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Optimization of digitization based Kater pendulum for precision gravity measurement

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Abstract

The gravitational acceleration is the derivative value of other quantity. It is well know in the scientists especially the physicists. Some Institute or application such as calibration laboratory, National Institute of Metrology, Physics Department and Geophysics Department in university must to know the accurate gravity value. An accurate g contributes to greater accuracy among equipment that requires g values as an input. However for research or application purposes, access to accurate g measurement equipment is expensive. Indeed, because there is no access to highly accurate equipment in Thailand, researchers here must rely on equipment from abroad, such as China or European countries, which is very costly. This research aims to construct and develop the device for measure the gravitational acceleration value, namely the Kater pendulum. It has 2 pivots and moveable mass, by measure the periods of oscillation with a little angular displacement. At the position of moveable mass which give the periods of the both pivots are equal. By using the laser sensor technology and connect to the computer to helping analyst the periods of oscillation. The accuracy of the measurement is 10^{-4} m/s^2 when compare with the reference value is 9.7829 m/s^2. This result is accurate more than using the commercial light sensor, with the same Kater pendulum prototype, about 1.15x10^{-2} m/s^2. And It accurate more than the simple pendulum model of undergrad laboratory about 0.63x10^{-2} m/s^2. Not only can the Kater pendulum provide the required accuracy, it also is relatively easier to design with a limited budget. The authors would be to express their indebt gratitude toward the National Institute of Metrology (Thailand) for financial support via the scholarship of Nittaya Arksonnarong (Cooperation on Science and Technology Researcher Development Project by Office of the Permanent Secretary Ministry of Science and Technology.

***Keywords**:* Kater pendulum, Pendulum, the gravity measurement.

1. Introduction

The gravitational acceleration (*g*) is known sine in the undergrad laboratory, is equal to 9.8 m/s^2. It is an important in scientific and industrial applications. It is used, both directly and indirectly, for example in calibration laboratory use it as derivative value such as force, torque and pressure laboratory, in geophysical surveys use it to determine variation of geological rock densities, change in latitude and elevation [1]. Depending its applications, the requirement of gravity measurement accuracy could be ranging from 10^{-2} m/s^2 to 10^{-6} m/s^2.

There are many type of the gravimeter such as the free fall methods system [3 , 4], the torsion balance, the mass on spring system [2] and the pendulum method [5]. a number of available methods to measure the gravitational acceleration. For this research consider the pendulum-based systems because it is easier to operate, low-cost maintenance and design. Additionally with advantages from computing technology, the pendulum systems can be improved and considered a good candidate for educational and industrial purposes where cost-per-performance is a key factor.

In figure 1, consider a basic physical pendulum, starting from force equation

$$-mgd\sin\theta = I\ddot{\theta} \quad , \qquad (1)$$

where

m is total mass of the pendulum

g is the gravitational acceleration

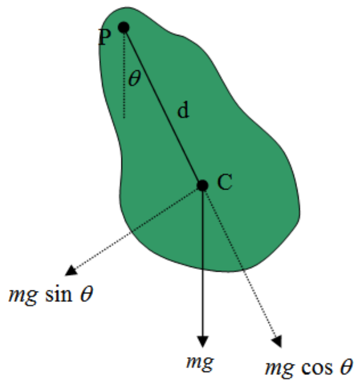


Figure 1: Physical pendulum.

d is the distance from the center of mass to the pivot

I is the moment of inertia about the center of mass and equal to mk_0^2 where k_0 is the radius of gyration) and

θ is the angular displacement.

