A New Approach on Modeling the Gradient of Reynolds Shear Stress in Reynolds-Stress Models for Fully-Developed Turbulent Channel Flow

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Abstract

A new approach on modeling the gradients of the Reynolds shear stresses in Reynolds-stress models is presented. The gradients of the Reynolds shear stresses are modeled in terms of the gradients of the product of the root-mean-square of the velocity fluctuations times a model constant [1, 2, 3], instead of being calculated from the solutions of the transport equations of the Reynolds shear stresses in Reynolds-stress models. Therefore, only four transport equations for the Reynolds normal stresses and the dissipation rate of turbulence kinetic energy are needed for three-dimensional flows, instead of seven transport equations for all Reynolds stresses and the dissipation rate of turbulence kinetic energy. In other words, the present gradient-of-Reynolds-shear-stress algebraic model can reduce the number of transport equations in Reynolds-stress models. The Direct Numerical Simulation (DNS) data of fully-developed turbulent channel flow is used to evaluate the present algebraic model in comparison with the Reynolds-stress model of Shima [4]. The computed results of the algebraic model agree well with the DNS data, except in the centerline region of the channel. Therefore, the present algebraic model is used to model the gradient of the Reynolds shear stress only in the near-wall region and the high-Reynolds-number $k-\varepsilon$ turbulence model is used in the outer region. The computed results of the combination of the algebraic model and the high-Reynolds-number $k-\varepsilon$ turbulence model show very good agreement with the DNS data and those of the Reynolds-stress model of Shima.

Keywords: Turbulence modeling, Reynolds-stress model, High-Reynolds-number $k-\varepsilon$ turbulence model, Channel flow

1 Introduction

Turbulence is one of the most important and unsolved problems in science and engineering. Turbulence modeling has played a key role in the process of predicting the mean-velocity field in turbulent flows. Conventionally, there are two ways of turbulence modeling: the eddy-viscosity concept and the Reynolds-stress transport phenomena. The eddy-viscosity concept can be classified by the number of transport equations into three levels: zero-equation, one-equation and two-equation turbulence models. The $k-\varepsilon$ model, which is the two-equation type and requires the transport equations for the turbulence kinetic energy k and the dissipation rate of turbulence kinetic energy ε, is the most widely-used turbulence model in the Computational Fluid Dynamics (CFD) commercial software. This is because the $k-\varepsilon$ model can predict a wide variety of flows accurately and it does not require high computational time. Nevertheless, it cannot accurately predict the complex flows where there are regions of separation and reattachment. The Reynolds-stress transport phenomena which is the turbulence model based on the transport equations of all Reynolds stresses can predict the complex flows more accurately. However, the Reynolds-stress model is not widely used in the CFD commercial software because the higher number of transport equations is required compared to the $k-\varepsilon$ model.

The present work is aimed to propose the approach to reduce the number of transport equations in Reynolds-stress models by modeling the gradients of the Reynolds shear stresses algebraically. In the present algebraic model, the gradients of the Reynolds shear stresses are modeled in terms of the gradients of the product of the root-mean-square of the velocity fluctuations times a model constant [1, 2, 3]. In other words, this gradient-of-

Reynolds-shear-stress algebraic model is used to obtain the Reynolds shear stresses in Reynolds-stress models. Therefore, the number of transport equations required in Reynolds-stress models for three-dimensional flows can be reduced from seven transport equations for all Reynolds stresses and the dissipation rate of turbulence kinetic energy to four transport equations for the Reynolds normal stresses and the dissipation rate of turbulence kinetic energy. The present algebraic model is evaluated in comparison with the Reynolds-stress model of Shima [4] using the fully-developed turbulent channel flow of Kim et al [5] as a test case.

2 Gradient-of-Reynolds-Shear-Stress Algebraic Model

The governing equation for the time-averaged streamwise velocity of steady incompressible fully-developed turbulent channel flow is given as follows:

$$\nu \frac{d^2\overline{u}}{dy^2} = \frac{1}{\rho}\frac{d\overline{p}}{dx} + \frac{d\overline{u'v'}}{dy} \tag{1}$$

where $\overline{u}$ is the time-averaged streamwise velocity, $\overline{p}$ is the time-averaged pressure, ν is the kinematic viscosity, ρ is the fluid density, and $\frac{d\overline{u'v'}}{dy}$ is the gradient of the Reynolds shear stress which is unknown and needs modeling.

