การประชุมวิชาการเครือข่ายวิศวกรรมเครื่องกลแห่งประเทศไทยครั้งที่ 17
15-17 ตุลาคม 2546 จังหวัดปราจีนบุรี

Simulation of a Two-Dimensional Channel Flow around Arbitrary Obstacle with the Lattice Boltzmann Method

Phadungsak Ratanadecho
Department of Mechanical Engineering, Faculty of Engineering,
Thammasat University (Rangsit Campus), Pathumthani, 12121
Tel (02) 564-3001 Fax (02) 564-3010

Abstract

It is shown that the lattice Boltzmann automata (LBA) provides a viable numerical method for the study of two-dimensional channel flow around arbitrary obstacle. The LBA is built up on the D2Q9 model and a single relaxation time method called the lattice-BGK method. Numerical results based on an advanced lattice Boltzmann technique for a discrete microscopic description of a low Reynolds number flow in a two-dimensional channel flow are reported and its practical relevance is investigated by comparing it with the analytical result. It is found that this approach allows to improve the understanding of the flow pattern in highly complex geometries and to obtain a reliable database for their operating behaviour and design.

1. Introduction

Recently, lattice gas automata (LGA) and lattice- Boltzmann automata (LBA) approaches (Frich et al. [1] and Frich et al. [2]) have been shown to be attractive alternatives to classical methods in CFD, e.g., finite volume methods and Finite element methods for the solution of the partial differential equations (PDE), i.e, Navier-Stokes equations (Noble et al. [3] and Noble et al. [4]). Partial differential equations (PDE) have been the only and most tractable way to describe dynamical and spatially extended system, for a long time.

However, as more difficult problems are considered, PDE may be less adequate and cannot always be formulated when complicated local dynamics involving thresholds or discontinuity are studied. Finally, the sophisticated numerical schemes used to solve PDE often screen out the nature of the process being analyzed and prevent their generalization to the new phenomena. In these situations, a description based on a simple model of reality, instead of an exact equation, is quite powerful. The solution procedure is the replaced by a direct computer simulation of the model, from which predictions can be made, as in a laboratory experiment. The crucial justification of this methodology is the observation that in many field of sciences, there are several levels of reality.

The LBA is a derivative of the lattice gas automata method which was first proposed about a dozen years ago by a number of physicists. Nowadays, the method has quickly found its way in dealing with a number of engineering flow problems. Unlike classical methods which solve the discretized macroscopic Navier-Stokes equations, the LBA is based on microscopic particle models and mesoscopic kinetic equations. The fundamental concept of the LBA is to "construct simplified kinetic models that incorporate the essential physics of microscopic or mesoscopic processes so that the macroscopic averaged properties obey the desired macroscopic equations".

The LBA is especially useful for modeling interfacial dynamics, flows over porous media, multi-phase flows, flow problems in highly complex geometries and various thermodynamic properties of a fluid system, such as multiphase flows problem (Ratanadecho [5] and Bernsdorf et al. [6]), in a relatively straightforward way. In addition, the LBA algorithm tends to be very simple, allowing parallelism in a straightforward manner.

The objective of the study is to develop algorithm based on lattice- Boltzmann (BGK) automata (Qian et al. [7]) to investigate a two-dimension flow around arbitrary obstacle mounted in a channel for a range of Reynolds numbers between 80 and 300. In order to check the accuracy, the calculations from present LBA model are compared with the theoretical result for the single phase channel flow problem.

2. Description of Numerical Method

The concept of LBA is that treats the fluid on a statistical level, simulating the movement and interaction of single particles or ensemble-averaged particle density distributions by solving a velocity discrete Boltzmann-type equation. The lattice-Boltzmann method has been shown to be a very efficient tool for flow simulation in highly complex geometries discretized by up to several million grid points.

2.1 Numerical schemes

All numerical simulations presented in this paper will be briefly described here.

For simplicity, an equidistant orthogonal lattice is chosen for common LBA computation. This could be done without a significant loss of memory and performance, since the LBA requires much less memory and CPU time than classical methods. On every lattice node r_*, a set of *i* real numbers, the particle density distributions f_i, is stored. The updating of the lattice consists of basically two step: a streaming process, where the particle densities are shifted in discrete time steps *t,* through the lattice along the connection lines in direction c_i to their next neighboring nodes $r_* + c_i$, and a relaxation step, where locally a new particle distribution is computed by evaluating an equivalent to the Boltzmann collision integrals (Δ_i^{Boltz}). For every time step, all quantities appearing in the Navier-Stokes equations (density, velocity, pressure gradient and viscosity) can locally be computed in terns of simple functions of this density distribution and (for the viscosity) of the relaxation parameter ω.

