A Robust Control with Scheduling Control Input Reduction

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Abstract

Using a robust stability analysis theorem, we proposed an original control input reduction scheme for an existing robust linear control. The resulting control was nonlinear, and was applicable to linear systems with nonlinear uncertainties simultaneously appeared in both the system matrix and the input matrix. Without perturbations, our robust control guaranteed global exponential stability for the system of interest. With perturbations, it assured input-to-state stability. It appeared in our studies that 15% reduction in magnitude of control input had insignificant effects on tracking performance of the control system.

Keywords: stability, control input reduction, gain scheduling, robustness, performance

1. Introduction

Saturations of amplifiers and actuators exist for certain in their physical implementations. Because these are important components in control system, such phenomenon is a subject of interest in the literatures. Among these, founding results have emerged. It were shown in [1] and later in [2] that, in general, linear control laws are insufficient for linear systems subjected to input saturation to be globally asymptotically stable. In addition, this type of stability may be achieved by nonlinear controls if and only if the system matrix and the input matrix are stabilizable when no eigenvalue of the system matrix has positive real part [3]. An example may be found in [4] for a chain of integrators.

It was shown in [5] that linear control laws could ensure semiglobal asymptotic stability for linear systems with input saturation. A low-gain control law was proposed to ensure that the resulting control input never saturates for any bounded set of initial conditions. For linear systems with known system matrix and input matrix, this technique was extended in [6] to construct a composite low-and-high gain control law. In the same fashion as [5], the low-gain part was responsible for semiglobal asymptotic stability about the origin. The additional high-gain part was added to achieve performance, and robustness. In all these works, uncertainties in the two matrices were not addressed.

The problem of robust stability analysis (RSA), in which allowable bounds on nonlinear uncertainties in the system matrix and the input matrix are computed for stable linear control systems, has been considered in parallel with the problem of robust controller design [7-9]. Matrix algebra and geometry were employed in [10] to develop a new RSA theorem, and a technique for extending the uses of RSA theorems over robust controller design for single-input linear systems. It was shown that the resulting allowable uncertainty bounds could be less conservative than those resulting from [11]. In all these, control input saturation was not considered.

In this paper, we extend the linear controller design technique for linear systems subjected to nonlinear uncertainties in both the system matrix and the input matrix such as those in [10-11] to acquire an additional distinct feature of reducing control input. Using a state-feedback control law, we assure that the system is input-to-state stable when perturbation exists. Without perturbation, the system is globally exponentially stable. The corresponding gain matrix is constructed by combining a constant gain matrix and a state-dependent gain matrix. The former is for achieving robustness and performance in smaller neighborhoods about the origin. The latter is for reducing control input by continuously lowering the magnitude of the resulting gains to account for saturations of amplifiers and actuators in larger neighborhoods.

2. Mathematical Description

In this paper, we are interested in computing control laws that guarantee input-to-state stability for the perturbed linear systems with nonlinear uncertainties:

$$\dot{x} = [A + \Delta A(x)]x + [B + \Delta B(x)]u + f(x,t) \quad (1)$$

where $x \in \Re^n$ is the state vector, the system matrix $A \in \Re^{n\times n}$ is known, the input matrix $B \in \Re^{n\times m}$ is known, $u \in \Re^m$ is the control input vector, the bounded vanishing nonlinear time-varying perturbation vector $f(x,t) \in \Re^n$ is unknown, and Δ denotes nonlinear uncertainties with appropriate dimensions and known bounds for system matrix A and the input matrix B. The origin is an equilibrium point of the system. We impose that the control inputs vector, the perturbation vector, and all uncertainty matrices are globally Lipschitz. Now we introduce the nonlinear state-feedback control law $u(x) = -K(x)x$, where $K(x) \in \Re^{m\times n}$ is the state-dependent gain matrix to obtain. We impose that $K(x) = K_n + K_\Delta(x)$ where K_n and $K_\Delta(x)$ are the nominal gain matrix, and the scheduling gain matrix respectively. Note that $K_\Delta(x)$ will be treated as psudo-uncertainties in the following development. We denote the elements of K(x), K_n, and $K_\Delta(x)$ by k_{ij}, k_{nij} and $k_{\Delta ij}$ respectively. Using the above structure for $K(x)$, we may write the control input as $u(x) = u_n(x) + u_\Delta(x)$. These are the resultant control input, the nominal control input, and the scheduling control input respectively.