With an assumption of small amplitude of oscillation (θ is less than 5 degrees), one could obtain a general solution and show that the period of the pendulum is given by

$$g = \frac{4\pi^2 I}{mdT^2} . \qquad (2)$$

Equation (2) can be used to provide the value of the gravitational acceleration. However the requirement to determine the center of mass and the exact value of moment of inertia prohibits the uselessness of this simple result. Invented by Henry Kater in 1817 [6], these problems have been circumvented by using a pendulum with two pivots at both ends, and an adjustable mass bob. By hanging the pendulum from the first pivot, the period is recorded as T_1. Then turning upside down the pendulum from the second pivot and the period is timed again as T_2. From each pivot, the period could be expressed by

$$T_{1,2} = 2\pi\sqrt{\frac{k_0^2 + d_{1,2}^2}{gd_{1,2}}} \qquad (3)$$

where

d_1 is the distance between the first pivot to the center of gravity

d_2 is the distance between the second pivot to the center of gravity

If the mass bob's position can be adjusted, one can readjust its position such that $d = k_0$ and when this condition is met, the equation (3) is simplied into $T_{1,2} = 2\pi\sqrt{\frac{d}{g}}$, resulting in the physical pendulum behaving like a simple pendulum with physical length *d*.

By adjusting the mass bob's position such that the periods measured from the two pivots are equal, the distance between two pivots will be equivalent to the length of the simple pendulum and the gravitational acceleration can be determined without knowing the center of mass and the moment of inertia:

$$T_1 = 2\pi\sqrt{\frac{k_0^{\ 2} + d_1^{\ 2}}{gd_1}} \text{ and}$$
$$T_2 = 2\pi\sqrt{\frac{k_0^{\ 2} + d_2^{\ 2}}{gd_2}}. \qquad (4)$$

If T_1 = T_2 then either d_1 = d_2 and $d_1 d_2 = k_0^{\ 2}$.

Again from equation (4),

$$\frac{gd_1T_1^{\ 2}}{4\pi^2} = k_0^{\ 2} + d_1^{\ 2} \text{ and}$$

$$\frac{gd_2T_2^{\ 2}}{4\pi^2} = k_0^{\ 2} + d_2^{\ 2} \qquad (5)$$

Equation (5), it can be simplified into

$$\frac{gd_1T^2{}_1}{4\pi^2} - \frac{gd_2T_2^{\ 2}}{4\pi^2} = \left(k_0^{\ 2} + d_1^{\ 2}\right) - \left(k_0^{\ 2} + d_2^{\ 2}\right) \qquad (6)$$

With some algebra straight forwardly, we can arrive,

$$g = \frac{8\pi^2}{\left(\frac{T^2{}_1 + T^2{}_2}{d_1 + d_2}\right) + \left(\frac{T^2{}_1 - T^2{}_2}{d_1 - d_2}\right)} \qquad (7)$$

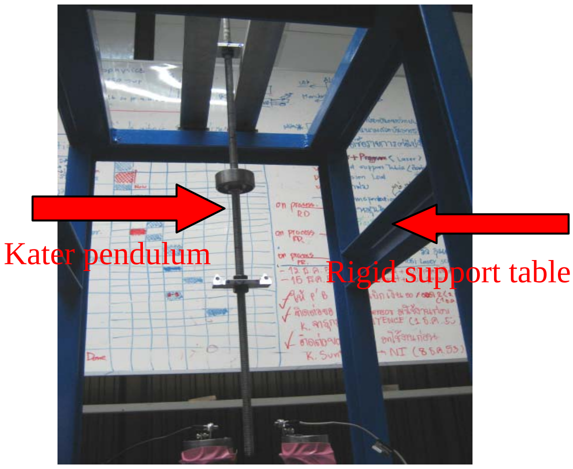
For the limit of T_1 approaches T_2, as T_1 = T_2, the second term of the denominator of Equation (7) will became negligibly vanished, i.e., $\left(\frac{T^2{}_1 - T^2{}_2}{d_1 - d_2}\right)$ converges to zero subsequently, gravitational acceleration is then determined by $g = \frac{8\pi^2}{\left(\frac{T^2{}_1 + T^2{}_2}{d_1 + d_2}\right)}$. Because d_1 and d_2 are distances from to the pivots to the center of mass and lie in the opposite site of the length of pendulum, the sum of d_1 and d_2 is therefore the physical length between the two pivots. Also using calculus, it can be shown that the period is minimum when *d* is equal to k_o.