For the present computation, a 2D nine-speed (D2Q9) lattice-Boltzmann automata with single time Bhatnagar-Gross-Krook (Bhatnagar et al. (8)) relaxation collision operator Δ_i^{Boltz} proposed by Qain et al. (7) is used:

$$f_i\left(t_*+1, r_*+c_i\right) = f_i\left(t_*, r_*\right) + \Delta_i^{Boltz} \quad (1)$$

$$\Delta_i^{Boltz} = \omega\left(f_i^{eq} - f_i\right) \quad (2)$$

with a local equilibrium distribution function N_i^{eq}:

$$f_i^{eq} = t_p \rho \left\{ 1 + \frac{c_{i\alpha} u_\alpha}{c_s^2} + \frac{u_\alpha u_\beta}{2c_s^2}\left(\frac{c_{i\alpha} c_{i\beta}}{c_s^2} - \delta_{\alpha\beta}\right)\right\} \quad (3)$$

This local equilibrium distribution function f_i^{eq} has to be computed every time step for every node from the components of the local flow velocities u_α and u_β, the fluid density ρ, a lattice geometry weighting factor t_p and the speed of sound c_s, which we chose to recover the incompressible time-dependent Navier-Stokes equations (Qian et al. [7]):

$$\partial_t \rho + \partial_\alpha\left(\rho u_\alpha\right) = 0\,, \quad (4)$$

$$\partial_t\left(\rho u_\alpha\right) + \partial_\beta\left(\rho u_\alpha u_\beta\right) = -\partial_{\alpha P} + \mu \partial_\beta\left(\partial_\beta u_\alpha + \partial_\alpha u_\beta\right) \quad (5)$$

In addition, the left side of the Eq. (1) is analogous to the "translation" stage in LBA, and the right to the "collision" stage. For example, in the two-dimensional "D2Q9" model, there are 9 velocities (c_i) on a square lattice: one has speed=0 and corresponds to a "rest" particle; four have speed=1 and are at 0, 90, 180 and 270 degrees; and four have speed= $\sqrt{2}$ at 45, 135, 225 and 315 degrees (1).

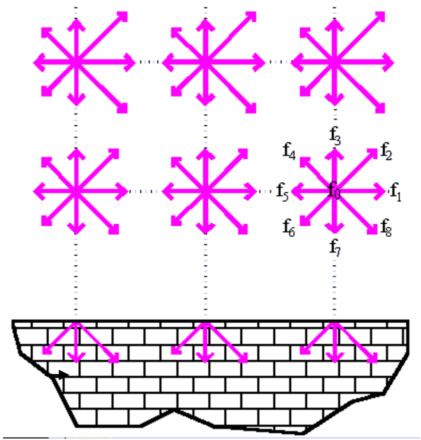

Fig. 1 Schematic 2D lattice Boltzmann calculation on a square lattice, after the translation step. Shown are 6 fluid sites and 3 wall sites (the wall is shown with a brick pattern).

Through careful choice of the equilibrium distribution, the macroscopic quantities (density, velocity, pressure gradient and viscosity) fulfilling the Navier-Stokes equation can be obtained in the term of the moments of the particle distribution function $f_i(t,r)$ at each site, e.g. for the D2Q9 model:

Density: $$\nu = \sum_i f_i(r,t) \quad (6)$$

Flow velocity: $$\nu = \sum_i f_i(r,t)\, c_i / \rho \quad (7)$$

Pressure: $$p = \rho c_s^2 \quad (8)$$

Viscosity: $$\nu = \frac{1}{6}\left(\frac{2}{\omega} - 1\right) \quad (9)$$

2.2 Boundary conditions

Wall boundary conditions

There is a long and still ongoing discussion on the proper use of boundary conditions within the framework of LBA. Although it is known that simple bounce-back wall boundary conditions are of first-order accuracy whereas the lattice-Boltzmann equation is of second order, these bounce-back conditions are the most efficient ones for arbitrary complex geometries. Previous investigations showed that the error produced by the bounce-back boundary conditions is sufficiently small if the relaxation parameter ω is close enough to 2. Therefore, we believe that the bounce-back conditions can still be used without any influence on the order of the LBA scheme, if ω is chosen within a suitable range. Furthermore, the bounce-back boundary conditions is the most efficient one for arbitrary complex geometries, which are most typical for the application of LBA.

Inlet and outlet boundary conditions

In order to simulate a fully developed laminar channel flow, a parabolic velocity profile with a maximum velocity $u_{\max}$ is prescribed at the channel inlet whereas the fixed pressure outlet boundary conditions are chosen.