In the present approach, the gradient of the Reynolds shear stress, $\frac{d\overline{u'v'}}{dy}$, is modeled in terms of the gradient of the product of the root-mean-square of the velocity fluctuations times a model constant as follows [1, 2, 3]:

$$\frac{d\overline{u'v'}}{dy} = C\frac{d(u'_{rms}v'_{rms})}{dy} \tag{2}$$

where u'_{rms} and v'_{rms} are the root-mean-square of the velocity fluctuations in the streamwise and cross-stream directions respectively, and C is the model constant and equal to –1/2.

The root-mean-square of the velocity fluctuations in the streamwise and cross-stream directions, u'_{rms} and v'_{rms}, can be obtained by solving the transport equations of the Reynolds normal stresses, $\overline{u'u'}$ and $\overline{v'v'}$, in Reynolds-stress models and then taking the square root of these Reynolds normal stresses to obtain the root-mean-square of the velocity fluctuations.

3 Simplification of Reynolds-Stress Models with the Gradient-of-Reynolds-Shear-Stress Algebraic Model

In order to solve Eq.(1) with the aid of Eq.(2), the transport equations for $\overline{u'u'}$, $\overline{v'v'}$, $\overline{w'w'}$ and ε are required as follows:

Transport equations for $\overline{u_i'u_j'}$ *only where* $i = j$:

$$\frac{D}{Dt}\left(\overline{u_i'u_j'}\right) = D_{ij} + T_{ij} + \phi_{ij} + P_{ij} - \varepsilon_{ij} \tag{3}$$

where each term on the right-hand side denotes the following transport phenomena for $\overline{u_i'u_j'}$: $D_{ij} = \frac{\partial}{\partial x_k}\left(\nu \frac{\partial}{\partial x_k}\left(\overline{u_i'u_j'}\right)\right)$ is the viscous diffusion, $P_{ij} = -\left(\overline{u_j'u_k'}\frac{\partial \overline{u_i}}{\partial x_k} + \overline{u_i'u_k'}\frac{\partial \overline{u_j}}{\partial x_k}\right)$ is the production, T_{ij} is the turbulent diffusion, ϕ_{ij} is the velocity-pressure-gradient correlation and ε_{ij} is the dissipation. Except D_{ij} and P_{ij} which in principle need no modeling, each term on the right-hand side is modeled by Shima (1993) as follows:

$$T_{ij} = \frac{\partial}{\partial x_k}\left(C_s \frac{k}{\varepsilon}\overline{u_k'u_m'}\frac{\partial}{\partial x_m}\left(\overline{u_i'u_j'}\right)\right) \tag{4}$$

$$\phi_{ij} = \phi_{(1)ij} + \phi_{(2)ij} + \phi_{(w1)ij} + \phi_{(w2)ij} \tag{5}$$

where

$$\phi_{(1)ij} = -C_1\frac{\varepsilon}{k}\left(\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}k\right) \tag{6.1}$$

$$\phi_{(2)ij} = -C_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P\right) \text{ with } P = -\overline{u_k'u_m'}\frac{\partial \overline{u_k}}{\partial x_m} \tag{6.2}$$

$$\phi_{(w1)ij} = C_{w1}\frac{\varepsilon}{k}\left(\overline{u_k'u_m'}n_k n_m \delta_{ij} - \frac{3}{2}\overline{u_k'u_i'}n_k n_j - \frac{3}{2}\overline{u_k'u_j'}n_k n_i\right)\frac{k^{3/2}}{C_\ell \varepsilon y} \tag{6.3}$$

$$\phi_{(w2)ij} = C_{w2}\left(\phi_{(2)km}n_k n_m \delta_{ij} - \frac{3}{2}\phi_{(2)ik}n_k n_j - \frac{3}{2}\phi_{(2)jk}n_k n_i\right)\frac{k^{3/2}}{C_\ell \varepsilon y} \tag{6.4}$$

$$\varepsilon_{ij} = -\frac{2}{3}\delta_{ij}\varepsilon \tag{7}$$

with the following model constants and functions:

$$C_1 = 1 + 2.58AA_2^{1/4}\left(1-\exp\left(-\left(0.0067R_T\right)^2\right)\right) \tag{8}$$