3. A Strengthened Class Gamma Theorem

Using the proposed control input, the system of interest can now be written as:

$$\dot{x} = [A + \Delta A(x)]x - [B + \Delta B(x)][K + K_\Delta(x)]x + f(x,t) \quad (2)$$

When the perturbation is removed from Eq. (2), and K_n as well as all the associated uncertainty specifications are available, the resulting system can always be written as:

$$\dot{x} = \bar{A}x + \textstyle\sum_{j=1}^{r}[h_j(x)E_j]x \quad (3)$$

where $\bar{A} \equiv A - BK_n$ is known, $E_j \in \Re^{n\times n}$ is known, and $h_j(x) \in \Re$ is a nonlinear uncertain function, for which upper bound h_{uj} and lower bound h_{lj} are known. We now present a robust stability analysis theorem that can be employed to determine if the system in Eq. (3) is globally exponentially stable.

Theorem 1 If the dynamical system in Eq. (3) is globally Lipschitz with matrix $\bar{A}$ being Hurwitz and $\max(\lambda(Z)) < 0$, then the origin is globally exponentially stable. The matrix $Z = Z^T \in \Re^{n\times n}$ is obtained by:

1) Specified Q > 0 and $\bar{A}$ to compute P from the Lyapunov equation.
2) Compute $\bar{A}_l = \bar{A} + \sum_{j=1}^{r} h_{lj}E_j$, and $\Phi = P\bar{A}_l + \bar{A}_l^T P$.
3) Compute $\Psi_j = [PE_j + E_j^T P] = \Psi_j^T$.
4) Compute $\Lambda_{\Psi_j} = T_{\Psi_j}^T \Psi_j T_{\Psi_j}$, where the matrix $T_{\Psi_j} = [\, v_{\Psi_j 1} \mid \cdots \mid v_{\Psi_j n} \,]$, $\{v_{\Psi_j 1}, \; \ldots \; , v_{\Psi_j n}\}$ is the set of n orthonormal eigenvectors of Ψ_j.
5) Set all negative elements of Λ_{Ψ_j} to zero to get $\Lambda_{\Psi_j}^{\geq 0}$
6) Compute $\Psi_j^{\geq 0} = T_{\Psi_j}\Lambda_{\Psi_j}^{\geq 0}T_{\Psi_j}^T$.
7) Compute $Z \equiv \Phi + \sum_{j=1}^{r}[(h_{uj} - h_{lj})\Psi_j^{\geq 0}]$.

Proof We write for $h_j(x)$, j = 1, 2, ..., r that $h_j(x) = h_{lj} + h_j(x) - h_{lj} \equiv h_{lj} + l_j(x)$, where $l_j(x)$ $h_j(x) - h_{lj}$. Since $h_j(x) \in [h_{lj}, h_{uj}]$, $l_j(x) \in [0, h_{uj} - h_{lj}]\forall j$. Substituting $h_{lj} + l_j(x)$ for $h_j(x)$ in Eq.(2) yields $\dot{x} = \bar{A}_l x + \sum_{j=1}^{r} l_j(x)E_j x$.

Now put Q > 0 into the Lyapunov equation $-Q = (1/2)[P\bar{A} + \bar{A}^T P]$, then solve for P, and obtain $\Phi = P\bar{A}_l + \bar{A}_l^T P$. Note that the Lyapunov function V(x)=$(1/2)x^T Px$ is such that $P = P^T > 0$ and $(1/2)\min(\lambda(P))\|x\|^2 \leq V(x) \leq (1/2)\max(\lambda(P))\|x\|^2$.