Initial boundary conditions

For the validation test cases, the equilibrium distribution function f_i^{eq} is computed from given velocity fields for uniform pressure distribution and taken as the initial solution for the density distribution function f_i. The flow field for the arbitrary obstacle is initialized with the equilibrium distribution function f_i^{eq} for zero velocity and uniform pressure, and the inlet velocity had slowly been increased during the first few thousand, iterations, to avoid the generation of pressure waves.

3. Result and Discussion

In order to check the accuracy, the calculations from present LBA model are compared with the theoretical result for the single phase channel flow problem. Comparison of the velocity profile in Fig. 2 shows the same trend although the spatial variation of velocity profile near the center of channel predicted by our model is slightly higher than theoretical result. This might be due to the initialization of the densities with equilibrium distribution f_i^{eq} because of lower iteration numbers.

The good results for the above test case clearly show the possibility of performing accurate numerical simulation for various single phase channel flow problem with the present implementation of the lattice Boltzmann method. Especially, as is already know from the lattice Boltzmann theory, where this quantity could be taken into account for a proper definition of viscosity, no problems with the numerical dissipation have been observed.

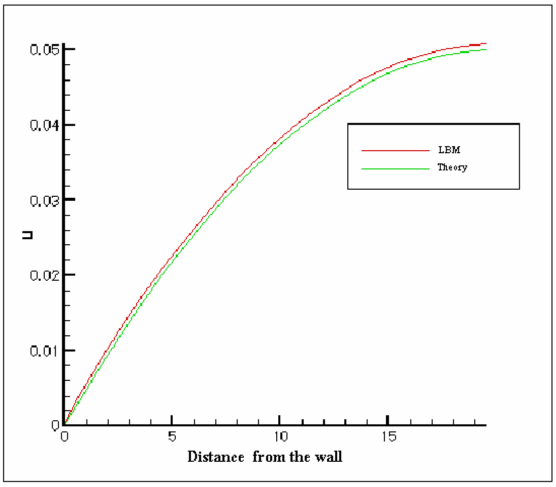

Fig. 2 Single phase channel flow

3.1 Flow around a square obstacle

The flow around a square obstacle positioned inside a channel was simulated for a range of Reynolds number *Re* between 80 and 300, can be defined by the length of the obstacle *d*, the maximum flow velocity $u_{\max}$ of the parabolic inflow profile and the dynamic viscosity ν as

$$\mathrm{Re} = \frac{u_{\max} d}{\nu} \tag{10}$$

In this region, it is known from experiments and other numerical studies that vortex shedding is observed and a two-dimensional time dependent flow evolves. At a Reynolds number Re above approximately 300, the flow might become three-dimensional, and two-dimensional computations will therefore produce unphysical results.

Considering the computational domain as shown in Fig. 3, obstacles of sizes ranging from *d* x *d* = 10 x 10 up to *d* x *d* = 40 x 40 lattice units are positioned vertically centered in the first third section of the computational domain with sizes between *l* x *h* = 500 x 80 and *l* x *h* = 2000 x 320 lattice units.

For the wall, a no-slip boundary condition is realized by particle density bounce-beck. A parabolic velocity inflow profile is applied, and the outlet pressure is fixed.

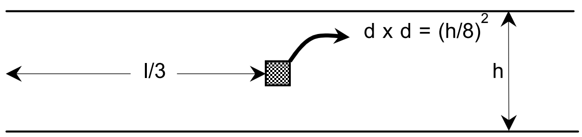

Fig. 3 Obstacle of size d×x in channel of size l×h

The only quantity taken into account in the present analysis is the Strouhal number St, computed from the obstacle diameter *d* , the measured frequency of the wakes *f* and the maximum velocity $u_{\max}$, as defined in Eq. (11):

$$St = \frac{fd}{u_{\max}} \tag{11}$$

All computations are done on one processor of the Pentium III. Starting with zero flow velocity and uniform pressure, after a sufficient number of iterations, time-dependent flow evolves with a fixed frequency *f* . This frequency *f* is determined by spectral analysis of the temporal evolution of the *v*-component of the flow velocity at several points in the wake behind the obstacle.

For this quantity, the numerical convergence of the scheme with respect to grid resolution is investigated first. What is known from fluid mechanics, and can be reproduced very well by our simulations (see Fig.4), is the fact that the topology of the vortex shedding behind a square obstacle changes significantly with the Reynolds number. Whereas for a Reynolds number of 80 the separation point of the vortices is observed to be the rear edge of the obstacle, it moves from the rear to the front edge of the obstacle for higher Reynolds numbers. At Re = 266, small secondary vortices can be found at the top and bottom of the obstacle. A sufficient resolution of this secondary vortex appears to be crucial for the development of a correct shedding frequency f, which results in the necessity for finer grids for higher Reynolds numbers.