$$C_2 = 0.75A^{1/2} \tag{9}$$

$$C_{w1} = -\frac{2}{3}C_1 + 1.67 \tag{10}$$

$$C_{w2} = \frac{1}{C_2}\max\left(\frac{2}{3}\left(C_2 - 1\right) + 0.5, 0\right) \tag{11}$$

$$A = 1 - \frac{9}{8}A_2 + \frac{9}{8}A_3 \tag{12}$$

$$A_2 = a_{ij}a_{ji} \tag{13}$$

$$A_3 = a_{ij}a_{jk}a_{ki} \tag{14}$$

$$a_{ij} = \frac{\overline{u_i'u_j'}}{k} - \frac{2}{3}\delta_{ij} \tag{15}$$

$$R_T = \frac{k^2}{\nu\varepsilon} \tag{16}$$

$$C_s = 0.22 \tag{17}$$

$$C_\ell = 2.5 \tag{18}$$

Transport equation for ε:

$$\frac{D\varepsilon}{Dt} = \frac{\partial}{\partial x_k}\left(C_\varepsilon \frac{k}{\varepsilon}\overline{u_k'u_m'}\frac{\partial \varepsilon}{\partial x_m} + \nu\frac{\partial \varepsilon}{\partial x_k}\right) + C_{\varepsilon 1}^{*}\frac{\varepsilon}{k}P - C_{\varepsilon 2}\frac{\varepsilon}{k}\tilde{\varepsilon} \tag{19}$$

where

$$\tilde{\varepsilon} = \varepsilon - 2\nu\left(\frac{\partial\sqrt{k}}{\partial x_m}\right)^2 \tag{20}$$

$$C_{\varepsilon 1}^{*} = C_{\varepsilon 1} + \psi_1 + \psi_2 \tag{21}$$

$$\psi_1 = 1.5A\left(\frac{P}{\varepsilon} - 1\right) \tag{22}$$

$$\psi_2 = 0.35\left(1 - 0.3A_2\right)\exp\left(-\left(0.002R_T\right)^{1/2}\right) \tag{23}$$

$$C_\varepsilon = 0.18 \tag{24}$$

$$C_{\varepsilon 1} = 1.45 \tag{25}$$

$$C_{\varepsilon 2} = 1.90 \tag{26}$$

The turbulence kinetic energy k is defined as:

$$k = \frac{1}{2}\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right) \tag{27}$$

The Reynolds shear stress $\overline{u'v'}$ appearing in the transport equations of $\overline{u_i'u_j'}$ and ε can be evaluated from Eq.(2) as follows:

$$\int d\left(\overline{u'v'}\right) = C\int\left(\frac{d}{dy}\left(u'_{rms}v'_{rms}\right)\right)dy \tag{28}$$

where the integration on the right-hand side can be determined by the trapezoidal method using the numerical values of $\frac{d}{dy}\left(u'_{rms}v'_{rms}\right)$.

4 Results and Discussion

The present algebraic model is evaluated in comparison with the Reynolds-stress model of Shima [4] and the DNS data of the fully-developed turbulent channel flow of Kim et al [5] at Reynolds number $Re_\tau \equiv u_\tau h/\nu$ of 180 where $u_\tau \equiv \sqrt{\tau_w/\rho}$ is the friction velocity and h is the channel half-height.

Figures 1-6 are the profiles of mean velocity, Reynolds shear stress $\overline{u'v'}$, Reynolds normal stress in the streamwise direction $\overline{u'u'}$, Reynolds normal stress in the cross-stream direction $\overline{v'v'}$, Reynolds normal stress in the spanwise direction $\overline{w'w'}$ and dissipation rate of turbulence kinetic energy ε respectively. All the results are presented in scale of wall units.