We compute the Lyapunov time derivative, which may be written as $\dot{V}(x) = (\partial V/\partial x)\dot{x}$ $= (1/2)x^T\Phi x + (1/2)\sum_{j=1}^{r} l_j(x)x^T\Psi_j x$. Note that the definitions of Φ, and Ψ_j are given previously. Since $\Psi_j^T = \Psi_j$ $\forall j$, Ψ_j has a set of n real eigenvalues $\{\lambda_{\Psi_j 1}, \; \ldots \; , \lambda_{\Psi_j n}\}$ and the corresponding set of n orthonormal eigenvectors $\{v_{\Psi_j 1}, \; \ldots \; , v_{\Psi_j n}\}$. Using the linear transformation $x = T_{\Psi_j}z$, we now write $x^T\Psi_j x = z^T[T_{\Psi_j}^T \Psi_j T_{\Psi_j}]z \equiv z^T\Lambda_{\Psi_j}z$. Notice that $\Lambda_{\Psi_j} = \mathrm{diag}[\,\lambda_{\Psi_j 1} \; \ldots \; \lambda_{\Psi_j n}\,]$.

We set all negative elements of Λ_{Ψ_j} to zeros to obtain $\Lambda_{\Psi_j}^{\geq 0}$. Because $z^T[\Lambda_{\Psi_j}^{\geq 0}]z \geq 0$, it follows that $z^T[\Lambda_{\Psi j}^{\geq 0}]z \geq z^T\Lambda_{\Psi j}z = x^T\Psi_j x$. Accordingly, $z^T[\Lambda_{\Psi_j}^{\geq 0}]z = x^T[T_{\Psi_j}^{-1}]^T[\Lambda_{\Psi_j}^{\geq 0}][T_{\Psi_j}^{-1}]x \equiv x^T\Psi_j^{\geq 0}x \geq 0$, where $\Psi_j^{\geq 0} = [T_{\Psi_j}^{-1}]^T[\Lambda_{\Psi_j}^{\geq 0}][T_{\Psi_j}^{-1}]$. Because matrix T_{Ψ_j} is orthogonal, it follows that $T_{\Psi_j}^{-1} = T_{\Psi_j}^T$, and $\Psi_j^{\geq 0} = [T_{\Psi_j}][\Lambda_{\Psi_j}^{\geq 0}][T_{\Psi_j}^T]$. Because $[\Psi_j^{\geq 0}]^T = \Psi_j^{\geq 0}$ and $(h_{uj} - h_{lj}) \geq l_j(x) > 0$, we now have that $l_j(x)[x^T\Psi_j x] \leq (h_{uj} - h_{lj})[x^T\Psi_j^{\geq 0}x]\ \forall x$.

Applying the last inequality in $\dot{V}(x)$ shows that $\dot{V}(x) \leq \frac{1}{2}x^T\Phi x + \frac{1}{2}\sum_{j=1}^{r}((h_{uj} - h_{lj})[x^T\Psi_j^{\geq 0}x])$. By letting $Z \equiv \Phi + \sum_{j=1}^{r}[(h_{uj} - h_{lj})\Psi_j^{\geq 0}]$, we have shown now that $\dot{V}(x) \leq (1/2)x^T Zx$. If $\max(\lambda(Z)) < 0$, then it follows that $\dot{V}(x) \leq (1/2)\max(\lambda(Z))\|x\|^2$. This indicates that the origin of Eq. (3) is a globally exponentially stable equilibrium [12]. The proof is now completed. ⊗

Theorem 1 gives the conditions that guarantee exponential stability of the origin of the unperturbed system in Eq. (3). Although not shown explicitly, this type of stability implies exponential rate of convergence for all trajectories [12], which should provide satisfactory level of performance in many applications. With the existence of perturbation vector $f(x,t) \in \Re^n$ in the system of interest, we can arrive at a weaker type of stability as shown in the following Corollary 1.

Corollary 1 If Theorem 1 is satisfied, then the origin of the system in Eq. (2) is input-to-state stable.