2. Objective

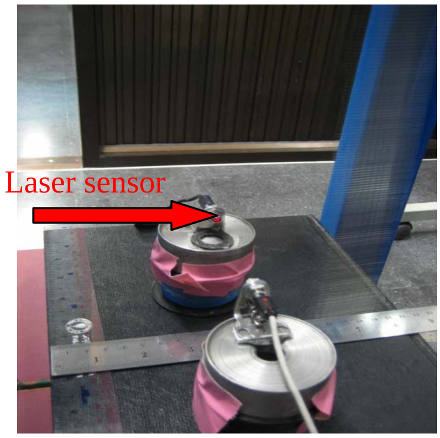
To develop and design a digitization based Kater pendulum and to comparing with the commercial light sensor with the same prototype.

3. Methods and Materials

In Figure 2(A) shows parts and components of a pendulum prototype used in this research. The pendulum is made of a threaded rigid metal rod with triangular-blade pivots and the mass bob can be further inserted and adjusted. The portable stand for support the pendulum made of rigid metal frame and insert the deadweight of steel block for increase its rigidity and to reduce possible wobbling during the swing of the pendulum. In order to detect the motion of the pendulum, a pair of narrow beam laser and a commercial light sensor are used to detect motion of the pendulum are shown in Figure 2(B,C). Once the pendulum swing through its equilibrium position, the detector sends the signal to a data acquisition card and then to the computer are shown in Figure 3(A,B). The period of oscillation of the pendulum is measured over 200 cycles.



A



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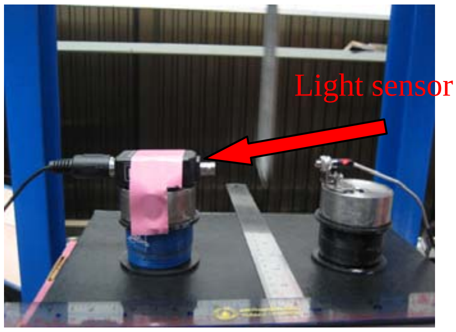
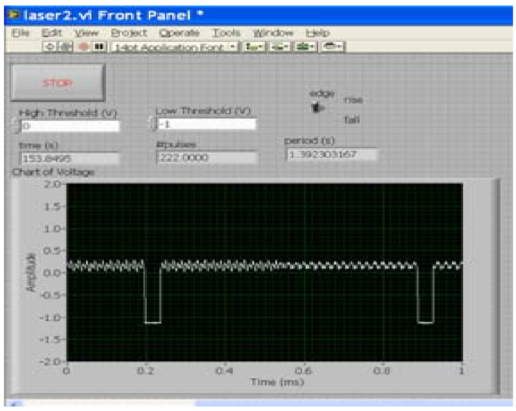
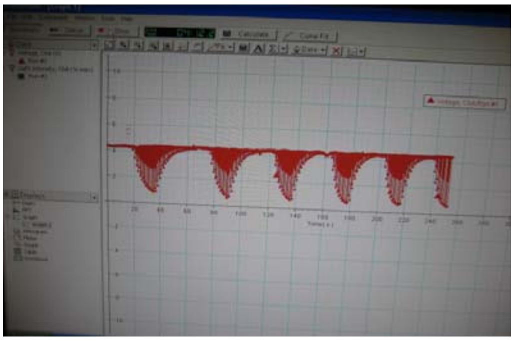


C

Figure 2: A) The Kater pendulum with rigid support table
B) Laser-based detector
C) Commercial Light-based detector



A



B

Figure 3: A) The analysis software by using with the laser sensor.
B) The analysis software by using with the commercial light sensor

For the dimension of the first prototype of Kater pendulum is shown in 4;

