCC} = 8^\circ\text{C}$, AR = 1.33 and $Ra = 1.14 \times 10^{11}$; (a) Velocity contour and (b) Velocity vector

5.2 Temperature distribution

Temperature is another important parameter of human thermal comfort [16]. Too large vertical temperature difference will yield discomfort within the air conditioned room. In the present work, the considered vertical temperature gradient is measured between the height levels of 0.4 and 1.9 m. In the following sections, either T_{RCC} or T_{RFC} profiles is displayed for each case due to the space limitation.

5.2.1 Square room (AR = 1)

For the cooling panel temperatures from 4 – 10 °C, the vertical temperature difference of the room with radiant floor cooling is found to be higher than the room with radiant ceiling cooling. The vertical temperature gradient of the room with radiant floor cooling is around 1.60 – 3.16 °C/m while the vertical temperature gradient of the radiant ceiling cooling room is around 0.04 – 0.19 °C/m (Fig. 8).

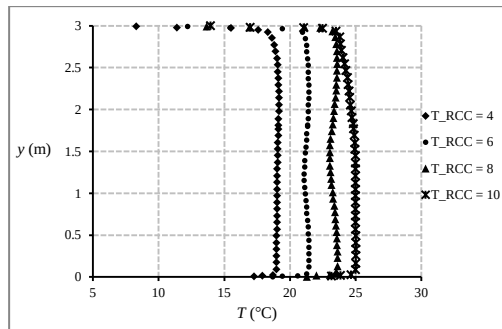


Fig.8 The vertical temperature of the room with radiant ceiling cooling at mid-width level

5.2.2 Wide room (AR = 0.6)

The cooling panel temperature is varied from 10 – 16 °C. The vertical temperature gradient of the room with radiant floor cooling is around 0.64 – 1.31 °C/m (Fig. 9) while the vertical temperature gradient of the room with radiant ceiling cooling is around 0.09 – 0.17 °C/m.

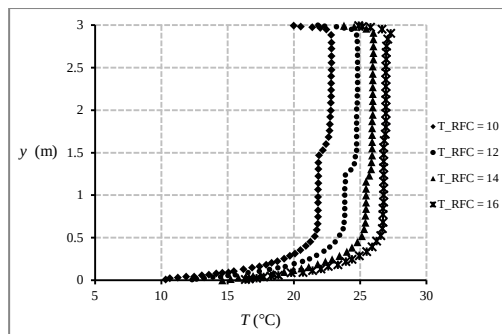


Fig. 9 The vertical temperature of the room with radiant floor cooling at mid-width level

5.2.3 Tall room (AR = 1.33)

The cooling panel temperature is varied from 4 – 10 °C. Similar to the previous cases, the vertical temperature difference of the room with radiant floor cooling is found to be greater than the room with radiant ceiling cooling. The vertical temperature gradient of the room with radiant floor cooling is around 3.14 – 4.02 °C/m while the vertical temperature gradient of the room with radiant ceiling cooling is around 0.03 – 0.17 °C/m (Fig. 10).

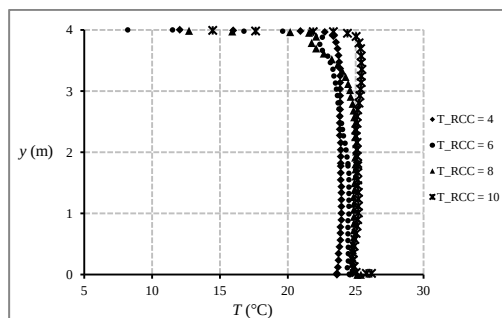


Fig. 10 The vertical temperature of the room with radiant ceiling cooling at mid-width level

5.3 Thermal comfort analysis

In the present study, the predicted mean vote (PMV) index is employed to evaluate thermal comfort. The PMV values range from -3 to +3, where the positive values stand for hot feeling and the negative for cold feeling with 0 being a neutral value [16]. Activity level is set to a constant value at 70 W/m² or 1.2 Met, and Clo-value is set to 0.64 (typical office clothing). The range of radiant cooling panel temperature considered here is between 4°C to 16 °C for all room aspect ratios.

The PMV values range of -0.2 to 0.2 are considered to be an acceptable range for thermal comfort [4]. Thus, for the square room case, the temperature of cooling panel should be 8 °C for both rooms (Fig. 11). At the cooling panel temperature of 8 °C, PMV values for the rooms with radiant floor and ceiling cooling are -0.16 and -0.01, respectively.

In the case of the wide rectangular room with AR of 0.6, the appropriate cooling panel temperature should be 12 °C for the radiant floor cooling room with PMV of -0.15, while the cooling panel temperature should be 14 °C for the room with radiant ceiling cooling with PMV of 0.08 (Fig. 12).

Fig. 13 shows PMV for the tall room with AR of 1.33. For human thermal comfort, the temperature of the floor cooling panel should be 6°C with PMV of -0.16 and the panel temperature for the radiant ceiling cooling room should be 4°C with PMV of 0.06.