Proof Following the proof of Theorem 1, we have along the trajectory of Eq. (2) that $\dot{V}(x,t) = (1/2)x^T\Phi x + (1/2)\sum_{j=1}^{r} l_j(x)x^T\Psi_j x + (\partial V/\partial x)f(x,t)$. Then, $\dot{V}(x,t) \leq (1/2)x^T Zx + (\partial V/\partial x)f(x,t)$. Now, let $f(x,t)$ be bounded by $\phi \in \Re^+$ and notice that $\partial V/\partial x = x^T P$. Because $P = P^T > 0$, it follows that $\dot{V}(x,t) \leq (1/2)x^T Zx + \max(\lambda(P))\phi\|x\|$. If given that $\max(\lambda(Z)) < 0$ as in Theorem 1, then $\dot{V}(x,t) \leq (1/2)\max(\lambda(Z))\|x\|^2 + \max(\lambda(P))\phi\|x\|$. For any given bound ϕ on $f(x,t)$, the last inequality implies the existence of the corresponding region $\Omega \in \Re^n$ in which $\|x\| > -2\phi\max(\lambda(P))/\max(\lambda(Z))$ and $\dot{V}(x,t) < 0$ in $\Omega\ \forall t$. With $P = P^T > 0$, this implies that the trajectory is finally contained in $\Re^n - \Omega$ for all initial conditions. Input-to-state stability is asserted, and the proof is completed. ⊗

Theorem 1 and Corollary 1 are central to our controller design technique in the sense that they can be employed to confirm stability of a robust linear control when the control input is reduced. To do this, however, we require not only a stabilizing gain matrix K_n, but also an appropriate matrix Q.

4. Control Law

In addition to being theoretically applicable to the dynamic system of interest in Eq. (2), the robust linear controller design technique in [10] yields both K_n and Q simultaneously as a stabilizing solution pair. This is a distinctively desirable property for us because these two matrices are required in Theorem 1 to show global exponential stability. Accordingly, we decide to extend this robust control technique by the use of Theorem 1 to acquire an original capability of reducing control input for it. It turns out that the resulting robust control is nonlinear. It guarantees input-to-state stability for the perturbed system in Eq. (2), and exponential stability for the unperturbed system in Eq. (3), with reduced magnitude of control input when compared to that resulting from [10]. Our technique is composed of two portions. The former is to find K_n - the nominal linear gain matrix for stabilization, and the latter is to find $K_\Delta(x)$ - the scheduling nonlinear gain matrix for control input reduction. These are given in the followings.

4.1 Computing K_n

For convenience, we now take from [10] the following procedure:

1) Define a two dimensional domain of $\rho > 0$ and $\eta \geq 1$, and select a grid size for this domain.
2) Select coordinate (ρ, η), then complete step 2 – 5.
3) solve for P from $-2I = PA + A^T P - 2\rho PBB^T P$.
4) Compute K_n from $K_n = \eta\rho B^T P$.
5) Compute Q from $Q = I + (\eta - 1)N$.

Now, for each pair of (K_n, Q) obtained from the above procedure, compute $\max(\lambda(Z))$ in Theorem 1 using the available uncertainty specifications on h_{lj}, h_{uj} and $E_j\ \forall j$. The procedure terminates successfully when Theorem 1 is satisfied by a stabilizing pair of (K_n, Q). In this case, we have the nominal gain matrix K_n, Q, and the corresponding allowable bounds on all nonlinear uncertainties in A and B. These uncertainties do not include allowable bounds on the nonlinear scheduling gain matrix $K_\Delta(x)$, which will be discussed in the next section.

4.2 Computing K_Δ (x)

While preserving the required type of stability resulting from K_n determined previously, we now employ the scheduling control input $u_\Delta(x) = -K_\Delta(x)x$ to reduce the magnitude of the resultant control input $u(x) = -K(x)x$, that would otherwise equals to the nominal control input $u_n(x) = -K_n x$. We know by continuity of the maximum eigenvalue of matrix Z in Theorem 1 that this is always possible. Now, let the (i, j) element of $K_\Delta(x)$ and the corresponding upper bound be denoted by, $k_{\Delta ij}(x) \in \Re$ and $h_{\Delta ij} \in \Re^+$ respectively. We treat $k_{\Delta ij}$ as psudo-uncertainties that enter the system of interest via u(x). Without loss of generality, we arrange the psudo-uncertainty $k_{\Delta ij}$ such that the lower bound is zero for convenience in later development.