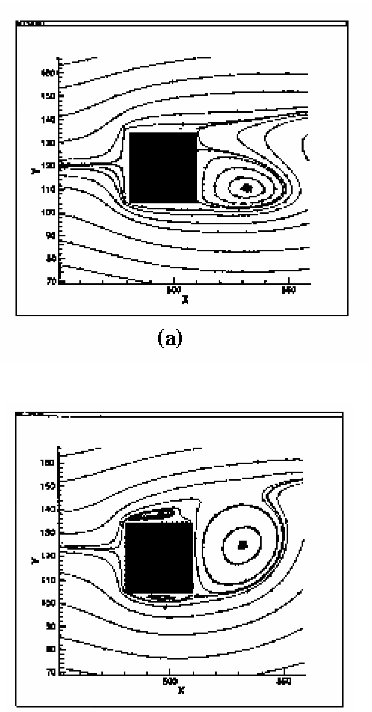

Fig. 4 Flow around a square obstacle at (a) Re=80 and (b) Re=266, for the higher Reynolds number, secondary vortices above and below the square obstacle are displayed

The dependence of the Strouhal number St on grid resolution can be seen for Reynolds numbers between 80 and 266 in Fig.5. The values indicate second-order convergence of the scheme, and lattice sizes of *l* x *h* = 2000 x 320 for obstacles of dimension *d* = 40 product results with good accuracy for Reynolds numbers up to 300. For Reynolds numbers < 100, nearly dependence of Strouhal number on grid resolution can be observed, which is in accordance with our observations concerning secondary vortices.

For one full period, the streamlines of a shedding vortex are shown in Fig.6 at *Re* = 80. One can see a small vortex developing at the rear top edge of the square obstacle, which is moving downwards while growing, and moves upwards while growing, to separate finally from the top rear edge of the obstacle.

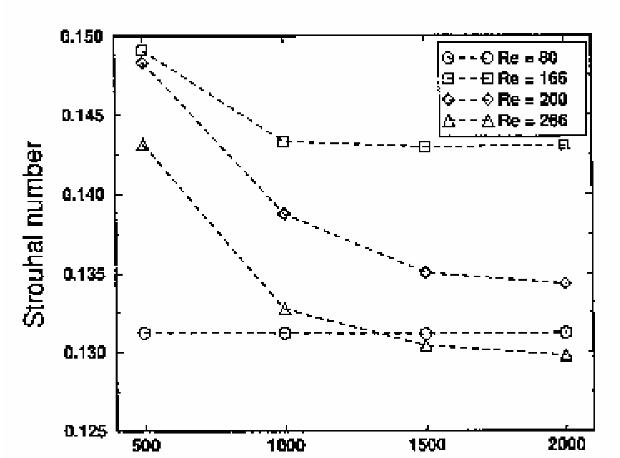

Fig. 5 Strouhal number St as a function of linear lattice dimension l for different Reynolds number Re

3.2 Flow around a highly complex obstacle

For practical applications, this simple procedure as explained in previous section allows for an easy implementation of arbitrary complex structures (e.g. the flow simulation through a porous structure as presented in Fig. 7) or to change the obstacle structure during the computation, which is necessary for problems with time varying flow geometry. To illustrate the capabilities of LBA, the flow contours and velocity vector fields during fluid flow through a highly complex porous structure are presented in Fig. 8. It is evident from the figure that regardless of the complexity of the pores, the flow features expected are well captured by using LBA simulation.