It is found that the mean velocity profile predicted by the present algebraic model is in closer agreement with the DNS data than that of the Reynolds-stress model of Shima. However, the present algebraic model fails to capture the physics of the flow in the region very close to the centerline of the channel. In principle, the normal gradients of all variables and the normal velocity become zero at the centerline of the channel because of the presence of the symmetry plane. Therefore, at the centerline of the channel the present algebraic model, or the right-hand side of Eq.(2), behaves as

$$\left(\frac{d\left(u'_{rms}v'_{rms}\right)}{dy}\right)_{y=h} = v'_{rms}\left(\frac{du'_{rms}}{dy}\right)_{y=h} + u'_{rms}\left(\frac{dv'_{rms}}{dy}\right)_{y=h} = 0 \quad (29)$$

whereas, in fact, the gradient of the Reynolds shear stress, or the left-hand side of Eq.(2), behaves as

$$\left(\frac{d\overline{u'v'}}{dy}\right)_{y=h} \neq 0 \quad (30)$$

as shown by the DNS data in Figure 2. Therefore, the present algebraic model is used to model the gradient of the Reynolds shear stress only in the near-wall region ($y/h \leq 0.5$) and the high-Reynolds-number $k-\varepsilon$ turbulence model is used in the outer region ($y/h > 0.5$) where

$$\frac{d\overline{u'v'}}{dy} = -\frac{d}{dy}\left(\nu_T \frac{d\bar{u}}{dy}\right) \quad (31)$$

and ν_T is the eddy viscosity which is defined as

$$\nu_T = C_\mu \frac{k^2}{\varepsilon} \quad (32)$$

and $C_\mu = 0.09$ is the model constant. It is found that the computed results of this combination, especially the mean velocity profile, show very good agreement with the DNS data and those of the Reynolds-stress model of Shima.

5 Conclusions

The algebraic approach on modeling the gradients of the Reynolds shear stresses in Reynolds-stress models is presented in the current work in order to reduce the number of transport equations required in Reynolds-stress models. The DNS data of fully-developed turbulent channel flow and the predicted results from the Reynolds-stress model of Shima [4] are used for assessment. It is found that the combination of the present gradient-of-Reynolds-shear-stress algebraic model in the near-wall region ($y/h \leq 0.5$) with the high-Reynolds-number $k-\varepsilon$ turbulence model in the outer region ($y/h > 0.5$) gives the very good agreement with the DNS data and the Reynolds-stress model of Shima.

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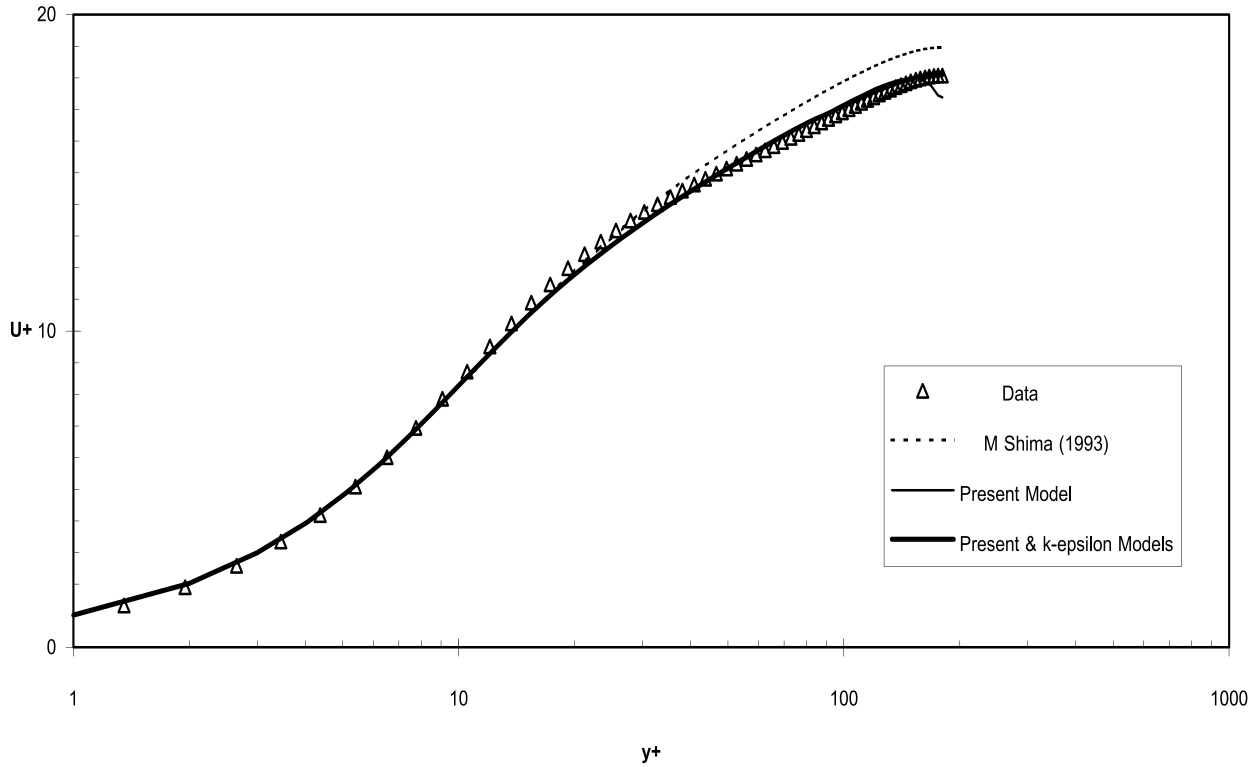

