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A Simple Checkerboard Suppression Technique for Topology Optimisation Using Simulated Annealing

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Abstract

In this paper, a simple technique to prevent checkerboard problem in topology optimisation is presented. The technique is employed along with Simulated Annealing (SA) the universal optimisation method. The objective function used in SA search strategy is obtained from the weighted sum of structural weight, compliance and checkerboard penalty. The developed design approach is implemented on two design case studies. The influence of weighting factors on the obtained optimum results is illustrated.

1. Introduction

In structural design process, topology optimisation is said to be a powerful tool in responding to the need to seek a better structural configuration. To design a structure for a particular propose, one may need to know what his structure should look like at the preliminary stage as illustrated in Figure 1. The classical design problem of topology optimisation is to find structural form such that minimizing its compliance whilst fulfilling predefined criterion e.g. mass and equilibrium constraints [1]. Apart from compliance or system strain energy, many designers have used other structural characteristics, such as natural frequency, as an objective function [2&3]. Design constraints may include natural frequency, buckling factor [4] and stress criterion [5]. In numerical point of view, by the use of Finite Element Method (FEM) for structural analysis, topology optimum design can be performed by discretising a structure into a number of connected finite elements. Design variables determine the distribution of element thickness, which means that elements with nearly zero – thickness represent holes on the structure whereas other elements indicate the existence of structural material. Figure 2 shows how element thickness distribution of a structural finite element model be converted to a structural configuration. Aside from element thickness, topology design variables can also be thought of as element density and modulus of elasticity [6].

The most traditionally used optimisation method for topology design is Optimality Criteria Method (OCM) [7] as it is arguably the most powerful method for this task. However, some may still fancy using classical gradient-based optimisation methods like Sequential Linear Programming (SLP) [8] and the Method of Moving Asymptotes (MMA) [9]. Moreover, random – directed optimisation methods e.g. Genetic Algorithms (GA) and Simulated Annealing (SA) have also been implemented on this type of design problem [10&11]. Despite the capability of reaching a global optimum of GA and SA, the methods seem to lose their attraction when used in topology design because they are time consuming and have no consistency. This is due to the large number of topological design variables. Nevertheless, these renowned universal optimisation methods cannot be overlooked due to their ease to use. Probably, they could be the tool to break the barrier of traditional design approach in the future.

Checkerboard along with mesh dependency and local optimum are inevitable problems in performing topology optimisation. Mesh dependency describes the various optimal results from different grid densities on one design domain. The local optimum, as we know, can result in the less effective design. Checkerboard patterns usually occur on an optimum structure particularly when using four – node quadrilateral membrane element for compliance minimisation. It has been recognised that such patterns come from numerical instability as shown in [1]. More importantly, it can lead to a so-called 1-node connected hinge problem in compliance mechanism synthesis [12]. Since the problem was stated, there have been several ways developed to alleviate the checkerboard patterns. The filtering technique as used in image processing [7&9], defining checkerboard constraint function [12], perimeter control method [13], using higher order finite element formulation e.g. 8-node quadrilateral membrane [6], and the use of nodal thickness as design variables instead of element thickness (known as Q4/Q4 strategy) [14].

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This paper presents a simple technique to suppress a checkerboard pattern in topology optimisation. The technique is developed to be employed with simulated annealing method with binary string representing structures. The optimisation problem is solved by means of multi-objective optimisation using weighted sum technique. In this search strategy, checkerboard problem can be prevented by taking checkerboard penalty for a structure as one of the objective functions. A number of topology optimisation of plate structures are assigned and solved using the developed technique. With a variety of weighting factor sets, the optimum solutions are obtained and compared. The influence of weighting factors on the optimum results is shown. It is eventually illustrated that the present design can effectively perform a checkerboard free topology optimisation.