Note that it is not necessary to use all $k_{\Delta ij}$ for control input reduction. Experience with the system of interest under the nominal control is helpful to identify that using a certain $k_{\Delta ij}$ may be more effective than the others. The same goes for setting positive values for $h_{\Delta ij}$, later which we will see that it indicates the degree of control input reduction. With all the selected $k_{\Delta ij}$ and the corresponding $h_{\Delta ij}$ available, we now have an additional set of psudo-uncertainty specifications. We then write the system with all uncertainty specifications as in Eq. (3), and employed Theorem 1 again to determine if the required control input reduction does not destabilize the system. Adjustments on $k_{\Delta ij}$ and $h_{\Delta ij}$ may be required for Theorem 1 to be satisfied.

Now that the allowable bound $h_{\Delta ij}$ on an elected scheduling gain $k_{\Delta ij}$ is found, we propose the following function for $k_{\Delta ij}$:

$$k_{\Delta ij} = -\mathrm{sign}(k_{nij}) h_{\Delta ij} \left(1 + 0.5 \begin{pmatrix} \tanh(\beta_{ij} u_{ni} - \gamma_{ij}) \\ -\ \tanh(\beta_{ij} u_{ni} + \gamma_{ij}) \end{pmatrix} \right) \quad (4)$$

where k_{nij} is the (i, j) element of the nominal gain matrix K_n, $\beta_{ij} > 0$ and $\gamma_{ij} > 0$ are real parameters. Note also that u_{ni} denotes the i-th component of the nominal control input vector u_n. Similarly, we use $u_{\Delta i}$ and u_i to denote the i-th component of u_Δ and u respectively.

We propose the function in the RHS of Eq. (4) primarily because it has desirable distribution that can be preserved over large domains. This will be discussed in the followings. The function is continuous and is approximately zero in the neighborhood about the origin of u_{ni}. The size of this neighborhood is primarily determined by the parameter β_{ij}. As $|u_{ni}|$ increases, $|k_{\Delta ij}|$ increases accordingly at a rate primarily determined by β_{ij}, and is finally bounded by $h_{\Delta ij}$. To see how $k_{\Delta ij}$ may be employed to reduce the resulting control input, consider Fig. 1, in which $k_{\Delta ij}$ and k_{ij} is plotted versus u_{ni} when $k_{nij} = -2$, $h_{\Delta ij} = 1$, $\beta_{ij} = 1$, and $\alpha_{ij} = 4$. Notice that the magnitude of $k_{ij} = k_{nij} + k_{\Delta ij}$ decreases as u_{ni} increases. Accordingly, the magnitude of u_i may be reduced by introducing $u_{\Delta i}$ appropriately.

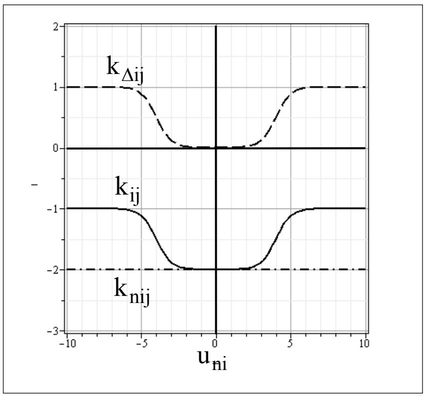

Fig. 1 Typical distribution of $k_{\Delta ij}$ and its effects on k_{ij}

If a nominal operating region of the system is known, then an upper bound $\bar{u}_{ni}$ on magnitude of nominal control input may be obtained accordingly. In this case, it can be shown that we may achieve similar distribution of $k_{\Delta ij}$ over u_{ni} as in Fig. 1 by choosing parameter $\beta_{ij} = 10/\bar{u}_{ni}$, and fixing parameter $\alpha_{ij} = 4$.

5. Example

A 5-joint modified SCARA (Selective Compliant Articulated Robot Arm) with independent joint control is to move a tool along a fixed trajectory. During normal operations, the magnitude of control input for the revolute joint 1 is tightly bounded by 100 V. Because the amplifier is near saturation at 110 V, we desire to lower the control input without affecting robot performance.