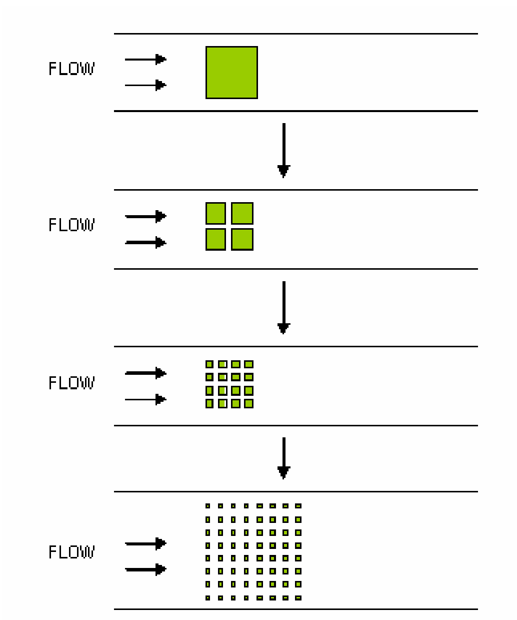

Fig. 7 Obstacle structure with increasing complexity

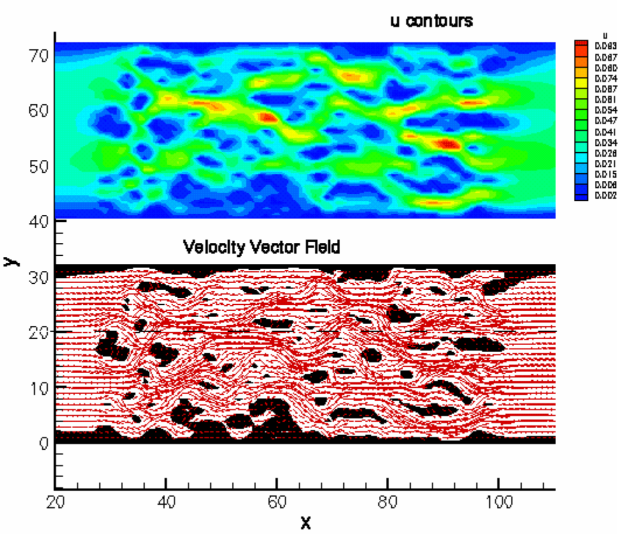

Fig. 8 Flow through 2D porous media

5. Conclusion

With two classical flow studies, in this paper is able to show that our implementation of the lattice BGK automata yields reliable results for time-dependent flows. Strouhal numbers St for two-dimensional channel flows around a square obstacle with a blockage ratio of b = 0.125 and Reynolds numbers between 80 and 300 are measured numerically. It could be shown that for a correct evaluation of the Strouhal number higher grid resolutions are necessary for higher Reynolds numbers owing to the generation of small secondary vortices below and above the obstacle, which have to resolved numerically.

In addition, concerning complex geometries, the CPU time for the LBA first decreased with increasing complexity of the obstacle structure (e.g. the flow simulation through a porous structure as presented in Fig. 8) and become almost independent from it for highly complex structure. In summary, the LBA method strengthen the often stressed opinion that this method is quite competitive tools with respect to the application of CFD especially for problems involving complex geometries such as porous media.

The next step in research in this area is to measure the performance of LBA model against experiment.

Acknowledgment

The author thanks Professor H. Ted Davis, for some helpful discussions and provided the two-dimensional plotting software. This work was supported by Department of Chemical Engineering & Materials Science University of Minnesota, Twin Cities under Post Doctoral Grant.

References

1. Frisch, U., d'Humieres, D., Hasslacher, B., Lallemand, P., Pomeau, Y. and Rivet, J.P., "Lattice Gas Hydrodynamics in Two and Three Dimension, *Complex Systems* **1**, 1987, pp. 649-707.(1987).

2. Frisch, U.; Hasslacher, B. and Pomeau, Y., " Lattice-Gas Automata for the Navier-Stokes Equation, Phys. Rev. Lett. 56, 1986, pp. 1505-1508.

3. Noble, D.R.; Georgiadis, J.G. and Buckius, R.O., " Direct Assessment of Lattice Boltzmann Hydrodynamics and Boundary Conditions for Recirculating Flows," Jour. Stat. Phys. 81, 1995 , pp. 17-33.

4. Noble, D.R.; Georgiadis, J.G. and Buckius, R.O., "Comparison of Accuracy for Lattice Boltzmann and Finite Difference Simulations of Steady Viscous Flow," Inter. Jour. Numerical Meth. Fluids 23, 1996, pp. 1-18.

5. P. Ratanadecho "The Numerical and Experiment Investigation of Heat Transport and Water Infiltration in Granular Packed Bed due to Supplying Hot Water (One- and –Two Dimensional Models)," ASCE Engineering Mechanics J., 2003 (Accepted)

6. Bernsdorf, J. and Schäfer, M., "Camparison of Cellular Automata and Finite Volume Techniques for Simulation of Incompressible Flow in Porous Media," ERCOFTAC Bulletin **28**, 1996, pp. 24-27

7. Qian, Y.H.; d'Humières, D. and Lallemand, P., " Lattice BGK Models for Navier-Stokes Equation," Europhys. Lett. 17 (6 BIS), 1992, pp. 479-484.

8. Bhatnager, P.L., Gross, E.P. and Krook, M., "A model for Collision in Gases. I. Small Amplitude Processes in Charged and Natural One-Component System," Physical Review, 94(3), 1954, pp. 411-525.