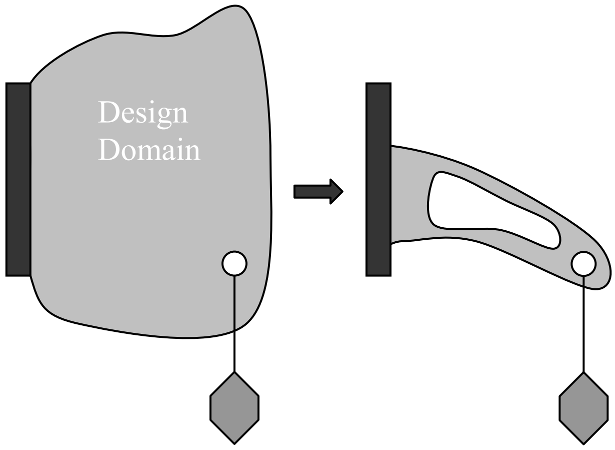


Figure 1 Structural conceptual design

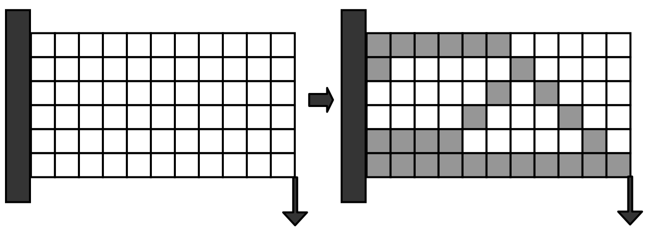


Figure 2 Discretised design domain

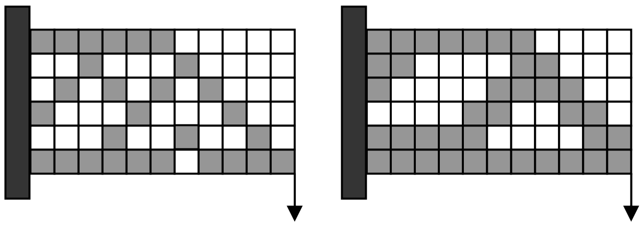


Figure 3 Structures with and without checkerboard form

2. Simulated Annealing

Simulated annealing is among the most popular random – directed methods like Genetic Algorithm (GA) and Evolutionary Programming (EP). The method is based upon mimicking the random behaviour of molecules during the annealing process, which involves slow cooling from a high temperature [15]. As the temperature cools, the atoms line themselves up and form a crystal, which is the state of minimum energy in the system. However, if the metal is cooled too quickly, the minimum energy state is not reached. The basic algorithm follows and is usually formulated as a minimisation problem.

The search procedure of SA is to start with a single initial solution with fitness f is taken and then adjusted in some manner to produce a candidate solution with fitness f'. If $f' < f$, then f' is taken onto the next iteration, however, in cases that $f' > f$, the candidate value may still be chosen depending upon the Boltzmann probability

$$\Pr = e^{(f'-f)/T} \tag{1}$$

where T is the annealing temperature.

The simple algorithm of SA is shown Figure 4. The key role of SA search is that the discovery of a new candidate. As traditionally SA is the method without using derivatives, the candidate is created, by mutating on a current solution. The more effective procedure is that creating a set of new candidates and then selecting the best of them to be compared with their parent [16].

As being a universal method, any aspect of design variables can be used in SA. For this work, topology of a structure is represented by binary string ‘1’ for material existence and ‘0’ for holes as shown in Figure 5. Also note that in this paper ‘1’ means 1 unit of thickness. A bit ‘0’ represents 0.000001 unit of thickness in stead of zero-thickness so as to prevent singularity in global stiffness matrix of structural system.