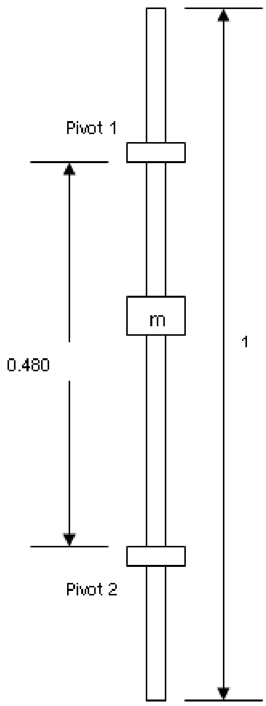


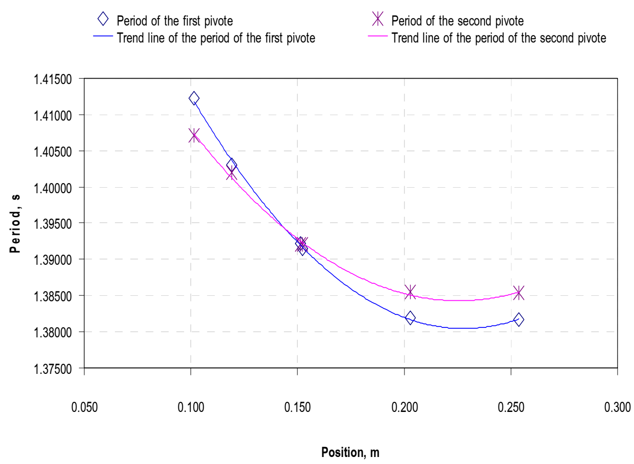
Figure 4: A diagram of the Kater pendulum with scale in meter.

4. Research procedure

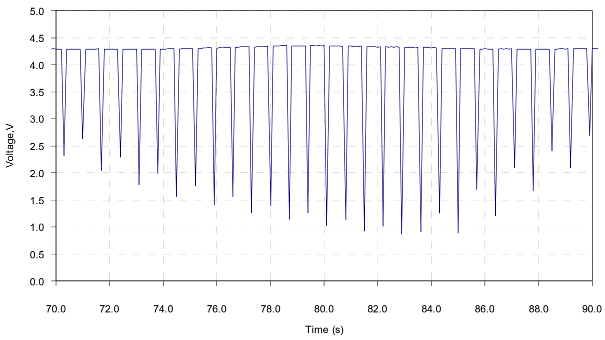
Hang the Kater pendulum at the support table with the first pivots and measure the periods of the oscillations (T_1). Then turned up-side down the Kater pendulum and measure the period of oscillation (T_2) again and recorded the results relative to the position of moveable mass. Then adjust the position of moveable mass and repeat the previous procedure again. Repeat the method until T_1 and T_2 are roughly identical or until its difference is less than 0.01 second. The mass bob is slighted adjusted so that the graph of T_1 and T_2 as a function of the relative position of the mass bob can be plotted. To achieve high accuracy of period or the intersection of $T_1$and T_2, the value of the identical period is interpolated. In this study the following aspects are investigated; repeatability and reproducibility of the measurement. After that at the position of moveable mass which T_1 and T_2, are equal. Exchange the laser sensor setup to be the commercial light sensor setup and measure the period at this position again. And record the data. Compare the results between 2 systems.

5. Results and conclusion

The measurement results are shown as Graph 1 and 2 respectively.



Graph 1: The Relation of the relative mass bob position and the period of both knife-edges.



Graph 2: The Relation of the relative mass bob position and the period of both knife-edges.

Graph 1 shows the result by using the laser sensor of period T_1 and T_2 as a function of the relative mass bob position. The exact matching period is found by using a linear interpolation of data. It is found that the identical period is 1.3942 second (g = 9.7486 m/s^2). Graph 2 shows the result of the period by using the commercial light sensor at T_1 and T_2 is equal. It is found that the identical period is 1.4000 second (g = 9.6682 m/s^2). As it is no data available for the exactly value of gravitational acceleration at the location where the experiment was conducted, thus according to the International Gravity Information System, PTB [23], it is presumed that the reference value should be 9.7829 m/s^2. So the deviation from the reference value of the laser sensor base is 0.35 % (absolute value) and for the commercial sensor base is 1.17 %. The results are shown that the optimization of digitization based Kater pendulum for precision gravity measurement is successfully. This result is accurate more than using the commercial light sensor, with the same Kater pendulum prototype, about 1.15 x10^{-2} m/s^2. And It accurate more than the simple pendulum model of undergrad laboratory about 0.63 x10^{-2} m/s^2. Not only can the Kater pendulum provide the required accuracy, it also is relatively easier to design with a limited budget.

6. Acknowledgement

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