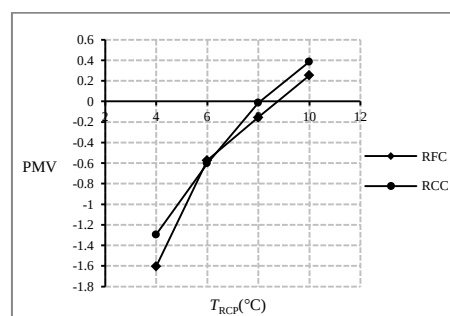


Fig. 11 Temperature of cooling panels and PMV index for the room with AR = 1

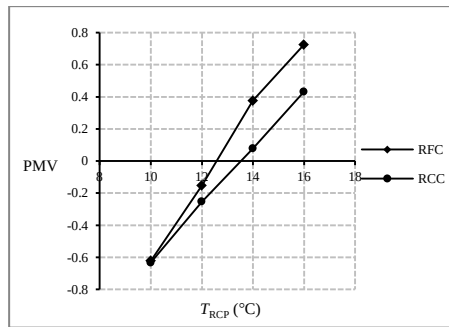


Fig. 12 Temperature of cooling panels and PMV index for the room with AR = 0.6

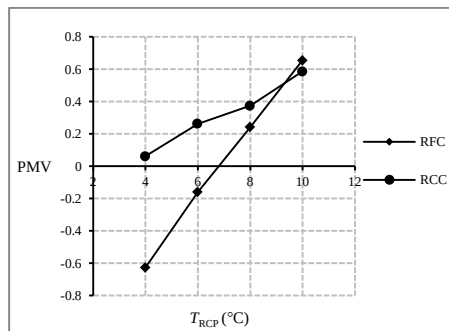


Fig. 13 Temperature of cooling panels and PMV index for the room with AR = 1.33

6. Conclusion

The numerical study of radiant cooling system in enclosed room with constant heat flux is carried out for the range of cooling panel temperature between 4°C to 16°C and aspect ratio of 0.6, 1 and 1.33.

It is found that air velocity distribution of the room with radiant ceiling cooling is more diffuse than the room with radiant floor cooling. The air velocity range is within 0.01 – 0.3 m/s and 0.01 – 0.1 m/s for the room with radiant ceiling cooling and the room with radiant floor cooling, respectively. For the room with AR of 1.33 with radiant ceiling cooling (cooling panel temperatures of 4, 6 and 8°C), discomfort occurs at feet/ankle level due to air velocity which is greater than 0.2 m/s.

Temperature distribution is considered in vertical temperature gradient form which is measured between the height levels of 0.4 and 1.9 m. It is seen that the room with radiant floor cooling has larger vertical temperature difference than the room with radiant ceiling cooling for all aspect ratios. This large vertical temperature gradient may cause discomfort to the room occupants.

Finally, thermal comfort analysis using PMV index as an indicator is performed. Within the human thermal comfort range of -0.2 – 0.2, the appropriate cooling panel temperature for the

square room is 8°C for both floor and ceiling cooling panel types, 12°C and 14°C for floor and ceiling cooling panel temperatures for the room with AR = 0.6, and 6°C and 4°C for floor and ceiling cooling panel temperatures for the room with AR = 1.33, respectively.

Nomenclature

AR	aspect ratio of the enclosed room, H/W
C_p	specific heat of the fluid (kJ/kgK)
$C_{\mu}, C_{\varepsilon 1}, C_{\varepsilon 2}, C_{\varepsilon 3}$	constant in the turbulence model
Clo	clothing insulation
g	acceleration due to gravity (m/s^2)
H	height of the enclosed room (m)
I	radiation intensity ($\text{W/m}^2\text{sr}$)
K	thermal conductivity of the fluid (W/mK)
K_{eff}	effective thermal conductivity (W/mK)
k	turbulent kinetic energy (m^2/s^2)
L	characteristic length of the room (m)
M	number of discrete directions
Met	Metabolic rate
P	pressure (Pa)
PMV	Predicted mean vote
q	heat flux (W/m^2)
r	position vector
Ra	Rayleigh number, $g\beta\Delta TL^3/\nu\alpha$
S	source term (W/m^3)
T	temperature (K)
T^*	dimensionless temperature, $(T - T_c)/\Delta T$
ΔT	temperature difference (K), $T_h - T_c$
u	horizontal velocity (m/s)
U^*	dimensionless horizontal velocity, uW/α
v	vertical velocity (m/s)
v^*	dimensionless vertical velocity, $v/\sqrt{g\beta\Delta TH}$
V^*	dimensionless vertical velocity, vH/α
W	width of the enclosed room (m)
X^*	dimensionless width of the room, x/W
Y^*	dimensionless height of the room, y/H

Greek

α	thermal diffusivity of the fluid (m^2/s)
β	coefficient of thermal expansion (K^{-1})
ε	dissipation rate of k (m^2/s^3), emissivity of the surface
κ	absorption coefficient (m^{-1})
μ	dynamic viscosity (Ns/m^2)
μ_t	turbulent viscosity (Ns/m^2)
ν	kinematic viscosity of the fluid (m^2/s)
ρ	density of the fluid (kg/m^3)
$\sigma_k, \sigma_T, \sigma_\varepsilon$	turbulent Prandtl of k , T and ε
σ_s	scattering coefficient (m^{-1})
ξ, η, μ	x -, y - and z -direction cosines
Φ	scattering phase function
ω	scattering albedo
Ω	direction vector

Subscripts

b	black body
c	cold wall
h	hot wall
k	index of sigma
m	discrete direction
n	iteration step

r	radiative term
RCC	radiant ceiling cooling surface
RFC	radiant floor cooling surface
w	wall

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