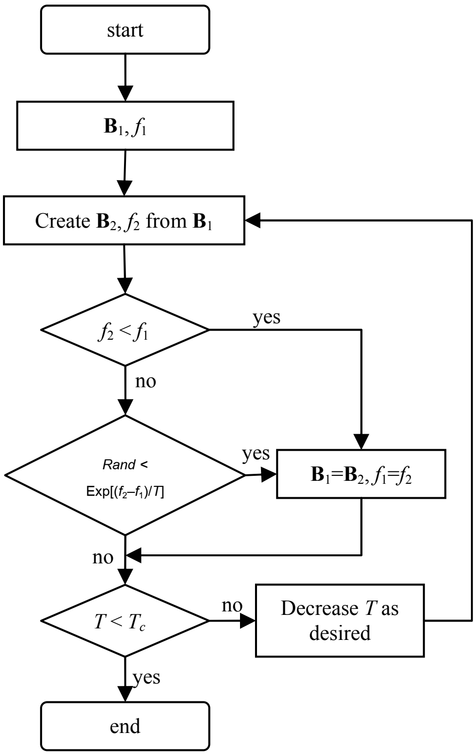


Figure 4 Flowchart of SA

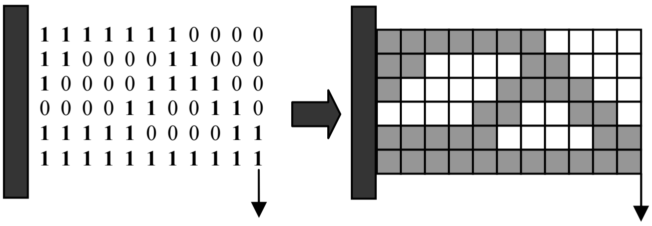


Figure 5 Binary strings representing a structure

3. Dealing with Checkerboard Patterns

In order to ease in explanation and computation, design of plate structures is used for demonstration of the technique whereas the design domain is rectangular as shown in Figure 6. The idea of preventing checkerboard here is the modification of that presented in [12]. Let the structure in Figure 6 have m+1×n+1 elements. Thus, there are m×n interior nodes as shown and, at each interior node, there are 4 elements surrounding it. If the 4-element pattern matches any of the two cases in Figure 7, the local checkerboard penalty value is one, otherwise, it is zero.

$$c_i = \begin{cases} 1, \text{case\#1 and \#2} \\ 0, \text{otherwise} \end{cases} \tag{2}$$

The total checkerboard penalty value, C, of a typical structure can be computed as

$$C = \sum_{1}^{m\times n} c_i \ . \tag{3}$$

It can be easily concluded that C = 0 represents a topology without checkerboard or even a 1-node connected hint and this is the minimum point of the penalty function. The important parameters in the design are checkerboard penalty C, structural weight M_s and system compliance V_e that is computed from FEA. The new objective function of the optimisation problem, which is the weighted sum of the three design parameters, can be written as a function of binary string **B** as

$$f(\mathbf{B}) = w_1 V_e + w_2 M_s + w_3 C \tag{4}$$

where w_1, w_2 and w_3 are the weighting factors for compliance, weight and checkerboard penalty respectively.

Note that the idea of using multi-objective optimisation technique in dealing with topology optimisation with the like of GA or SA is not unfamiliar by many researchers as it has been successfully implemented in [10] and [11].

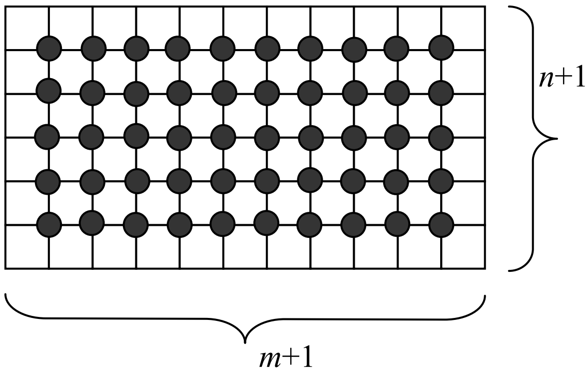


Figure 6 Discretised structural model

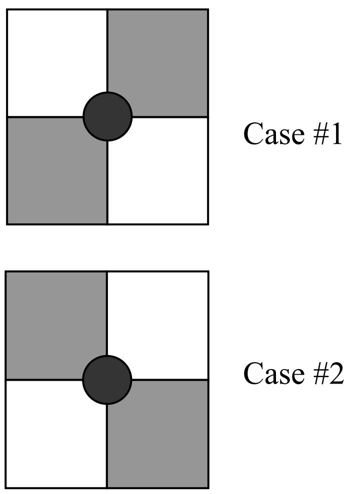


Figure 7 4-Element patterns to be penalised

4. Numerical Examples

Two topological optimisation problems that are the design of a classical MBB beam and cantilever beam are solved using the present method. The beams are show in Figure 8 and Figure 9. The beams are intuitively discretised as 20×10 elements for the MBB beam and 20×21 elements for the other. Both structures are optimised by using SA with 500 iterations, 20 candidates being created at each loop, initial annealing temperature 10 and the final temperature 0.001.