Figure 1 Profile of mean streamwise velocity in wall units at Re_τ = 180

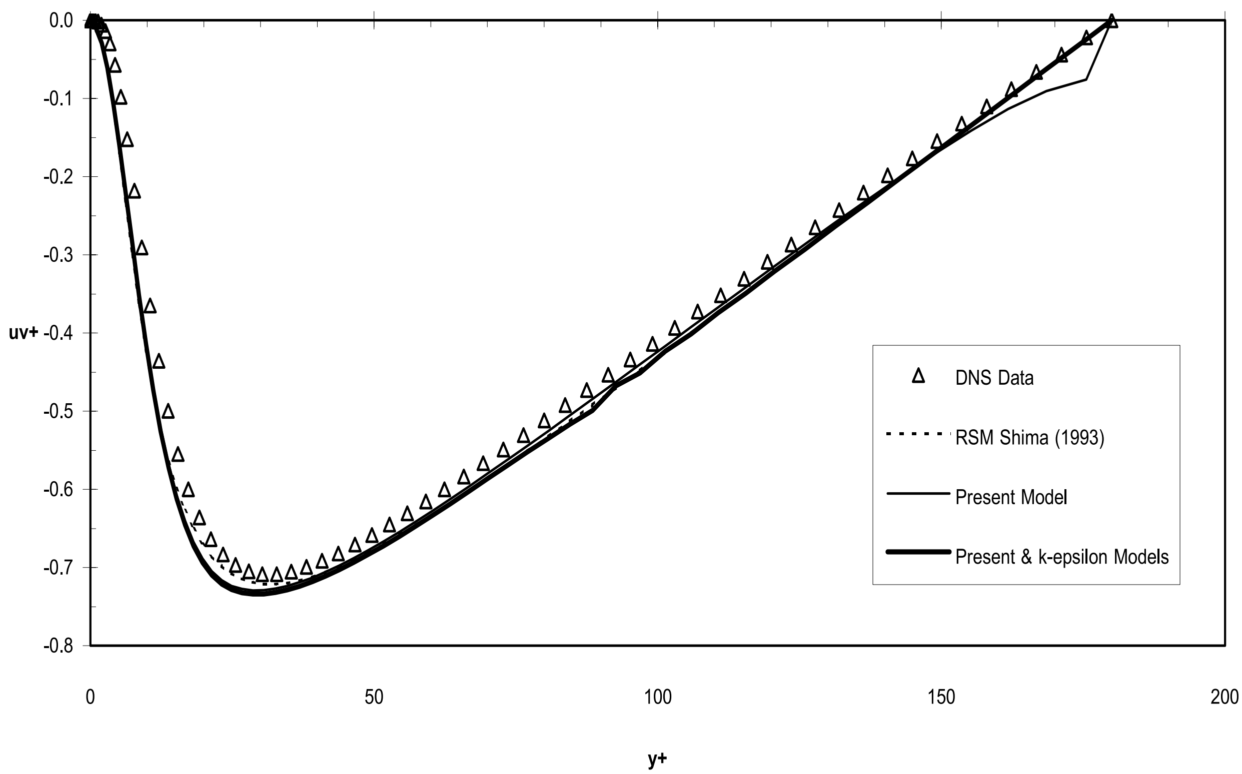

Figure 2 Profile of Reynolds shear stress