The control system for the DC joint motor is subjected to nonlinear uncertainties and perturbations resulting from robot operations. Now let e(t) be joint angle tracking error, and define the relevant state variables $\int_0^t e\,dt = x_1$, $e = x_2$, and $\dot{e} = x_3$. Dynamics of the control system may be represented by Eq. (5).

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & -(3.8+h_1(x))\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -(89.3+h_2(x))\end{bmatrix}u + \begin{bmatrix}0\\ 0\\ f(x,\bullet)\end{bmatrix} \quad (5)$$

The nonlinear functions $h_1(x) \in [0,\ 0.86]$, and $h_2(x) \in [0,\ \ 20.52]$ result from uncertainties in robot parameters and operations. The smooth function $f(x,\bullet)\ \in [-1500,\ 1500]$ in the perturbation vector results from joint interactions and changes in tracking signal. Because of their length and complexity, we do not show these functions here. However, the exact expressions for these may be obtained from [13].

Recall that our control law is given by $u(x) = -[K_n + K_\Delta(x)]x$ with $K_n = [k_{n1}\ \ k_{n2}\ \ k_{n3}]$, and K_Δ = $[k_{\Delta 1}(x)\ \ k_{\Delta 2}(x)\ \ k_{\Delta 3}(x)]$ in this example. For simplicity, we elect that $k_{\Delta 1}(x) = 0$, and $k_{\Delta 3}(x) = 0$. Following the procedure given in Section 4.1, we find that the pair $(\rho, \eta) = (0.5,\ 4)$ yields the nominal control $u_n(x) = -K_n x$ with $K_n = [-2.83\ \ -4.92\ \ -2.78]$ and

$$Q = \begin{bmatrix}4.0000 & 5.2206 & 2.9519\\ 5.2206 & 10.0848 & 5.1369\\ 2.9519 & 5.1369 & 3.9046\end{bmatrix}.$$

The above pair (K_n, Q) is a stabilizing solution because it yields $\max(\lambda(Z)) = -0.5335$ in Theorem 1. Many other stabilizing solutions exist, but we do not pursue them because the above already serves sufficiently as an example.

Because $\max(\lambda(Z)) < 0$, we know by continuity that the control system can tolerate some control input reduction. A little matrix manipulation reveals that our specified control input reduction can be written as additional uncertainty in element (3, 2) of the system matrix A. The upper and lower bounds for this element is 1.5(89.3 + 20.52), and zero respectively. For demonstration purpose, we arbitrarily pick $h_{\Delta 2} = 1.5$ and reapply Theorem 1. This additional uncertainty yields $\max(\lambda(Z)) = \ -0.3722$. Now, we have for control input reduction the scheduling nonlinear gain:

$$k_{\Delta 2} = 1.5(1 + 0.5(\tanh(0.1u_n - 4) - \tanh(0.1u_n + 4))),$$

and we have confirmed by using Theorem 1 that it does not destabilized the system. Clearly, we can increase the reduction bound $h_{\Delta 2}$ further to acquire more control input reduction before Theorem 1 is violated, although the number already serves sufficiently as an example.

Letting the tracking reference signal for the revolute joint be $r(t) = 390(t + 0.2\sin(4t))$ rad, we run numerical simulations for two cases. In the first case, no control input reduction is employed and $u(x) = u_n(x)$. In the other, we employed control input reduction and $u(x) = u_n(x) + u_\Delta(x)$. The simulation results in Fig. 2 show that the state is bounded in all cases. Tracking errors in both cases are almost the same, while the bound on magnitude of $u(x)$ in the second case is only 85% of that in the first case.