With a variety set of weighting factors $\{w_1, w_2, w_3\}$, the optimisation are then solved and the optimum results were obtained and illustrated. Figure 10 displays the optimum topologies of the MBB beam without checkerboard penalisation (w_3 = 0). The results in (a), (b), (c) and (d) belong to the weighting sets [0.5,1,0], [1,1,0], [1,5,0] and [0.5,5,0] respectively. The results, as expected, show no escape from being checkerboard topologies.

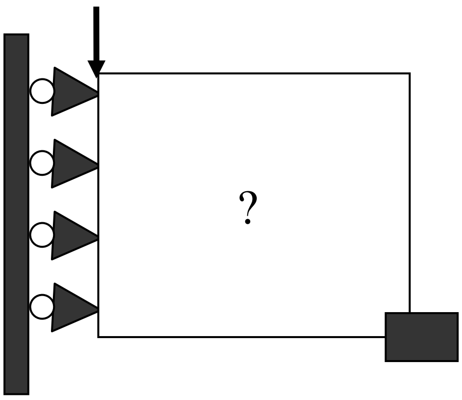


Figure 8 Design of an MBB beam

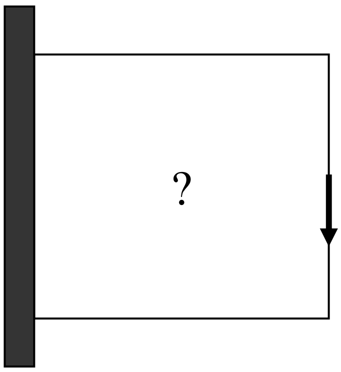


Figure 9 Design of a cantilever beam

(a) (b)

(c) (d)

Figure 10 Optimum results of the MBB beam without checkerboard penalty

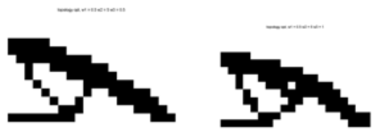
The more interesting optimum results are shown in Figure 11 for the MBB beam and Figure 12 for the cantilever beam. The sets of weighting factors that correspond to figures (a) to (p) are given in Table 1. The results of these two beams show that the strong penalty (w_3 = 5) can perfectly suppress checkerboards. However, the obtained topologies are unlikely to be realisable since they still need some refinement from the stage of shape and sizing optimisation.

Fig.	w_1,w_2,w_3	*Fig.*	w_1,w_2,w_3
a	0.5,0.5,5	i	1,1,0.5
b	0.5,1,0.5	j	1,1,1
c	0.5,1,1	k	1,1,5
d	0.5,1,5	l	1,5,0.5
e	0.5,5,0.5	m	1,5,1
f	0.5,5,1	n	1,5,5
g	0.5,5,5	o	5,5,0.5
h	1,0.5,0.5	p	5,5,1

Table 1 Weighting factors

(a) (b)

(c) (d)



(e) (f)

(g) (h)

(i) (j)

(k) (l)

(m) (n)

(o) (p)

Figure 11 Optimum results of the MBB beam with checkerboard penalty

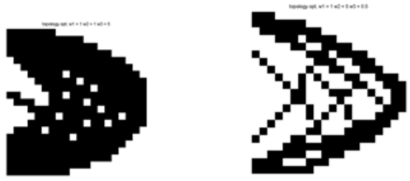
(a) (b)

(c) (d)

(e) (f)

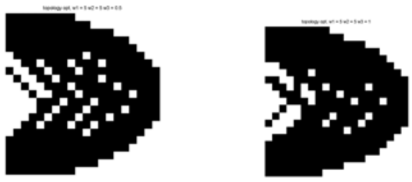
(g) (h)

(i) (j)



(k) (l)

(m) (n)



(o) (p)

Figure 12 Optimum results of the cantilever beam with checkerboard penalty

5. Conclusion

The present design strategy can prevent the formation checkerboard patterns if the proper set of weighting factors is used. It also shows that SA is a powerful tool for topology optimisation. The weighted sum technique is acceptable for the task. The extension of this concept to 3D topology optimum design is possible. Computation time is still far from matching the OCM. More work has to be made so that the convergence rate of the design approach is enhanced.

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