in wall units at Re_τ = 180

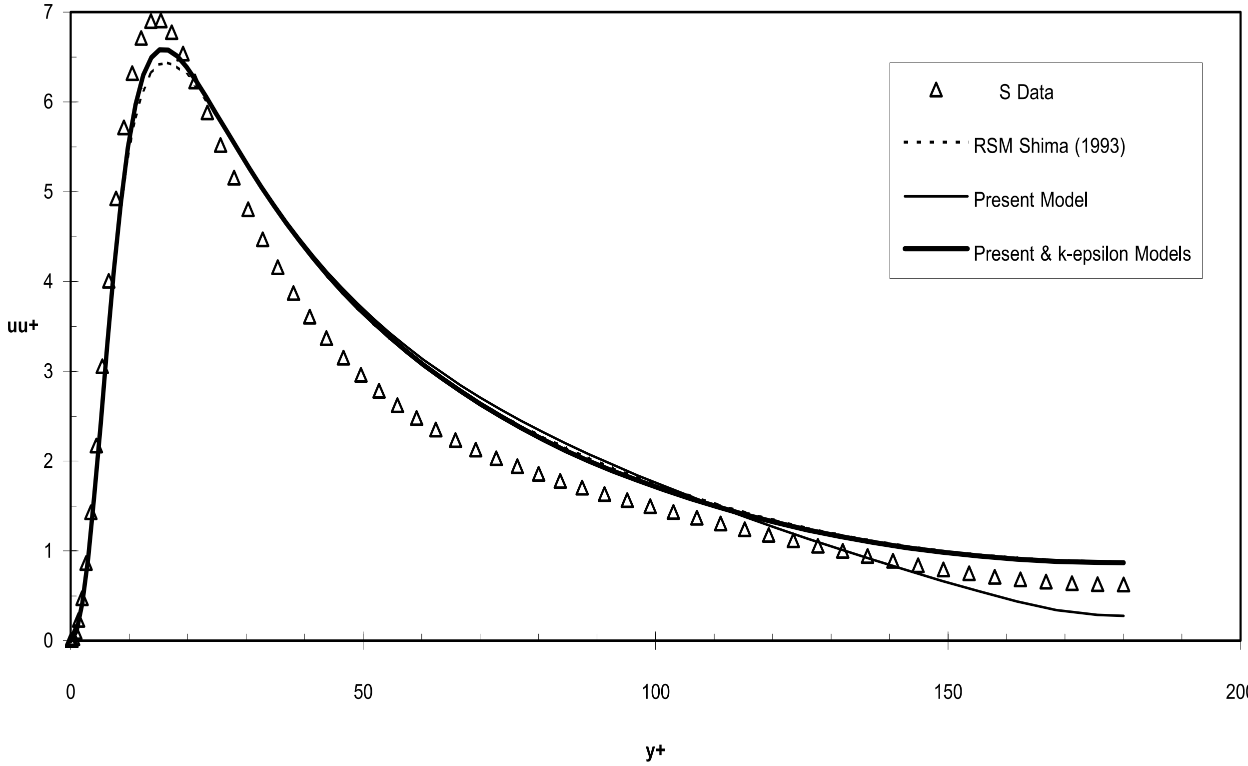

Figure 3 Profile of streamwise Reynolds normal stress in wall units at $Re_\tau = 180$