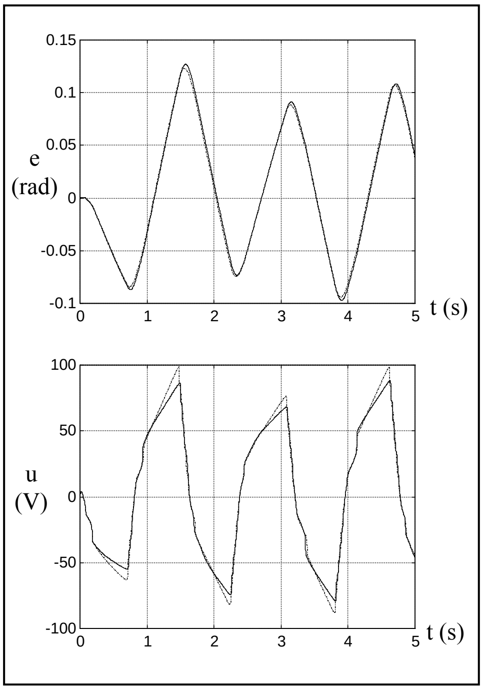

Fig. 2 Tracking error (upper) and control input (lower) Solid: with control input reduction, Dash: without

6. Conclusion

We are interested in linear systems subjected nonlinear uncertainties that enter the system matrix and the input matrix simulataneously. Also, nonlinear time-varying perturbation may exist. We propose a robust stability analysis theorem, which becomes a portion of our original nonlinear controller design technique. Our technique can generate a nonlinear control law that offers a guaranteed level of performance through global exponential stabilization of the unperturbed system. The resulting control law simultaneously guarantees that, when the perturbation is bounded, the state of the perturbed system is also bounded. Finally, our control law can reduce the magnitude of control input from the nominal value in large regions about the equilibrium point at the origin. It turns out in studies that 15% reduction in magnitude of control input has insignificant effects on tracking performance of the control system.

7. References

[1] Esfandiari, F. and Khalil, H. K. (1969), In-the-large stability of relay and saturating control systems with linear controller, *Int. J. Control*, Vol. 10, No. 4, , pp. 457-480.

[2] Sussmann, H. J. and Yang, Y. (1991), On the stabilization of multiple integrators by means of bounded feedback control, in *Proc. 30th IEEE CDC*, pp. 70-72.

[3] Sontag, E. D. and Sussmann, H. J. (1990), Nonlinear output feedback design for linear systems with saturating controls, in *Proc. 29th IEEE CDC*, pp. 3414-3416.

[4] Teel, A. R. (1992), Global stabilization and restricted tracking for multiple integrators with bounded control, *System & Control Letters*, Vol. 18, No. 3, pp.165-171.

[5] Lin, Z. and Saberi, A. (1993), Semi-global exponential stabilization of linear systems subjected to input saturation via linear feedbacks, *System & Control Letters*, Vol. 21, No. 3, pp. 225-239.

[6] Saberi, A., Lin, Z., and Teel, A. R. (1996), Control of linear systems with saturating actuators, *IEEE Trans. Automatic Control*, Vol. 41, No. 3, pp. 368-378.

[7] Vedavalli, R. K., (1985), Perturbation bounds for robust stability in linear state space model", *Int. J. of Cont.*, Vol. 42, pp. 1507-1517.

[8] Zhou, K., and Khargonekar, P. P., (1987), Stability robustness bounds for linear state-space models with structured uncertainty, *IEEE Trans. on Automatic Control*, Vol. 32, pp. 621-623.

[9] Olas, A., and Ahmadkhanlou, F., (1994), Globally optimal quadratic Lyapunov functions for robust stability of systems with structured uncertainty, *Dynamics and Control*, Vol. 4, pp. 5-20.

[10] Ngamsom, P. and Hoberock, L. L., (2003), Using robust stability analysis theorems for robust controller design, *ASME Journal Of Dynamic Systems, Measurement, and Control*, Vol. 125, No. 4, pp. 669-671.

[11] Khargonekar, P. P. et al, (1990), Robust Stabilization of uncertain linear systems: Quadratic stabilizability and H^{∞} control theory, *IEEE Trans. on Automatic Control*, Vol. 35, pp. 356-361.

[12] Khalil, H. K. (2000), Nonlinear Systems 3rd Edition, Prentice Hall.

[13] Ngamsom, P. et al (2004). Design of Low-Cost Robot Arm for Paint Spraying, MTEC research report no. MT-S-47-MAC-58-156-G.