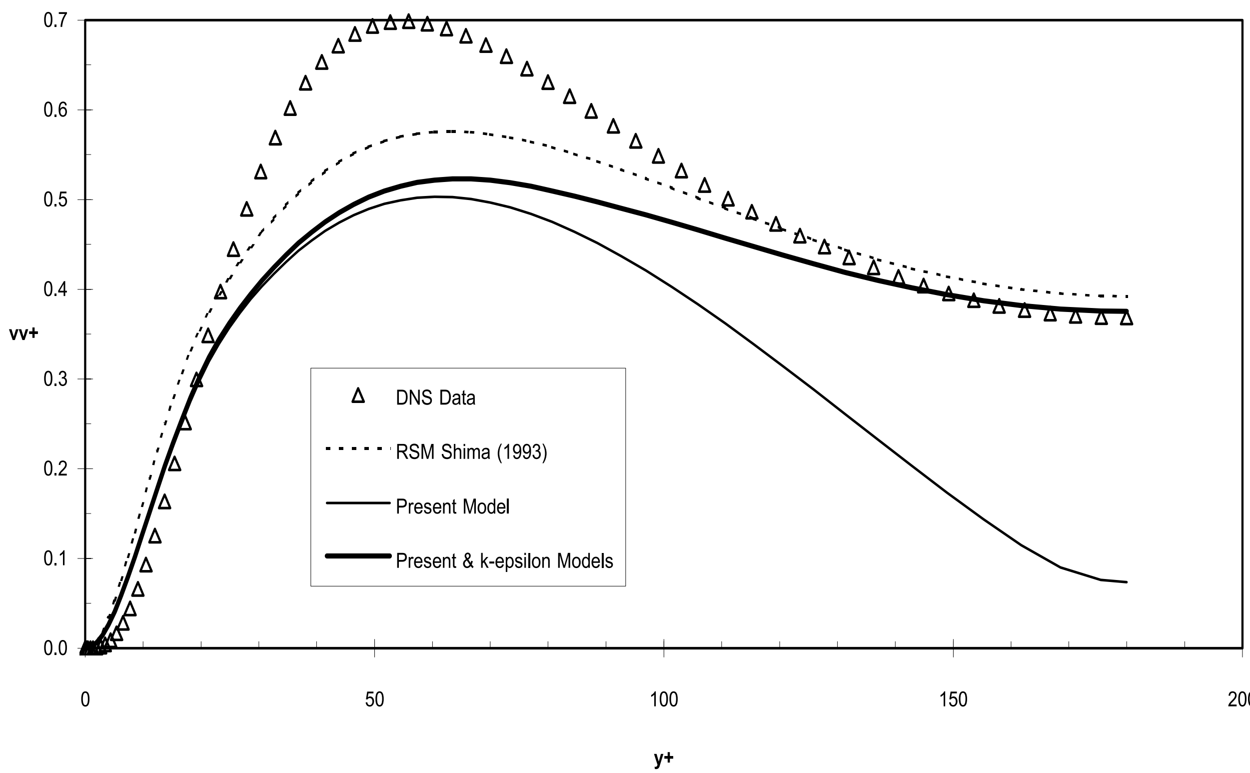

Figure 4 Profile of cross-stream Reynolds normal stress in wall units at $Re_\tau = 180$

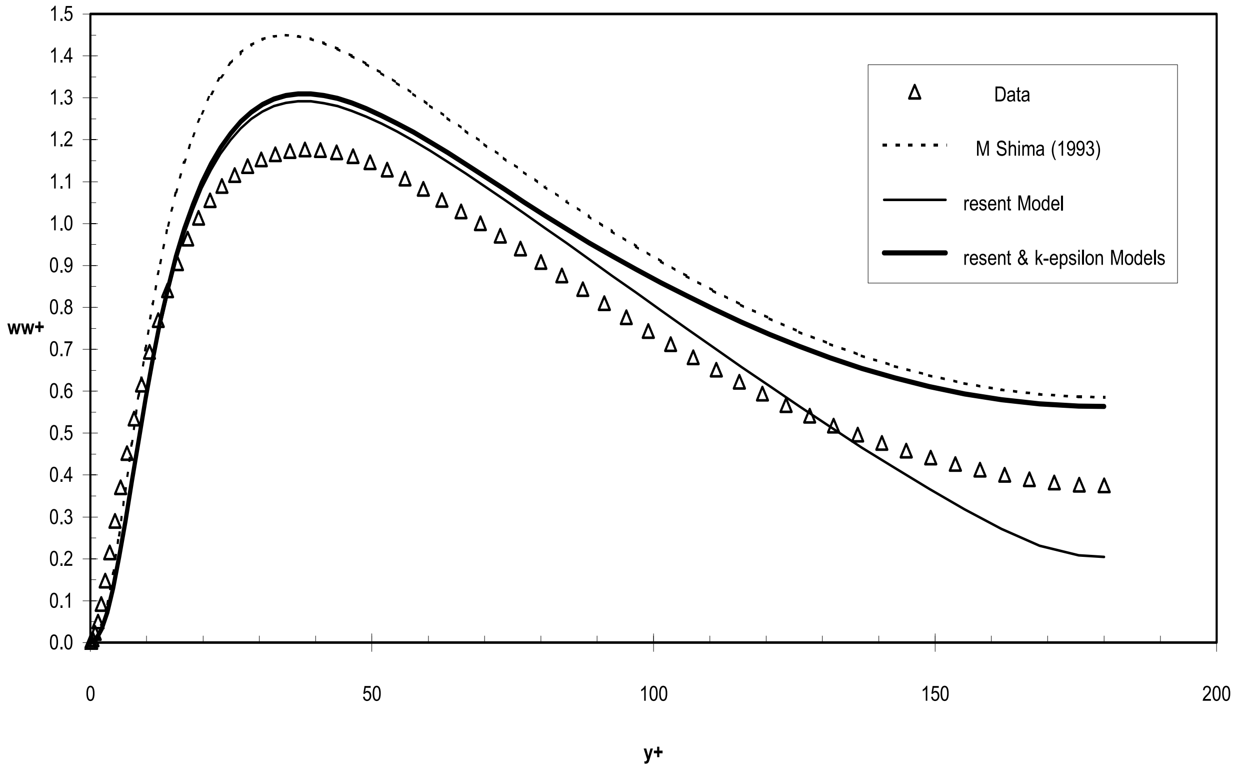

Figure 5 Profile of spanwise Reynolds normal stress in wall units at $Re_\tau = 180$

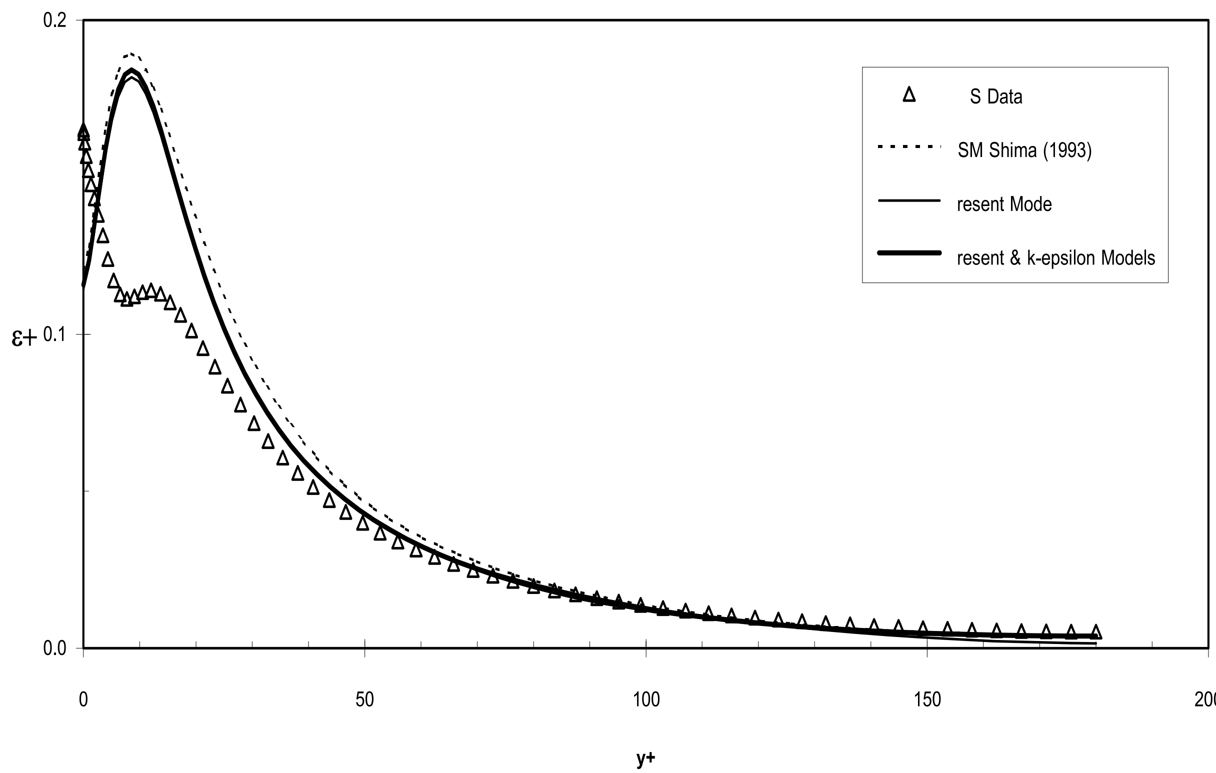

Figure 6 Profile of dissipation rate of turbulence kinetic energy in wall units at $Re_\tau = 180$